Sets & Steps
Math Practice & Worked Solutions

Precalculus

Sequences, Exponentials, Inverses & Logs | 25-Topic Precalculus Bundle

25-topic bundle | 2,400 problems | Four activity formats

Included topics

The following pages list all 25 included resources in the suggested order, with a link to each product. Each resource retains its complete preview.

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

200 PDFs and 25 READMEs; 1,904 buyer PDF pages across the 25 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Precalculus

Included topics

25-topic bundle | 2,400 problems | Four activity formats

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25 included resources | Complete resource previews follow this guide.

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23 **Solving Logarithmic Equations**

View individual product and its full preview

24 **Exponential and Logarithmic Inequalities**

View individual product and its full preview

25 **Linearizing Exponential Data**

View individual product and its full preview

25 included resources | Complete resource previews follow this guide.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Arithmetic vs Geometric Sequences

Arithmetic vs Geometric Sequences | APPC-U02-P01

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for terms, safe explicit form, and exponential limitation, explicit rule, signs, and real-domain limitation, all ratios and sequences, possible completions, two valid completions and conclusion and classification with failure evidence.

8 PDFs and a README; 51 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Arithmetic vs Geometric Sequences

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Classify the consecutive terms $1, 4, 9, 16$ as arithmetic, geometric, or neither. For the answer bank, record the classification. Complete and justify all parts of the original task in your work. Bank label: _____	Determine the sequence type for $-4, -9, -14, -19$ and state its step change. For the answer bank, record the common difference. Complete and justify all parts of the original task in your work. Bank label: _____
Translate $a_0=12$ and $a_n=a_{n-1}-7$ into a direct rule for whole-number n. For the answer bank, record the expression for a_n. Complete and justify all parts of the original task in your work. Bank label: _____	Convert $b_n=125(0.2^{n+1}), n = 0, 1, 2,...$, to a recurrence with the correct zeroth term. For the answer bank, record the zeroth term b_0. Complete and justify all parts of the original task in your work. Bank label: _____
Classify $7, 14, 28, 56$ and state the common change. For the answer bank, record the common ratio. Complete and justify all parts of the original task in your work. Bank label: _____	Write an explicit formula for $b_1=100$ and $b_n=b_{n-1}-15$ for $n \geq 2$. For the answer bank, record the expression for b_n. Complete and justify all parts of the original task in your work. Bank label: _____
For $a_n=\frac{7-2n}{5}$ at whole-number $n \geq 0$, find a_0 and an exact recurrence. For the answer bank, record the zeroth term a_0. Complete and justify all parts of the original task in your work. Bank label: _____	An arithmetic sequence has $a_{10}=1$ and decreases by 2 each step. Write a_n from $n = 1$ onward and find a_1. For the answer bank, record the first term a_1. Complete and justify all parts of the original task in your work. Bank label: _____

AP Precalculus

Arithmetic vs Geometric Sequences

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For indices 1 through 4, the terms are $11, 11, 11, 11$. Classify every arithmetic or geometric description that applies and state its invariant.	The terms at $n = 0, 1, 2, 3$ are all 0. If a geometric classification requires defined consecutive ratios, classify the sequence and explain the zero case.
For a_n=$5 - 3(n - 1), n = 1, 2,...$, give the necessary initial term and a recurrence.	An account begins with \$120, and each completed deposit adds \$25. Let n count completed deposits. Write the balance sequence and list its first three graph points.
Complete the arithmetic sequence a_1=4, a_2=?, a_3=16, a_4=22, and justify the missing term.	Convert g_1=48 and g_n=$(\frac{1}{2})g_{n-1}$ for $n \geq 2$ into an explicit sequence rule.

AP Precalculus

Arithmetic vs Geometric Sequences

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Given $a_0=7$ and $a_n=a_{n-1}+4$ for $n \geq 1$, write a direct rule for whole-number n.

Your response: _____

Incoming: d = 5 and $a_2=-1$.

An arithmetic sequence has $a_3=14$ and $a_8=34$. Find its common difference and a_1.

Context/domain: sequence

3	14
8	34

Your response: _____

Incoming: r = 3 and $a_1=-9$.

A geometric sequence with positive ratio satisfies $a_2=12$ and $a_5=324$. Recover r and a_1.

Your response: _____

Incoming: d = −3 and $a_1=31$.

An arithmetic sequence satisfies $a_6=-5$ and $a_{10}=19$. Determine d and extrapolate backward to a_1.

Your response: _____

Incoming: r = −3, uniquely.

For an arithmetic sequence indexed from 0, $a_2=5$ and $a_7=-10$. Find the common difference and a_0.

Context/domain: sequence

2	5
7	−10

Your response: _____

Incoming: Geometric with ratio 0.8.

A positive-ratio geometric sequence has $a_4=54$ and $a_7=2$. Recover the common ratio and a_1.

Your response: _____

AP Precalculus

Arithmetic vs Geometric Sequences

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The exact terms are $2, 10, 50, 245$. A student calls them geometric with ratio 5. Evaluate the claim and state the fourth term that would make it true. Student work/claim: The first two ratios equal 5, so all four given terms form a geometric sequence. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A student labels $10, 7, 4, 1, -3$ arithmetic with $d = -3$. Decide whether the complete exact list supports the claim and repair the last term if needed. Student work/claim: Every difference in the given list is -3. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Can the blank in 4, ?, $14, 19, 25$ be filled so the entire exact list is arithmetic? Explain which evidence is decisive. Student work/claim: Filling the blank with 9 makes the entire list arithmetic. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Arithmetic vs Geometric Sequences

Full Worked Solutions - excerpt

Worked example

Problem. Given $a_0 = 7$ and $a_n = a_{n-1} + 4$ for $n \geq 1$, write a direct rule for whole-number n.

Answer. $a_n = 7 + 4n$ for $n = 0, 1, 2, \ldots$

Explanation and check.

From index 0 to index n there are n additions of 4, so the term is $7 + 4n$.

Worked example

Problem. Starting at $a_3 = 81$, each later term is one third of the previous term. Write the explicit rule for $n \geq 3$.

Answer. $a_n = 81\left(\frac{1}{3}\right)^{n-3}$.

Explanation and check.

The exponent is zero at the first allowed index and counts the subsequent divisions by 3.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sequences to Linear and Exponential Models

Sequences to Linear and Exponential Models | APPC-U02-P02

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for differences, ratios, interpretation, domain judgment, domain distinction, two compatible models, existence and reason and class and amount.

8 PDFs and a README; 62 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Sequences to Linear and Exponential Models

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Extend $a(n) = -4 + 1.5n$ from whole-number indices to all real x and evaluate $f(-2)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	An account begins with \$140 and receives exactly \$25 each week with no interest. Is the weekly pattern linear or exponential, and what is the balance after one week? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
A sequence starts at 18 and increases by 6 per index. If the constant rate is extended between indices, what is its value at index 2.5? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The starting value is the value at index 0. Bank label: _____	A material starts at $250g$ and loses 12% of current mass per hour. Give its factor and model. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____

AP Precalculus

Sequences to Linear and Exponential Models

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

The sequence rule is $a(n) = 3 + 2n$ for whole-number n at least 0. Extend the same linear rule to real x at least 0, then find $f(2.5)$.

Exact samples at $x = 0, 1, 2, 3$ are $5, 8, 11, 14$. Which model class is consistent, and what does the finite table not prove?

Context/domain: real-input data

0	5
1	8
2	11
3	14

A balance starts at \$200 and rises each hour by 10% of the initial \$200, not its current value. Construct a real-time rule and classify it.

For exact points $(0, 1)$, $(2, 7)$, and $(5, 16)$, compare change per input unit rather than raw differences. Which class is supported?

Context/domain: real-input data

0	1
2	7
5	16

$L(x) = 4 + 16x$ and $E(x) = 4(3^x)$ share $(0, 4)$ and $(2, 36)$. Compare their values at $x = 1$.

Context/domain: matched models

0	4
2	36

A linear table gives $10, 15, 20$ and an exponential table gives $10, 15, 22.5$ at $x = 0, 1, 2$. After the shared +5, what differs?

AP Precalculus

Sequences to Linear and Exponential Models

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Production starts at 12 units and adds 6 units per minute. Write the model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: Both give factor 0.8: $\sqrt{0.64} = 0.8$ and cube root of $0.512 = 0.8$.

A sequence has value -2 at index 0 and adds 5 at each next whole-number index. Extend its constant-rate rule to real x and evaluate it at $x = -1.5$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: L keeps +5 while ratios fall; E keeps ratio 2 while increments grow $5, 10, 20$.

At consecutive inputs $-2, -1, 0, 1$, the exact outputs are $-7, -2, 3, 8$. Classify the data from their change pattern. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context/domain: integer samples

-2	-7
-1	-2
0	3
1	8

Your response: _____

Incoming: Linear: 105. Exponential: 15.

A population starts at 90 and grows 7% of current size per year. Write the model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

AP Precalculus

Sequences to Linear and Exponential Models

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A display begins with 46 items and staff add exactly 7 items each day. Classify the model and find the count after four days. Student work: exponential; 74 items. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use $L(x) = 12 + 6x$ and $E(x) = 12(2^x)$ to refute "same starting value means same change." Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

Pallet count is $p(n) = 40 + 6n$ after n completed trucks. Does $p(2.5) = 55$ describe a sequence term in this context? Student work: No. The algebraic line gives 55, but 2.0 completed trucks is outside the stated event domain. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

An amount starts at $400mg$ and loses 5% of its current amount each hour. Classify the change and find the amount after two hours. Student work: linear decay; $361mg$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Sequences to Linear and Exponential Models

Full Worked Solutions - excerpt

Worked example

Problem. A tank starts with $500L$ and drains at a fixed $30L$ per day. Write the model and name its change type.

Answer. $V(t) = 500 - 30t$, a linear decrease while $V(t) \geq 0$.

Explanation and check.

A fixed amount per time unit gives slope -30 rather than a percentage.

Worked example

Problem. An investment starts at $\$600$ and grows 5% of its current balance per year. State its factor and continuous-time rule.

Answer. The factor is 1.05, so $B(t) = 600(1.05^t)$.

Explanation and check.

Current-value growth preserves $a1.05$ multiplier while dollar increments change.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Parameters and Reference Points

Exponential Parameters and Reference Points | APPC-U02-P03

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for adjusted ratios and limitation, adjusted sequences and conclusion, actual constant, base sign and consequence, degenerate class and failed condition and domain failures.

8 PDFs and a README; 57 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exponential Parameters and Reference Points

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Classify $h(x) = 4(1^{x-7}) - 2$ and give its actual value for every x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	Does $f(x) = (\frac{1}{2})(\frac{5}{4})^x$ define a real nonconstant exponential on all real inputs? Explain. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
Is $f(x) = -8(6^x)$ a real nonconstant exponential on all real x? Explain. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	For $N(t) = 40(0.9^{t-6}) + 12$, what happens at day 6, and what does 40 measure there? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
Translate the base 0.875 in $q(x) = 90(0.875^x)$ into a percent change per unit x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	For which real p is $g(x) = 7(5 - 2p)^x$ a real nonconstant exponential on all real inputs? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____

AP Precalculus

Exponential Parameters and Reference Points

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For which real m does $f(x) = 4(m+2)^x$ define a real nonconstant exponential for every real x?	For $f(x) = 7(2^x) + 5$, what ratio stays constant when x increases by 1?
A student says $P(t) = 800(1.04^t)$ grows by 1.04% per year. Correct the statement.	Does $g(x) = 2(-3)^x$ define a real exponential for every real x? Give a decisive input.
For $f(x) = 6(2^{x-3})$, identify the input where the coefficient 6 appears as an output, then find $f(0)$.	Interpret the base 0.73 in $A(t) = 320(0.73^t)$ as a percent change per unit time.

AP Precalculus

Exponential Parameters and Reference Points

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $f(x) = 9(2^x) + 4$, find $f(0)$ and interpret the coefficient 9 relative to the baseline. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $f(0) = 13$; the coefficient 9 is the displacement $13 - 4$ from baseline 4.

For $g(x) = -11(3^x)$, find $g(0)$ and interpret the coefficient. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $g(0) = -11$; the coefficient is the zero-input output.

Is $q(x) = 7((2 - \sqrt{3})^x)$ a valid real nonconstant exponential for all real x? Justify the base bounds. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: Yes. Since $1 < \sqrt{3} < 2$, the base $2 - \sqrt{3}$ lies strictly between 0 and 1 and is not 1.

For $f(x) = 4(3^x) - 10$, state the constant ratio involving displacement from the baseline. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $\frac{f(x+1)+10}{f(x)+10} = 3$.

For $D(t) = 22(0.75^{t-3}) - 5$, find $D(3)$ and express $D(0)$. What does 22 represent? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $D(3) = 17$ and $D(0) = 22(\frac{4}{3})^3 - 5$; 22 is displacement above baseline -5 at day 3.

Compare $f(x) = 12(0.6^x)$ and $g(x) = 6(0.6^x)$. How does the coefficient change affect every output? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

AP Precalculus

Exponential Parameters and Reference Points

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Compare $f(x) = 8(1.2^{x-2}) + 5$ and $g(x) = 8(0.7^{x-2}) + 5$. Give their shared reference point and describe their directions to its right. Student work: Both pass through $(3, 13)$. To the right, f's displacement grows while g's displacement decays. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

A table of integer inputs makes $u(x) = 5(-2)^x$ look defined. Why is it not a real exponential on all real inputs? Student work: Fractional inputs such as $x = \frac{1}{2}$ produce nonreal values, so integer evidence does satisfy the all-real domain requirement. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Interpret factors 1.12 and 0.12 as percent changes of a positive quantity. Distinguish percent of the prior value from percent increase or decrease. Student work: Factor 1.12 means 112% remains and $a12\%$ increase. Factor 0.12 means 12% remains and an 87% decrease. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

For $f(x) = 3(4^x) - 2$, compare raw output ratios and baseline-adjusted ratios at $x = 0, 1, 2$. Student work: Outputs $1, 10, 46$ have raw ratios 10 and 4.6. Adding 2 gives displacements $3, 12, 47$ with ratio 4. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Exponential Parameters and Reference Points

Full Worked Solutions - excerpt

Worked example

Problem. For $h(x) = 5(4^{x-1}) - 3$, compute $h(1)$ and $h(0)$. Which quantity is represented directly by the coefficient 5?

Answer. $h(1) = 2$ and $h(0) = -\frac{7}{4}$. The coefficient 5 is the displacement $h(1) - (-3)$, not either displayed output by itself.

Explanation and check.

At $x = 1$ the factor is 1, then the baseline -3 must still be added.

Worked example

Problem. For which real c is $h(x) = (c - 5)(2^x)$ a nonconstant exponential?

Answer. All real c except c = 5.

Explanation and check.

The base is already valid; only the coefficient must be nonzero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Features and Concavity

Exponential Features and Concavity | APPC-U02-P04

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for monotonicity and concavity, monotonicity, concavity, rate trend, monotonicity, concavity, slope trend, concavity and differences, differences and concavity and differences, direction, concavity.

8 PDFs and a README; 99 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential Features and Concavity

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

State the range and horizontal asymptote of $f(x) = -3(2^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	Find the absolute minimum and maximum of $f(x) = 4(\frac{1}{2})^x$ on $[1, 3]$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
Is $f(x) = 6(1.2^x) + 3$ increasing or decreasing? Describe the exponential displacement from 3. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	State the range of $g(x) = 8(\frac{1}{3})^x$ and its limit as $x \to \infty$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
Compare $f(x) = 3(1.4^x)$ and $g(x) = 12(1.4^x)$ for all real x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	A nonconstant $f(x) = a(2^{cx}) + k$ stays below $y = k$ and decreases. What signs must a and c have? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____

AP Precalculus

Exponential Features and Concavity

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A known exponential has exact outputs $3, 6, 12, 24$ at $x = 0, 1, 2, 3$. Use first differences to describe its concavity.

Precision: Given values are exact

x	f(x)
0	3
1	6
2	12
3	24

AP Precalculus

Exponential Features and Concavity

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Is $h(x) = -10(0.8^x) - 2$ increasing or decreasing, and how does it approach its baseline? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: It increases toward -2 from below.

On $[2, \infty)$, determine the minimum and whether a maximum exists for $f(x) = 3(2^x) - 5$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: Minimum 7 at $x = 2$; no maximum.

Find the range and both tail limits of $g(x) = 10 - 7(0.2^x)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: Range $(-\infty, 10)$; $g(x) \to -\infty$ as $x \to -\infty$ and $g(x) \to 10$ from below as $x \to \infty$.

Describe the monotonicity and concavity of $f(x) = 5(1.5^x) - 20$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: It is increasing and concave up.

Find the effective one-unit factor and monotonicity of $f(x) = 10(2^{-3x}) + 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: The effective factor is $\frac{1}{8}$, so f decreases toward 1 from above.

Find the absolute extrema of $f(x) = 5 - 2(\frac{1}{2})^x$ on $[-1, 2]$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

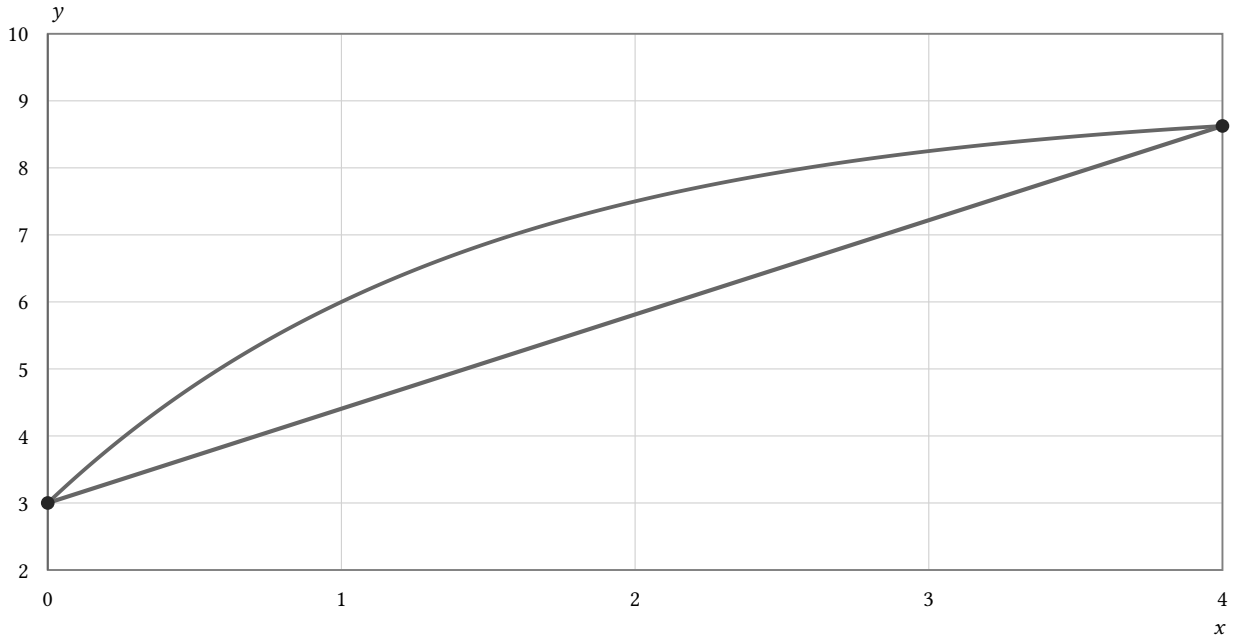
AP Precalculus

Exponential Features and Concavity

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $f(x) = 9 - 6(0.5^x)$, a chord joins $x = 0$ and $x = 4$. Does the chord overestimate or underestimate interior values? Explain. Student work: It underestimates them because f is concave up and lies above its chord in the interior. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.



Exponential Features and Concavity

Full Worked Solutions - excerpt

Worked example

Problem. A calculator rounds $f(12)$ to 0 for $f(x) = 0.1^x$. Explain why this does not mean the graph reaches its asymptote.

Answer. $f(12) = 10^{-12} > 0$. The value may round to 0, but no finite input makes a positive-base exponential equal 0.

Explanation and check.

Display precision is not exact attainment.

Worked example

Problem. Find where $f(x) = 2^x + 5$ and $g(x) = 3(2^x) - 1$ intersect, then state which is larger on each side.

Answer. Equality gives $2^x = 3$, so $x = \log$ base 2 of 3. Before that $f > g$; after that $g > f$.

Explanation and check.

Different baselines and amplitudes create one order reversal.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Rewriting Exponential Expressions

Rewriting Exponential Expressions | APPC-U02-P05

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for general proof and counterargument, normalized formulas and graph distinction, vertical multiplier and interpretation, exact multiplier, multiplier and interpretation and decision and normalized comparison.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Rewriting Exponential Expressions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Rewrite $3(2^{x+2})$ in the form $A(B^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	Write 2^{-x} with x as a positive exponent. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
Put $2(5^{x-1})$ in the form $A(B^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	Write 10^{-x} using a positive exponent x and a base below 1. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
Rewrite $7(3^{x+1})$ as $A(B^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	Express (cube root of 125)^x without a radical. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
For $f(x) = 3^x$, express $f(x+2)$ as a constant multiple of $f(x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	Normalize $5(3^{2x-1})$ into $A(B^x)$, with an exact coefficient. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____

AP Precalculus

Rewriting Exponential Expressions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A student writes $3^{x+2} = 3^x + 9$. Identify the failed exponent law and give the correct rewrite.	Show that shifting $f(x) = 2^x$ right 3 units produces the same graph as a vertical scaling. State the scale factor.

AP Precalculus

Rewriting Exponential Expressions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Normalize $\left(\frac{1}{7}\right)^{2-x}$ as $A(B^x)$ with $B > 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $(\frac{1}{49})(7^x)$

Convert $256^{3\frac{x}{8}}$ to a single exponential with an integer base. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: 8^x

Express $f(x+\frac{1}{3})$ as a multiple of $f(x)$ when $f(x) = 8^x$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $f(x+\frac{1}{3}) = 2f(x)$.

Write $(\frac{1}{2})(81^{\frac{x}{4}+\frac{1}{2}})$ as $A(B^x)$ with B an integer. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $(\frac{9}{2})(3^x)$

Find a so that $a(2^{x+1}) = 14(2^x)$ for all real x. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: a = 7

Express 3^{2-x} as $A(B^x)$ with $0 < B < 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

AP Precalculus

Rewriting Exponential Expressions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Find h so that $2^{x+h} = 8(2^x)$ for every real x. Student work: $h = 4$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

The table shows $f(x) = 4^x + 2$ and $g(x) = 2^x + 2$ agree at $x = 0$. Are they identical? Use the normalized bases to explain. Student work: No. They share the translated anchor at $x = 0$, but bases 5 and 2 give different values for other inputs. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Precision: Given values are exact

x	f(x)	g(x)
-1	2.25	2.5
0	3	3
1	6	4

Rewriting Exponential Expressions

Full Worked Solutions - excerpt

Worked example

Problem. A student writes $3^{x+2} = 3^x + 9$. Identify the failed exponent law and give the correct rewrite.

Answer. Exponents add when same-base powers are multiplied, not added. The correct identity is $3^{x+2} = 9(3^x)$.

Explanation and check.
Use $3^{x+2} = 3^x \cdot 3^2$.

Worked example

Problem. Show that shifting $f(x) = 2^x$ right 3 units produces the same graph as a vertical scaling. State the scale factor.

Answer. $f(x-3) = 2^{x-3} = (\frac{1}{8})2^x$, so the right shift equals multiplying outputs by $\frac{1}{8}$.

Explanation and check.
The constant exponent -3 factors out as 2^{-3}.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Compound Growth and Decay Rates

Compound Growth and Decay Rates | APPC-U02-P06

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for three-year factors and ranking, one-year and five-year comparisons, model and exact factor, model and remaining factor, model and target factor and hourly and daily factors with percent interpretation.

8 PDFs and a README; 52 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Compound Growth and Decay Rates

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An exponential process multiplies by 2.5 over six months. Determine its monthly factor and approximate monthly percent increase. For the answer bank, record the exact monthly factor. Complete and justify all parts of the original task in your work. Bank label: _____	A quantity retains 96% every five days. Find its ten-day factor and cumulative percent decrease. For the answer bank, record the ten-day factor. Complete and justify all parts of the original task in your work. Bank label: _____
An index grows by factor 1.03 each quarter. Find its 18-month factor and total percent increase. For the answer bank, record the exact 18-month factor. Complete and justify all parts of the original task in your work. Bank label: _____	An exponential process has factor 9 over two days. Find the positive daily factor and daily percent increase. For the answer bank, record the positive daily factor. Complete and justify all parts of the original task in your work. Bank label: _____
A quantity retains 80% each week. What factor and total percent decrease apply after two weeks? For the answer bank, record the two-week factor. Complete and justify all parts of the original task in your work. Bank label: _____	A quantity has $a3$-year half-life. Write its model and give the factor remaining after 6 years. For the answer bank, record the fraction remaining after 6 years. Complete and justify all parts of the original task in your work. Bank label: _____
For $A(t) = A_0 e^{-0.05t}$, where t is in days, find the ten-day multiplier and percent decrease. For the answer bank, record the exact ten-day factor. Complete and justify all parts of the original task in your work. Bank label: _____	A process multiplies by 1.12 every three days. Determine its nine-day factor and percent increase. For the answer bank, record the nine-day factor. Complete and justify all parts of the original task in your work. Bank label: _____

AP Precalculus

Compound Growth and Decay Rates

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A quantity is multiplied by 1.20 each year. What factor and percent change apply over two years?	An exponential process has factor 4 over two hours. Find its positive hourly factor and hourly percent change.
The model $P(h) = 500(1.02^{h})$ uses hours. Rewrite the same process using d days.	A population doubles every 5 days. Write its factor model and state the factor after 10 days.
Process A grows by factor 1.20 every two years; process B grows by factor 1.09 each year. Which grows faster after normalizing to two years?	For $Q(t) = Q_0 e^{0.04t}$, with t in years, find the five-year multiplier and percent increase.

AP Precalculus

Compound Growth and Decay Rates

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

The model $T(s) = 18(0.97^{\frac{s}{15}})$ uses seconds and a15-second factor. Rewrite it with m measured in minutes. For the matching card, record the expression for $T(m)$. Complete and justify all parts of the original task in your work.

Your response: _____

Incoming: 1.2

Rewrite $C(d) = 60(1.5^{d})$, with d in days, using h measured in hours. For the matching card, record the expression for $C(h)$. Complete and justify all parts of the original task in your work.

Your response: _____

Incoming: $40(1.3)^{\frac{m}{60}}$

The model $F(m) = 200(1.03^{m})$ uses months. Rewrite the same process with y in years. For the matching card, record the expression for $F(y)$. Complete and justify all parts of the original task in your work.

Your response: _____

Incoming: $(\exp(-0.09), \exp(-0.36))$

A population has a monthly growth factor of 1.1. What is its growth factor over three months? For the matching card, record the three-month growth factor. Complete and justify all parts of the original task in your work.

Your response: _____

Incoming: 27

A medication level is multiplied by 0.6561 every four hours. Find the equivalent hourly multiplier. For the matching card, record the hourly multiplier. Complete and justify all parts of the original task in your work.

Your response: _____

Incoming: $0.4^{\frac{7}{2}}$

The model $Q(w) = 80(0.95^{w})$ uses weeks. Rewrite it as a function of elapsed days d. For the matching card, record the expression for $Q(d)$. Complete and justify all parts of the original task in your work.

Your response: _____

AP Precalculus

Compound Growth and Decay Rates

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student adds two 20% decreases and reports $a40\%$ loss. Show the correct compounded result and explain the discrepancy. Student work/claim: Two losses of 20% sum to $a40\%$ total loss. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A student divides $a30\%$ two-interval increase by 2 and reports 15% per interval. Find the correct repeated factor and quantify what two 15% increases would produce. Student work/claim: A two-interval gain of 30% means 15% in each interval. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A report says that $a6\%$ decrease every half-year means $a12\%$ decrease in one year. Determine the actual annual percent decrease and explain the discrepancy. Student work/claim: Two half-year losses of 6% make an annual loss of 12%. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A two-year investment gain is 21%. A student divides by 2 and reports 10.5% per year. Find the constant annual percent rate that actually compounds to the observed gain, and verify it. Student work/claim: Divide the two-year gain 21% by two to obtain the annual compound rate. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Compound Growth and Decay Rates

Full Worked Solutions - excerpt

Worked example

Problem. A balance grows by factor 1.08 every two weeks. What factor and total percent increase apply over six weeks?

Answer. The six-week factor is $1.08^3 = 1.259712$, giving $a25.9712\%$ increase.

Explanation and check.

Six weeks contains three of the stated two-week intervals.

Worked example

Problem. An exponential process has factor 4 over two hours. Find its positive hourly factor and hourly percent change.

Answer. The hourly factor is $\sqrt{4} = 2$, corresponding to $a100\%$ hourly increase.

Explanation and check.

The positive unit factor must square to the stated two-hour multiplier.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Growth and Decay Applications

Exponential Growth and Decay Applications | APPC-U02-P07

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for day-based formula, formula and day-fourteen value, formula and earlier value, formula and hour-four value, formula and modeled initial value and formula and prediction.

8 PDFs and a README; 54 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exponential Growth and Decay Applications

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An account starts with \$50 and increases by 12% of its current balance each year. Write its balance after t years. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	A subscriber count is 500 at week 6 and grows by factor 1.04 each week. Write a model $S(w)$ anchored at the known week. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
A count starts at 50 and doubles weekly through week 3. Thereafter it grows by factor 1.1 per week. Write a piecewise model with no jump. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	A fish population begins at 3500 and approaches a habitat capacity of 5000. Each year, 80% of the current difference from capacity remains. Write $P(t)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
Create a value table for $A(n) = 500(1.1)^n$ at $n = 0, 1, 2, 3$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	At week 5 a quantity is 70, and afterward its weekly multiplier is 0.8. Write an anchor-form model using week w. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____
A fund starts at 100, grows by factor 1.03 annually through year 5, and then retains factor 0.97 annually. Write a continuous piecewise model. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____	A temperature starts at 90°F and approaches a65°F environment. After each hour, 30% of its excess remains. Write $T(h)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: _____

AP Precalculus

Exponential Growth and Decay Applications

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

For $P(t) = 40(1.5)^t$, where t is a whole number of days from 0 through 4, construct a value table and state the initial value.	A room is 80°F when cooling begins toward a surrounding temperature of 20°F. Every ten minutes, half of the remaining temperature difference is retained. Build a model and find the temperature after thirty minutes.
A culture begins at 100, grows by factor 1.1 per day through day 3, and then decays by factor 0.8 per day. Construct a continuous-at-switch piecewise model through day 5 and evaluate day 5.	A quantity is 300 at $t = 4$ and grows by factor 1.25 every 2 time units. Audit the proposal $Q(t) = 300(1.25)^{\frac{t}{2}}$, repair it, and verify the anchor and one interval later.
A filter leaves 85% of the current impurity after each hour. If $200mg$ is present initially, write a model and state the percent removed per hour.	A process has value 250 at $t = 3$ and factor 1.2 per four time units. Construct its anchor form. The value is also reported as 300 at $t = 7$; determine whether that report is consistent and, if so, write the equivalent $t = 7$ anchor form.

AP Precalculus

Exponential Growth and Decay Applications

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $M(t) = 100 + 40(0.7)^t$.

A measured level starts at 30 and approaches a floor of 12. Each week, 40% of its excess above 12 remains. Write a model $L(w)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $Q(t) = 250(1.008)^{\frac{t}{60}}$.

A quantity equals 243 at $t = 5$ and triples every two time units. Find its value at $t = 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $A(t) = 625(0.8)^{\frac{t-8}{4}}$.

A machine worth \$1200 loses 7% of its current value each quarter. Write its value after q quarters. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $A(t) = 96(\frac{2}{3})^{\frac{t-6}{4}}$, and $A(14) = 96(\frac{2}{3})^2 = 42\frac{2}{3}$.

At day 20 a quantity is 81 and is multiplied by $\frac{1}{3}$ every ten days. Write an anchored exponential formula. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: $V(q) = 1200(0.93)^q$.

An investment starts at 250, grows by factor 1.1 per quarter through quarter 4, and then by factor 1.03 per quarter. Write a continuous piecewise model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: 11

A measurement starts at 140 and approaches baseline 100. Each day, 70% of its excess above baseline remains. Write $M(t)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

AP Precalculus

Exponential Growth and Decay Applications

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

List the table values of $Q(n) = 30(2)^n$ for $n = 0, 1, 2, 3$. Student work: $(0, 30), (1, 60), (2, 120)$, and $(3, 239)$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

An asset starts at 400 and loses 12% of its current value each day. Audit the proposed model $V(t) = 400(0.12)^t$ and repair it. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

An account starts at 600 and gains 15% every three months. With t measured in months, audit $A(t) = 600(1.15)^{3t}$, repair it, and find the correct month-6 value. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

A score begins at 15 and approaches a ceiling of 75. Every two weeks, 60% of its remaining signed difference from the ceiling is retained. Build the model and evaluate week 6. Student work: $S(w) = 75 + 60(0.6)^{\frac{w}{2}}$, and $S(6) = 62.04$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Exponential Growth and Decay Applications

Full Worked Solutions - excerpt

Worked example

Problem. A quantity is 96 at month 15 and has factor 1.2 every three months. Find its modeled value at month 6. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Answer. The value is $\frac{96}{1.2^3} = 55\frac{5}{9} \approx 55.56$.

Explanation and check.

Month 6 is three three-month intervals earlier, so undo three factors of 1.2.

Worked example

Problem. A value begins at 260 and approaches baseline 200, retaining 90% of its excess each week. Write $V(w)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Answer. $V(w) = 200 + 60(0.9)^w$.

Explanation and check.

The initial excess is 60; apply the weekly factor to that excess and add 200.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Functions from Two Points

Exponential Functions from Two Points | APPC-U02-P08

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for parameters and model, sufficiency decision and parameters, uniqueness and parameters, a and model, base and model and consistency and information decision.

8 PDFs and a README; 92 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exponential Functions from Two Points

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Can any exponential function fit exact rows $(1, 4)$ and $(1, 5)$? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Points:

x	f(x)
1	4
1	5

Bank label: _____

Relative to baseline 5, the exact outputs at $x = 0$ and $x = 1$ are 45 and 25. Determine the scale and positive base of their exponential displacement. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Points:

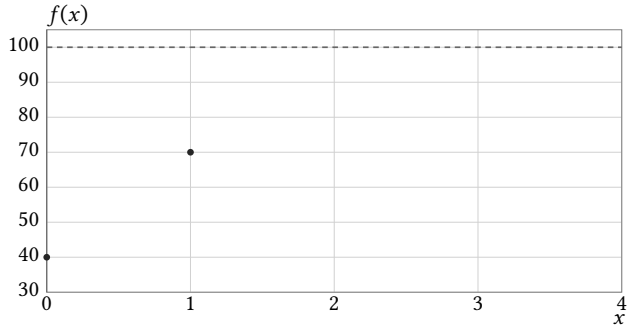
x	f(x)
0	45
1	25

Bank label: _____

For $f(x) = 10 + a(0.5)^x$, is $f(2) = 15$ sufficient to determine the model? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Bank label: _____

A graph on x: 0 to 4 by 1 and y: 30 to 105 by 10 has exact asymptote $y = 100$ and exact marks $(0, 40)$ and $(1, 70)$. Determine $f(x) = 100 + ab^x$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.



x	exact marked f(x)
0	40
1	70

Bank label: _____

AP Precalculus

Exponential Functions from Two Points

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Can one nonconstant real model $f(x) = ab^x$ with $b > 0$ pass through the exact points $(0, 5)$ and $(2, -20)$? Explain.

Points:

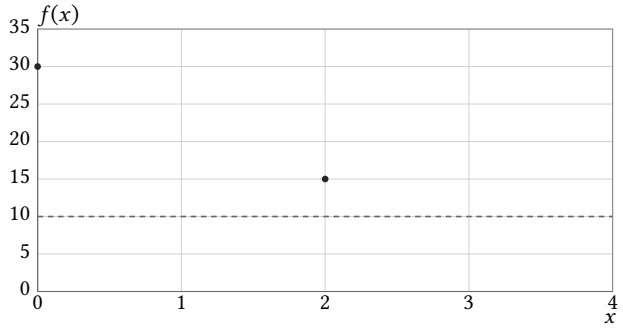
x	f(x)
0	5
2	−20

The exact equally spaced values are $f(0) = 20$, $f(1) = 35$, and $f(2) = 42.5$. Fit $f(x) = L + ab^x$ with $b > 0$ and $b \neq 1$.

Points:

x	f(x)
0	20
1	35
2	42.5

A graph on x: 0 to 4 by 1 and y: 0 to 35 by 5 has exact marked points $(0, 30)$ and $(2, 15)$ and horizontal asymptote $y = 10$. Estimate $f(x) = L + ab^x$ and state why these marks permit an exact result.



x	exact marked f(x)
0	30
2	15

Fit a pure exponential to exact points $(0, 50)$ and $(2, 72)$, then test the unused exact observation $(4, 103.68)$.

Unused Point: $(4, 103.68)$

x	f(x)
0	50
2	72

AP Precalculus

Exponential Functions from Two Points

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Fit $f(x) = ab^x$ through exact points $(-2, 9)$ and $(0, 36)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Points:

x	f(x)
-2	9
0	36

Your response: _____

Incoming: The two-unit ratio is 4, so $b = 2$. Since $f(0) = a, a = 36$; $f(x) = 36(2)^x$.

Fit a pure exponential to exact values $30, 45$, and then check the unused value 67.5 at successive inputs $0, 1, 2$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Unused Point: $(2, 67.5)$

x	f(x)
0	30
1	45

Your response: _____

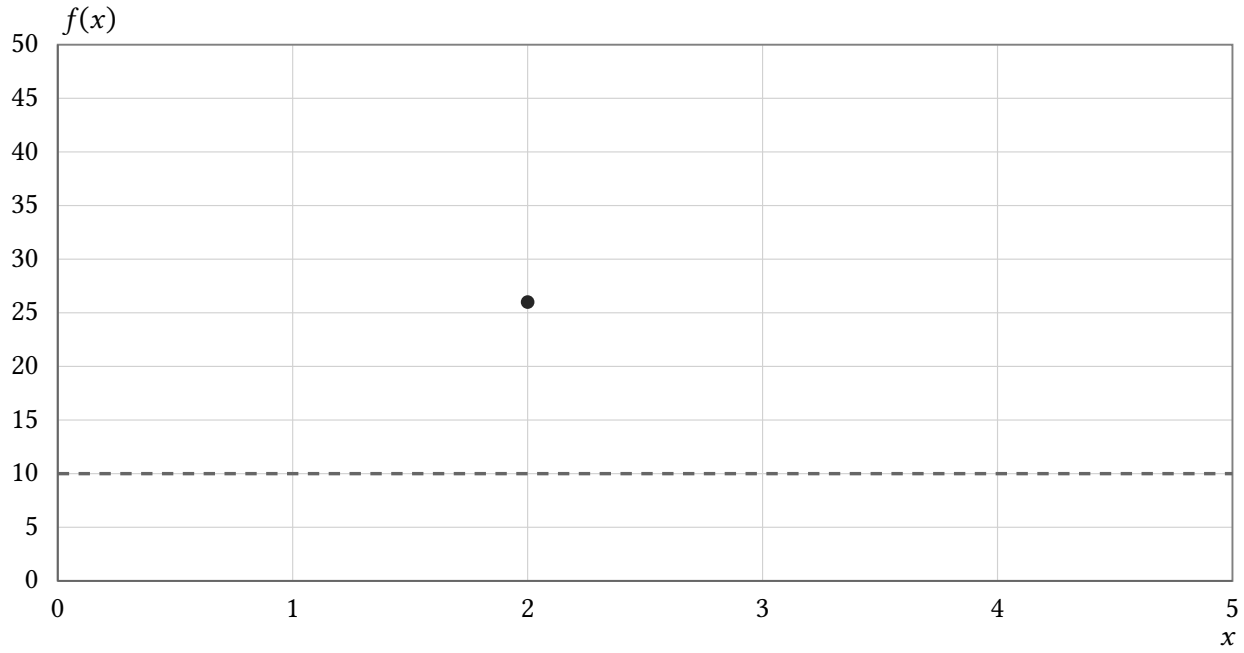
AP Precalculus

Exponential Functions from Two Points

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A graph has exact baseline $y = 10$ and only exact point $(2, 26)$. Can $f(x) = 10 + ab^x$ be uniquely determined? Student work: No. The mark gives a $b^2 = 16$, leaving infinitely many negative bases and matching scales. A second point at a different input is needed. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.



x	exact marked f(x)
2	26

Exponential Functions from Two Points

Full Worked Solutions - excerpt

Worked example

Problem. Does exact value $f(3) = 25$ determine $f(x) = 20 + a(0.5)^x$? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Answer. Yes. Since $20 + \frac{a}{8} = 25, a = 40$ and $f(x) = 20 + 40(0.5)^x$.

Explanation and check.

Only the scale is unknown once baseline and base are fixed.

Worked example

Problem. For $f(x) = a(3)^x$, does $f(2) = 45$ determine the model? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Answer. Yes. Since $9a = 45, a = 5$ and $f(x) = 5(3)^x$.

Explanation and check.

The fixed base leaves one scale parameter, so one nonzero observation suffices.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Regression

Exponential Regression | APPC-U02-P09

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for metrics and verdict, classification and verdict, metric and verdict, evidence statement, robustness conclusion and validity and weighting implication.

8 PDFs and a README; 88 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exponential Regression

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Format and interpret the regression using the reporting rule without rounding b before converting it to a percent.

Given: domain: months; payload: convention: raw-output least squares; report rule: coefficient a to nearest tenth and percent change to nearest tenth percent; technology result: a: 7.408; b: 0.9631

Bank label: _____

Write the original-scale model represented by the fitted excess.

Given: domain: hours; payload: baseline: 6; fitted excess: $e = 45(0.65)^t$; relation: $e = y - 6$

Bank label: _____

Interpret the base of this fitted colony model.

Given: domain: days; payload: convention: raw-output least squares; output unit: colonies; technology result: $N(t) = 18.4(1.065)^t$

Bank label: _____

The negative response is a displacement below zero, and technology fit its positive magnitude. Restore the signed model.

Given: domain: seconds; payload: baseline: 0; definition: $m = -y$; magnitude regression: $m(t) = 12.0(0.823)^t$; table: $(0, -12.0)$; $(2, -8.1)$; $(4, -5.5)$

Context/domain: seconds

0	−12.0
2	−8.1
4	−5.5

Bank label: _____

AP Precalculus

Exponential Regression

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

May all three rows be entered directly into a log-output exponential regression for $y = ab^t$? Explain before fitting.

Context/domain: minutes

0	8.2
2	4.1
4	0.0

AP Precalculus

Exponential Regression

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START Convert the two-hour regression factor to an equivalent hourly percent increase. Given: domain: 2-hour blocks; payload: input definition: n counts two-hour blocks; technology result: $Q(n) = 9.3(1.44)^n$ Your response: _____	**Incoming:** The relative residual magnitudes are at most about 1.2%, but their signs alternate at consecutive days 0 through 7 before another positive residual at day 8. Percentage errors are small; the ordered oscillation is a pattern, so small magnitude alone does not establish an adequate pattern-free model. Compare the week-12 residuals after rounding only the amplitude. Given: domain: weeks; payload: observation: x: 12; y: 27.4; predictions: retained: 26.98; rounded: 27.05; residual definition: observed-predicted; retained model: $y = 51.86(0.947)^x$; rounded model: $y = 52(0.947)^x$ Your response: _____
Incoming: The student's 0.011 is a log-scale residual. The required original-scale residual is $39.0 - 38.12 = 0.88$. Find both residuals at the origin. Given: domain: months; payload: input: 0; observed: 20.6; retained model: $y = 20.54(0.92)^x$; rounded model: $y = 21(0.92)^x$ Your response: _____	**Incoming:** Dividing hours by 24 gives $0, 1, 3, 5$ days, so the conversion is consistent. The fitted base is a factor per day, not per hour. A regression models the excess above a known level of 250. Restore the original response and state its long-run level. Given: domain: days; payload: baseline: 250; definition: $e = y - 250$; fitted excess: $e(t) = 84.5(0.91)^t$ Your response: _____

AP Precalculus

Exponential Regression

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The numerical regression inputs are years since 2018, although the source table lists calendar years. Is the fit invalid, and how should a be interpreted? A student says recoding calendar years invalidates the fit and a predicts calendar year zero. Diagnose both claims.

The last measured value equals the known baseline. Diagnose the proposed baseline-corrected log regression and give two defensible next steps without silently altering the row. A student enters $\log(0)$ for the baseline-equal reading. Diagnose the operation and explain two defensible next steps.

Context/domain: hours

0	31
2	19
5	12

Exponential Regression

Full Worked Solutions - excerpt

Worked example

Problem. The residual magnitudes are modest, but every sign is positive. Interpret this evidence.

Answer. The model underpredicts every observation, showing a one-sided level bias rather than centered residuals. Its calibration should be reconsidered.

Explanation and check.

Observed-minus-predicted is positive whenever prediction is low.

Worked example

Problem. A chamber warms toward 72 degrees Fahrenheit from below. Regression was performed on the positive shortfall $d(t) = 72 - y(t)$, producing $d(t) = 18.6(0.81)^t$. Restore a model for y and describe its approach.

Answer. $y(t) = 72 - 18.6(0.81)^t$; it increases toward 72 from below.

Explanation and check.

The fitted magnitude is the amount below equilibrium, so subtract it from 72; the decaying shortfall makes y rise.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Comparing Regression Models

Comparing Regression Models | APPC-U02-P10

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for exponential base and predictions, maximum losses and model, why selection is premature, predictions and plausibility, qualified candidates and costs and feasibility.

8 PDFs and a README; 100 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Comparing Regression Models

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Evaluate both candidate models at hour 3. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The exponential is $E(t) = 5(1.3)^t$. Evaluate at input 3. The quadratic is $Q(t) = 5 + t^2$.

Bank label: _____

Compute original-scale SSE for both candidates. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. For the exponential model, the predicted outputs in input order are 9.5, 14.5, 19. Use the inputs 0, 2, 4, in that order. For the linear model, the predicted outputs in input order are 11, 14, 18. The observed outputs, in input order, are 10, 14, 19.

Bank label: _____

Select the simplest exact family and state its rate. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given domain/context: years

0	50
2	44
4	38
6	32

Bank label: _____

Select the most discriminating feasible observation. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The feasible months are 1, 3. The gaps are month10.1; month30.8; month52.4. The measurement resolution is 0.5.

Bank label: _____

AP Precalculus

Comparing Regression Models

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Evaluate both fitted models at month 4. The exponential is $E(t) = 8.0(1.24)^t$. Use least squares on original output scale. The linear is $L(t) = 8.2 + 3.1t$. Evaluate at input 4.	Choose the cheaper forecast under the stated asymmetric loss. The actual value is 100. Use this loss rule: underprediction costs \$5 per unit; overprediction costs \$1 per unit. The model predictions are exponential 106; linear 96.
Which feasible input is most useful for distinguishing the models? The feasible inputs are $0, 1, 5$. Use measurement precision nearest unit. Use these candidate models: exponential $E(t) = 10(1.2)^t$; linear $L(t) = 10 + 2t$.	The models are close during years $0 - 3$. Compare their year-20 predictions and qualify the inference. The exponential is $E(t) = 50(1.08)^t$. Evaluate the future input 20. The linear is $L(t) = 50 + 5t$. The observations cover the input interval $0, 3$.

AP Precalculus

Comparing Regression Models

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: At 4, $L = 260$ and $E \approx 243.10$. At 25, $L = 575$ and $E \approx 677.27$. Their ordering reverses, so local superiority is horizon-specific.

Compare hour-3 outputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The exponential is $E = 12(1.3)^t$. Evaluate at input 3. The linear is $L = 12 + 5t$.

Your response: _____

Incoming: Exponential, $y = 64(0.75)^t$, because each output is three-fourths of the preceding output.

Choose the lower-cost forecast. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The actual value is 80. Use this loss rule: shortage $6 per unit; surplus $2 per unit. The model predictions are exponential 85; linear 76.

Your response: _____

Incoming: Linear; its points scatter around zero, whereas quadratic residuals trend upward from overprediction to underprediction.

Choose the best feasible week for distinguishing the candidates. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The last feasible week is 5. The model predictions are week220.1, 20.4; week532.0, 34.3; week850.2, 57.9. The measurement resolution is 1.

Your response: _____

Incoming: Linear reaches 30 at $t = 15$. Exponential reaches 30 when $0.95^t = 0.25$, so $t \approx 27.03$. Both are extrapolations far beyond the data.

Select the exact family and write its rule. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given domain/context: days

0	64
1	48
2	36
3	27

Your response: _____

AP Precalculus

Comparing Regression Models

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Apply the production loss to choose a model. Student work: Quadratic underpredicts by 3 and costs 20; exponential overpredicts by 4 and costs 4. Choose quadratic because its unsigned error, 3, is smaller. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. The actual value is 60. The model predictions are exponential 64; quadratic 57. Use underprediction beyond 2 units costs 20; otherwise absolute error.

Choose a staffing model using the interval-specific loss. Student work: exponential costs $1 + 30 + 2 = 33$. Quadratic costs $2 + 1 + 1 = 4$, so choose quadratic; its avoidance of critical noon underprediction is decisive. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. The actual value is $20, 35, 28$. The critical input interval is $11, 14$. The hours are $8, 12, 16$. For the linear model, the predicted outputs in input order are $19, 32, 30$. For the quadratic model, the predicted outputs in input order are $22, 36, 27$. Use inside critical interval, each understaffed unit costs 10; all other errors cost 1 per unit.

Which observation best tests the quadratic-turn claim? Student work: Day 10, because it lies after the proposed vertex and directly tests reversal; day 7 is before the turn where all three can still rise. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. Observations are available at inputs $7, 9$. For the exponential model, the prediction is continues rising faster. For the linear model, the prediction is continues rising at day 9. For the quadratic model, the prediction is peaks at day 8 then falls. The measurement resolution is sufficient at both.

Compare both candidates by absolute and percent error at the row. Student work: Linear error is 4 units or 4%; linear error is 3 units or 3%. Exponential is closer by both measures at this observation. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. For the exponential model, the prediction is 103. Evaluate at input 8. For the linear model, the prediction is 96. The observed outputs, in input order, are 100.

Comparing Regression Models

Full Worked Solutions - excerpt

Worked example

Problem. Compare hour-3 outputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The exponential is $E = 12(1.3)^t$. Evaluate at input 3. The linear is $L = 12 + 5t$.

Answer. $L(3) = 27$ and $E(3) = 26.364$.

Explanation and check.

Substitute 3 into each candidate.

Worked example

Problem. Compare the day-6 fitted values. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The exponential is $E = 18(0.86)^t$. Evaluate at input 6. The linear is $L = 18 - 2t$.

Answer. $L(6) = 6$ and $E(6) \approx 7.28$.

Explanation and check.

Evaluate each rule at the same elapsed day.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Composite Functions

Composite Functions | APPC-U02-P11

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for formulas and domains, rules and domains, rules and equality set, commutation conclusion on listed set, both composites and both ordered compositions.

8 PDFs and a README; 110 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Composite Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Read the inner table first and evaluate $f(g(0))$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given input: 0

input	f(input)
-4	10
1	-2
5	7

input	g(input)
-2	5
0	1
3	-4

Bank label: _____

Use the tables to compare the two composition orders at $x = 1$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given input: 1

input	f(input)
1	3
2	4
3	2
4	1

input	g(input)
1	2
2	1
3	4
4	3

Bank label: _____

AP Precalculus

Composite Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Determine the real domain of $f(g(x))$. The input condition is real numbers. Report interval notation and reason. Use f rule $f(u) = \frac{1}{u-4}$. Use g rule $g(x) = 2x + 1$.	Construct f composed with g. The input condition is real x. Report $f(g(x))$. Use f rule $f(u) = u^2 + 3$. Use g rule $g(x) = 2x - 1$.

AP Precalculus

Composite Functions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A tank fills according to $V(t)$, and its water depth is $d(V)$. Compose the models and state the physically allowed time domain. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The input condition is $0 \le t \le 30$ minutes. Report depth as a function of time and domain. Use depth model $d(V) = \frac{V}{120}$ centimeters. Use tank capacity 540 liters. Use volume model $V(t) = 18t$ liters.

Your response: _____

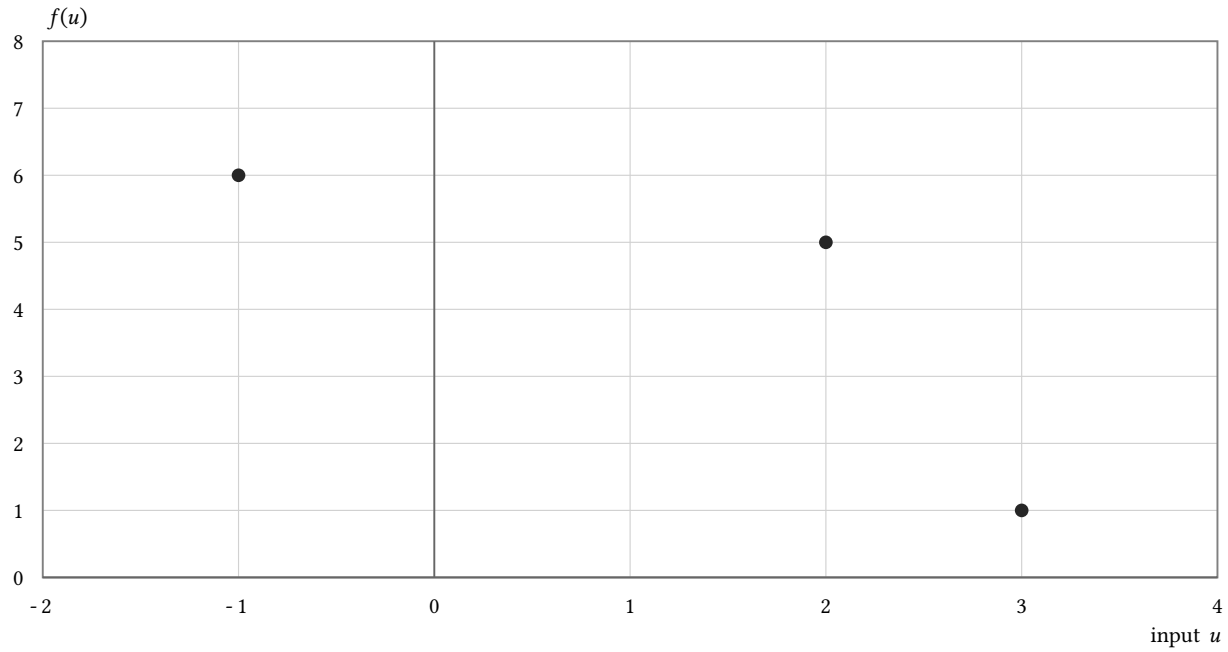
AP Precalculus

Composite Functions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use the table for g and the point-only graph for f to find $f(g(4))$. Student work: 4 Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.



input	g(input)
1	3
2	-1
4	2

Composite Functions

Full Worked Solutions - excerpt

Worked example

Problem. Construct $f(g(x))$ without losing the inner restriction. The input condition is real x with nonnegative inner radicand. Report composite and domain. Use f rule $f(u) = 3u - 2$. Use g rule $g(x) = \sqrt{x + 4}$.

Answer. $f(g(x)) = 3\sqrt{x + 4} - 2$, with $x \geq -4$.

Explanation and check.

Substitute the radical for u; its radicand controls the domain.

Worked example

Problem. Find the x-intervals for each branch of $f(x^2)$. The input condition is real x. Report piecewise composite on x. Use f rule $f(u) = u + 1$ if $u < 4$; $2u$ if $u \geq 4$. Use g rule $g(x) = x^2$.

Answer. $f(x^2) = x^2 + 1$ for $-2 < x < 2$, and $2x^2$ for $x \leq -2$ or $x \geq 2$.

Explanation and check.

The inner output is below 4 exactly when $x^2 < 4$; the threshold is visited on both sides.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Decomposing Functions

Decomposing Functions | APPC-U02-P12

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for actual composite domains, domain-faithful decomposition, validity and domain dependence, validity and intermediate images, validity of both pairs and validity relative to H domain.

8 PDFs and a README; 87 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Decomposing Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Split H at the affine expression inside the square. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Use the input condition real x.

Give two nonidentity components.

Let $H(x) = (5x + 1)^2 - 7$.

Bank label: _____

Write the visible two-stage rule as $H = f$ composed with g. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Use the input condition real x.

Give affine and absolute components.

Let $H(x) = |3x - 5| + 2$.

Bank label: _____

Use the prescribed outer rule to recover the missing g-values at the listed inputs. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given f rule: $f(u) = 2u + 1$

input	H(input)
-3	7
0	-1
4	11

Bank label: _____

List the outer pairs forced by the two aligned tables. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given g table:

input	H(input)
-6	0
0	5
2	12

input	g(input)
-6	3
0	-1
2	7

Bank label: _____

AP Precalculus

Decomposing Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recompose both proposed pairs and decide whether each is a substantive decomposition of H.

Use the input condition real x.

Give validity of both pairs.

Let $H(x) = (x + 2)^4$.

For pair A, use $g(x) = x + 2$, $f(u) = u^4$.

For pair B, use $q(x) = (x + 2)^2$, $p(v) = v^2$.

Recover the outer table values required for $H(x) = f(g(x))$.

Given g table:

input	H(input)
-2	7
0	2
4	9

input	g(input)
-2	1
0	3
4	5

Explain why the proposed f is not a function of its placeholder alone and repair it.

Use the input condition real x.

Give correct outer rule.

Let $H(x) = (x^2 + 1)^3 + 2(x^2 + 1)$.

The student proposes $f(u) = u^3 + 2x$.

The student proposes $g(x) = x^2 + 1$.

Construct the smallest outer table needed to express H as f after g.

Given g table:

input	H(input)
-3	6
1	-1
5	8

input	g(input)
-3	0
1	2
5	4

Find an inner rule whose reciprocal gives H, and retain every stated exclusion.

Use the input condition real x except $x = 1$ and $x = -2$.

Give inner g and domain.

Let $H(x) = \frac{x+2}{x-1}$.

Use the prescribed outer function $f(u) = \frac{1}{u}$.

Compare the recomposed rules and explain why domain declarations are part of the comparison.

Use the input condition $x \geq 0$.

Give validity relative to H domain.

Let $H(x) = x$ on $x \geq 0$.

For pair A, use $g(x) = \sqrt{x}$, $f(u) = u^2$.

For pair B, use $q(x) = x$, $p(v) = v$.

AP Precalculus

Decomposing Functions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Replace the prescribed inner expression consistently to recover f. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give outer polynomial.

Let $H(x) = (x-4)^3 - 2(x-4) + 6$.

Use the prescribed inner function $g(x) = x - 4$.

Your response: _____

Incoming: $f(u) = u^3 - 2u + 6$.

Undo the outer translation to recover g. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give inner function.

Let $H(x) = \sqrt{x^2+4} + 7$.

Use the prescribed outer function $f(u) = u + 7$.

Your response: _____

Incoming: $g(x) = \sqrt{x^2+4}$.

Recover f and state the intermediate inputs on which it must be defined. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give outer rule on image.

Let $H(x) = 4x^2 - 12$.

Use the prescribed inner function $g(x) = x^2$.

Your response: _____

Incoming: $f(u) = 4u - 12$ for $u \geq 0$.

Recover the pre-tax subtotal and tax stages. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition $0 \leq n \leq 80$ tickets.

Give subtotal and taxed-total maps.

Use the compound model $T(n) = 1.075(14n + 5)$ dollars.

Your response: _____

Incoming: $S(n) = 14n + 5$ dollars and $T(S) = 1.075S$ dollars.

Treat the recurring logarithm as one intermediate variable. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition $x > 0$.

Give log inner and polynomial outer.

Let $H(x) = (\ln(x))^2 - 6\ln(x) + 2$.

Your response: _____

Incoming: $g(x) = \ln(x)$, $x > 0$, and $f(u) = u^2 - 6u + 2$.

Remove the outer scaling to identify g. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give inner rule.

Let $H(x) = \frac{3}{x^2+1}$.

Use the prescribed outer function $f(u) = 3u$.

Your response: _____

AP Precalculus

Decomposing Functions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Check whether pulling the positive factor outside absolute value gives a second valid decomposition. Student work: Both are invalid for all real x because $|2(x+3)| = 2|x+3|$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use the input condition real x.

Give validity.

Let $H(x) = |2x+6|$.

For pair A, use $g(x) = 2x+6$, $f(u) = |u|$.

For pair B, use $q(x) = x+3$, $p(v) = 2|v|$.

Isolate the quadratic expression evaluated before absolute value. Student work: $g(x) = x^2 + 6x + 2$ and $f(u) = |u| + 5$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use the input condition real x.

Give quadratic inner and absolute outer.

Let $H(x) = |x^2 - 6x + 2| + 5$.

Decomposing Functions

Full Worked Solutions - excerpt

Worked example

Problem. Can a single f satisfy $x = f(\sin(x))$ for every real x? Give a decisive pair of inputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Answer. No. $\sin(0) = \sin(2\pi) = 0$, but $H(0) = 0$ and $H(2\pi) = 2\pi$.

Explanation and check.

The periodic inner map discards which cycle produced the value, while H retains that information.

Worked example

Problem. Formally undo the logarithm and verify whether the resulting inner function is admissible. Student work: $g(x) = e^{-x^2}$, which is negative for every real x and satisfies $\ln(g(x)) = -x^2$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Answer. $g(x) = e^{-x^2}$, which is positive for every real x and satisfies $\ln(g(x)) = -x^2$.

Explanation and check.

A logarithm may output negative values; only its input must be positive, which the exponential guarantees.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

One-to-One Functions and Domain Restrictions

One-to-One Functions and Domain Restrictions | APPC-U02-P13

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for displayed-table invertibility, finite versus continuous claim, limited conclusion, interval restriction containing both, whether interval restriction can contain both and whether full domain is one-to-one.

8 PDFs and a README; 110 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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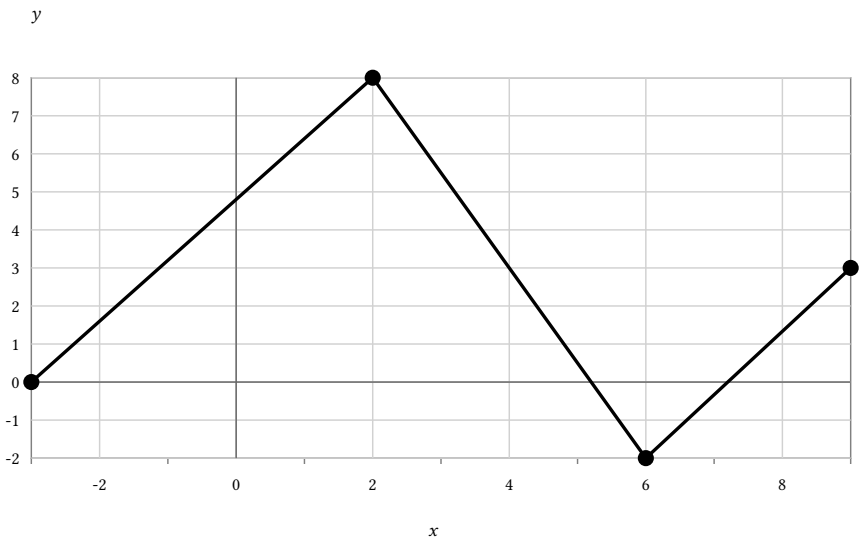
AP Precalculus

One-to-One Functions and Domain Restrictions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Choose the maximal one-to-one segment containing the marked input $x = 4$.



-3	0
2	8
6	-2
9	3

Bank label: _____

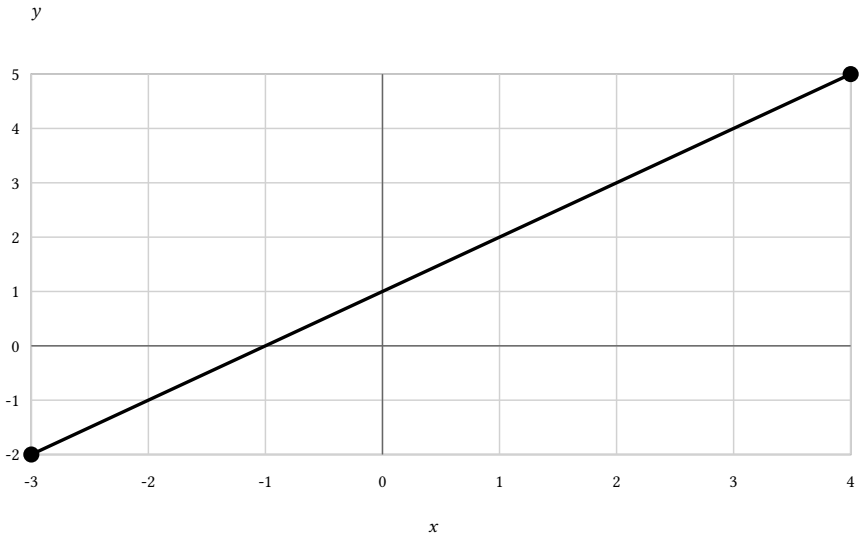
AP Precalculus

One-to-One Functions and Domain Restrictions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Decide whether the graphed function is one-to-one on its stated domain.



AP Precalculus

One-to-One Functions and Domain Restrictions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[-2, 3]$.

Select the time domain that makes height invertible for questions explicitly about the ascent.

Given: $0 \le t \le 8$ seconds

Height: $h(t) = 16 - (t - 4)^2$ meters; Question context: recover time during ascent

Your response: _____

AP Precalculus

One-to-One Functions and Domain Restrictions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Evaluate the cross-piece output ranges before accepting the claim. A student accepts the claim because each piece increases. Identify the invalid inference and give a cross-piece collision.

Given: $[-4, -2]$ union $[1, 3]$

Claim: Each piece is increasing, so the union is invertible.; Rule: $f(x) = x + 5$ on $[-4, -2]$; $f(x) = 2x$ on $[1, 3]$

Do the samples prove that the underlying model is invertible on all positive reals? Explain. A student says four distinct sampled outputs prove invertibility on all positive reals. Identify the unsupported extension.

Given domain/context: sampled inputs from an unspecified model

1	3
2	5
4	9
8	17

One-to-One Functions and Domain Restrictions

Full Worked Solutions - excerpt

Worked example

Problem. For which k is q_k one-to-one on the entire interval $[-2, 4]$?

Given: $-2 \le x \le 4$

Family: $q_k\ (x) = (x - k)^2$

Answer. $k \le -2$ or $k \ge 4$.

Explanation and check.

The vertex must not lie in the interval's interior.

Worked example

Problem. Test the proposed restriction and either prove uniqueness or provide two allowed inputs with one output.

Given: $x \ge -1$

Claim: Restricting to $x \ge -1$ makes f invertible.; Rule: $f(x) = (x - 2)^2$

Answer. It is not sufficient: $f(1) = f(3) = 1$, and both inputs satisfy $x \ge -1$.

Explanation and check.

The restricted interval still crosses the vertex at $x = 2$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Finding Inverse Functions

Finding Inverse Functions | APPC-U02-P14

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for eighth-root inverse and domain, inverse and bounded domain, inverse and domain, inverse and finite domain, inverse branch rules and intervals and inverse branches with open images.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Finding Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Reverse all three associations.

Required output: inverse mapping

Input	Output
-1	6
4	2
8	11

Bank label: _____

Solve the rational relation for its original input.

Domain: x is not -1

Given rule: $f(x) = \frac{x-2}{x+1}$

Bank label: _____

Construct the model that recovers travel time from distance.

Domain: $0 \le t \le 12$ hours

Given distance model: $D(t) = 55t$ miles

Bank label: _____

Translate both statements into inverse notation.

Domain: an invertible function

Given facts: [”$f(2) = 13$”, ”$f(9) = -4$”]

Bank label: _____

AP Precalculus

Finding Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Reverse the finite mapping and evaluate the requested inverse value.

Required output: inverse table and $f^{-1}(8)$

Input	Output
-2	5
0	1
3	8

AP Precalculus

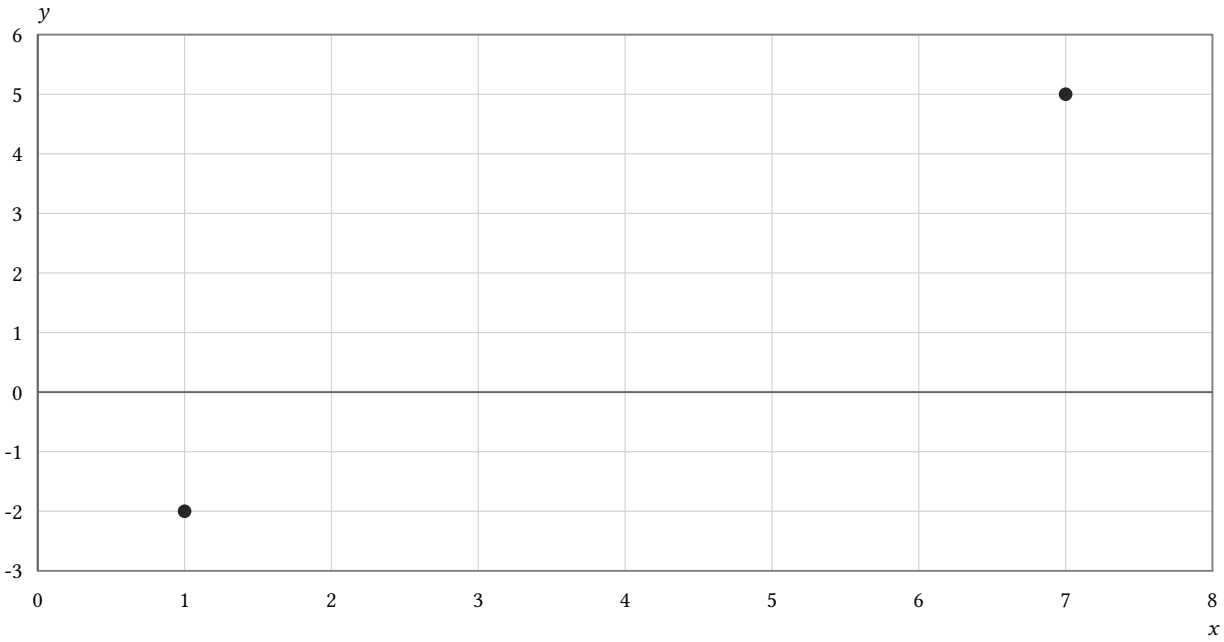
Finding Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Identify the two endpoints of the reflected inverse curve.



Your response: _____

AP Precalculus

Finding Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Construct the inverse of the decreasing odd-power chain. A student proposes $\sqrt[5]{x} - 2 - 7$. Locate the first incorrect inverse operation and repair the rule.

Domain: all real numbers

Given rule: $p(x) = 7 - (x + 2)^5$

Solve for the inverse and state its excluded input and output values. A student keeps $x \neq -\frac{5}{2}$ as the inverse input exclusion. Diagnose the interchange of domain and range.

Domain: x is not $-\frac{5}{2}$

Given rule: $f(x) = \frac{3x-2}{2x+5}$

Write the inverse pieces in increasing order of their new input intervals. A student reverses the formulas but keeps the original input intervals on the inverse pieces. Identify and repair the interval error.

Domain: $[-3, 0]$ union $[2, 6)$

Given rule: $f(x) = 4 - x$ for $-3 \le x \le 0$; $f(x) = 3x + 8$ for $2 \le x < 6$

Construct the inverse of the decreasing radical branch and retain its exact range. A student gives $9 - x^2$ as the inverse on every real input. Explain why the algebraic expression alone is insufficient.

Domain: $0 \le x \le 9$

Given rule: $r(x) = \sqrt{9 - x}$

Finding Inverse Functions

Full Worked Solutions - excerpt

Worked example

Problem. Reverse the root and translations in their operational order.

Domain: all real numbers

Given rule: $h(x) = \sqrt[3]{x-4}+2$

Answer. $h^{-1}(x) = (x-2)^3 + 4$.

Explanation and check.

Subtract 2, cube, and then add 4.

Worked example

Problem. Recover both inverse intercepts and one additional reflected point.

Domain: an invertible interval

Given facts: [" $f(0) = 9$ ", " $f(-4) = 0$ ", " $f(2) = 13$ "]

Answer. The inverse x-intercept is 9, its y-intercept is -4, and it contains $(13, 2)$.

Explanation and check.

Reflect $(0, 9), (-4, 0)$, and $(2, 13)$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Verifying Inverse Functions

Verifying Inverse Functions | APPC-U02-P15

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for complete membership audit, complete reflection verdict, component-level verdict, distinct identities, endpoint-sensitive verdict and equality-set and identities.

8 PDFs and a README; 113 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Verifying Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Check whether the candidate uses the correct scale inside the cube root.

Domain: both functions on all real numbers

Given f: $f(x) = 3x^3 + 2$

Given g: $g(x) = \sqrt[3]{\frac{x-2}{2}}$

Bank label: _____

Assess whether the candidate covers the full inverse input set.

Domain: f: $1 \le x \le 5$; proposed g: $3 \le x \le 20$

Given facts: ["f is one-to-one", "range$(f) = [3, 27]$", "candidate formula is algebraically correct"]

Bank label: _____

Check whether the candidate inverse is complete.

Required output: complete verdict and the supporting evidence requested above

Input	f(input)
-7	5
-2	0
4	13
10	-3

Input	g(input)
5	-7
0	-2
-3	10

Bank label: _____

Verify each composition on its stated domain.

Domain: f: $x \ge 2$; g: $x \ge 0$

Given f: $f(x) = (x-2)^2$

Given g: $g(x) = 2+\sqrt{x}$

Bank label: _____

AP Precalculus

Verifying Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Audit all reversed pairs and identify any failure.

Required output: complete verdict and the supporting evidence requested above

Input	f(input)
-4	7
0	2
5	-1
8	9

Input	g(input)
7	-4
2	0
-1	5
9	6

Identify why the successful composition is insufficient on the declared domain of f.

Domain: f on all real numbers; g on $x \geq 0$

Given claim: $f(g(x)) = x$ proves they are inverses

Given f: $f(x) = x^2$

Given g: $g(x) = \sqrt{x}$

The formulas reverse algebraically. Decide whether the declared domains make them inverse functions.

Domain: f: x is not 2; proposed g: all real x

Given f: $f(x) = \frac{1}{x-2} + 5$

Given g: $g(x) = 2 + \frac{1}{x-5}$

Explain why the successful direction does not certify the larger-domain function.

Domain: f on all real numbers; g on $x \geq 0$

Given claim: $f(g(x)) = x$ for $x \geq 0$ proves inverses

Given f: $f(x) = |x|$

Given g: $g(x) = x$

AP Precalculus

Verifying Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Determine whether the proposed inverse output set matches the original input set.

Domain: f: $-4 \le x \le 3$; proposed g range: $-4 \le y \le 4$

Given facts: ["f is one-to-one", "g domain equals range(f)", "candidate formula is correct"]

Your response: _____

Incoming: No. range(g) must end at 3, not 4; the candidate declares an output that is not in domain(f).

Check whether the extra row preserves a valid inverse function.

Required output: complete verdict and the supporting evidence requested above

Input	f(input)
-6	2
0	8
5	-1
9	12

Input	g(input)
2	-6
8	0
-1	5
12	9
8	4

Your response: _____

Incoming: No. g must have a hole at $(5, 1)$; filling it adds an inverse input absent from range(f).

Explain why the declared domain of f prevents certification.

Domain: f on all real numbers; g on $x \ge 1$

Given evidence: $f(g(x)) = x$

Given f: $f(x) = (x-4)^2 + 1$

Given g: $g(x) = 4+\sqrt{x-1}$

Your response: _____

Incoming: For $x < 4$, $g(f(x)) = 4 + |x-4| = 8\text{-}x$ rather than x, so f must be restricted to $x \ge 4$.

Use the nonpositive sign of $x - 1$ when checking the reverse composition.

Domain: f: $x \le 1$; g: $x \ge 2$

Given f: $f(x) = 2 + (x-1)^2$

Given g: $g(x) = 1-\sqrt{x-2}$

Your response: _____

AP Precalculus

Verifying Inverse Functions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Name the additional identity and set information required.

Domain: unspecified function domains

Given claim: This alone proves f and g are inverses

Given evidence: $f(g(x)) = x$ wherever g is defined

Test the claim using composition rather than product notation.

Domain: nonzero real x

Given claim: $f^{-1}(x) = \frac{1}{2x}$

Given function: $f(x) = 2x$

Compute both round trips and certify or reject the inverse claim. Student work/claim: Checking only $f(g(x))$ for $f(x) = 3x + 2$ and $g(x) = \frac{x-2}{3}$ is enough to certify the inverse claim. Identify the first invalid inference or omitted check, correct it, and complete the original task.

Verifying Inverse Functions

Full Worked Solutions - excerpt

Worked example

Problem. Explain why the evidence does not establish an inverse pair. Choose between testing one successful input and verifying both composition identities on the full declared domains. Use the sufficient method.

Answer. One successful input can neither prove $f(g(x)) = x$ for all allowed x nor verify $g(f(x)) = x$ on the other domain.

Explanation and check.

Inverse verification requires identities on the full relevant sets, not one sample.

Worked example

Problem. Explain why an algebraic identity alone may fail to define a valid composition everywhere claimed.

Answer. Outputs of g must lie in $\text{domain}(f)$, and $\text{domain}(g)$ must equal $\text{range}(f)$; otherwise the symbolic substitution can simplify where the actual composition is undefined.

Explanation and check.

Function composition is governed by declared sets as well as formulas.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Converting Logarithmic and Exponential Forms

Converting Logarithmic and Exponential Forms | APPC-U02-P16

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for base role and equivalent equation, base role, role and appropriate representation, role and operation, role of N and role of y and corresponding operation.

8 PDFs and a README; 49 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Recover the positive value represented by coordinate -3.

Given: domain: base-10 logarithmic scale; payload: axis: $u = \log_{10}(x)$; coordinate: -3

Bank label: _____

Determine the sign from the position of x relative to 1.

Given: domain: $x > 1$; payload: expression: logarithm base 2 of x

Bank label: _____

Record the exponent using logarithmic notation.

Given: domain: base 3; payload: statement: $3^4 = 81$

Bank label: _____

Find the multiplicative factor from A's original value to B's.

Context/domain: base-2 logarithmic coordinate

A	-2
B	1

Bank label: _____

Describe the logarithmic coordinate of the smaller amount.

Given: domain: relative multiplicative position; payload: factor per interval: 4; output ratio: $\frac{1}{4}$

Bank label: _____

Identify k's role and express it without evaluation.

Given: domain: known positive R; payload: relationship: $3^k = R$

Bank label: _____

Determine which side of exponent zero produces z.

Given: domain: $z > 1$; payload: argument: z; base: e

Bank label: _____

Convert the marked coordinates to exact original-scale values.

Context/domain: base-5 logarithmic scale

P	-0.5
Q	0
R	1.5

Bank label: _____

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Represent and explain the number of multiplicative intervals. Given: domain: successive equal intervals; payload: factor per interval: 3; output ratio: 27	Describe the unknown without evaluating it. Given: domain: positive quantities; payload: question: In $\log_5(18)$, what quantity is being requested?
Interpret the logarithm of the ratio, including why its sign is allowed. Given: domain: relative to a present amount; payload: earlier to present ratio: $\frac{1}{25}$; growth factor: 5	State which part of an exponential relationship is unknown. Given: domain: positive quantities; payload: question: How many factors of 6 turn 1 into 50?
Express what the exponent counts in this growth situation. Given: domain: equal growth intervals; payload: factor per interval: 2; final to initial ratio: 8	Identify the unknown role and express it without evaluating. Given: domain: positive quantities; payload: relationship: $7^x = 30$

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Rewrite the exponential fact using common logarithm notation.

Given: domain: common logarithm notation; payload: statement: $10^{-2} = 0.01$

Your response: _____

Incoming: The result equals zero.

Rewrite the relationship so the exponent is the subject.

Given: domain: valid base b and positive r; payload: statement: $b^m = r$

Your response: _____

Incoming: u = ln(5).

Describe the logarithmic interval count represented by the ratio.

Given: domain: continuous growth assumption; payload: factor per interval: 2; output ratio: $\sqrt{2}$

Your response: _____

Incoming: $(\frac{1}{3})^{-3} = 27$.

Describe what N and p represent in the equivalent exponential relationship.

Given: domain: $N > 0$; payload: statement: $\log_2(N) = p$

Your response: _____

Incoming: The original quantity is multiplied by 49.

Decide whether a logarithm or exponentiation directly answers the stated question.

Given: domain: positive N; payload: relationship: $2^5 = N$

Your response: _____

Incoming: $\log_5(\frac{1}{5}) = -1$.

Display the exponent-zero relationship explicitly.

Given: domain: base 7; payload: statement: $\log_7(1) = 0$

Your response: _____

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Diagnose the sign error and repair the statement.

Given: domain: real logarithms; payload: student claim: $\log_2(-8) = 3$ because $2^3 = 8$

Explain why $\log_3(20)$ would answer the wrong question.

Given: domain: positive b; payload: relationship: $b^3 = 20$

Explain why no real logarithm value can satisfy the claim and give a nearby valid example.

Given: domain: real logarithms; payload: student claim: $\log_1(5)$ is some large exponent

---

Explain why the student's logarithm does not represent the unknown.

Given: domain: positive b; payload: relationship: $b^3 = 12$; student expression: $\log_{12}(3)$

Converting Logarithmic and Exponential Forms

Full Worked Solutions - excerpt

Worked example

Problem. Identify the exponent that produces argument 1. Choose between using the base behavior and the argument position relative to 1, or approximating a decimal first. Determine the sign by the exact structural method.

Answer. The value is 0.

Explanation and check.

Any nonzero valid base raised to exponent 0 equals 1.

Worked example

Problem. Identify the original value located at coordinate $u = 2$.

Given: domain: base-10 logarithmic scale; payload: axis: $u = \log_{10}(x)$; coordinate: 2

Answer. x = 100.

Explanation and check.

Coordinate 2 means $10^2 = x$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Evaluating Logarithms and Domains

Evaluating Logarithms and Domains | APPC-U02-P17

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for domain, admissible x-values, corrected evaluation and verification, corrected exact value, corrected value with verification and correction.

8 PDFs and a README; 49 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Evaluating Logarithms and Domains

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

State the domain using the strict positivity of the argument. Given: domain: real x; payload: statement: $\ln(x+5)$ Bank label: _____	Find the exact logarithm by rewriting the argument as a power of the base. Given: domain: real logarithm; payload: statement: $\log_4(2)$ Bank label: _____
Determine whether the value is available exactly and explain the common-base comparison. Given: domain: real logarithm; payload: statement: $\log_{16}(8)$ Bank label: _____	Intersect the base and argument restrictions and remove the forbidden base value. Given: domain: real x; payload: statement: log base $(5\text{-}x)$ of $(x+1)$ Bank label: _____
Compare exponents after expressing both quantities as powers of 3. Given: domain: real logarithm; payload: statement: log base $(\frac{1}{3})$ of $\sqrt{27}$ Bank label: _____	Decide whether a rational exponent is available exactly and state it. Given: domain: real logarithm; payload: statement: $\log_{81}(3)$ Bank label: _____
Solve the strict positivity condition for the decreasing argument. Given: domain: real x; payload: statement: $\log(9-3x)$ Bank label: _____	Intersect the base restrictions with the sign condition on the quadratic argument. Given: domain: real x; payload: statement: log base $(-x)$ of (x^2-4) Bank label: _____

AP Precalculus

Evaluating Logarithms and Domains

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Determine the real domain, keeping the logarithm's boundary strict. Given: domain: real x; payload: statement: $\log(2x - 6)$	Use a sign chart for the entire logarithm argument. Given: domain: real x; payload: statement: ln((5-x)/(x+2))
Evaluate and verify with one exponential equation. Given: domain: real logarithm; payload: statement: $\log_5(125)$	Determine where the rational argument is strictly positive, noting its repeated zero. Given: domain: real x; payload: statement: $\log(\frac{(x-2)^2}{x+1})$
Build a sign chart and retain only intervals with a positive argument. Given: domain: real x; payload: statement: $\ln(\frac{(x+3)(x-1)}{x-4})$	Combine the linear base and argument constraints, including the base-one exclusion. Given: domain: real x; payload: statement: log base $(2x + 1)$ of $(5\text{-}x)$

AP Precalculus

Evaluating Logarithms and Domains

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: -3, since $\frac{1}{27} = 3^{-3}$.

Evaluate using the implied base of the common logarithm.

Given: domain: real logarithm; payload: statement: log(10000)

Your response: _____

Incoming: $x > 2$ with x not equal to 3

Intersect the split domain of the quadratic argument with the upper bound from the second logarithm.

Given: domain: real x; payload: statement: ln(x^2-4)+log(5-x)

Your response: _____

Incoming: The value is exactly 2 because the inputs are declared exact and $2.5^2 = 6.25$.

Find the parameter values that make the displayed base valid.

Given: domain: real x; payload: statement: log base $(x + 4)$ of 2

Your response: _____

Incoming: $\frac{1}{2}$ because $49^{\frac{1}{2}} = 7$.

Use the quadratic base's minimum and its two base-one points inside the argument interval.

Given: domain: real x; payload: statement: log base $(x^2 + x + 1)$ of $(4\text{-}x^2)$

Your response: _____

Incoming: No approximation is needed; the exact value is $\frac{2}{3}$.

Apply both validity conditions to the decreasing variable base.

Given: domain: real x; payload: statement: log base $(7\text{-}x)$ of 4

Your response: _____

Incoming: $(-4, -1)$ union (2,infinity)

Use the repeated factor without treating its zero as a sign change.

Given: domain: real x; payload: statement: $\log((x - 4)^2(x + 2))$

Your response: _____

AP Precalculus

Evaluating Logarithms and Domains

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Diagnose the claim before doing any power calculation.

Given: domain: real logarithm; payload: claim: $\log_3(-9) = 2$

A student reports the logarithm as exactly $\frac{3}{2}$. Decide what the table actually supports.

Given: domain: rounded positive measurements; payload: argument measurement: 27.0; argument precision: nearest tenth; base measurement: 9.0; base precision: nearest hundredth

---

Repair the exponent after accounting for the reciprocal in the argument.

Given: domain: real logarithm; payload: claim: $\log_5(\frac{1}{25}) = 2$ because $5^2 = 25$

Identify both the sign mismatch in the cited power and the real-logarithm restriction that settles the claim.

Given: domain: real logarithm; payload: claim: log base -2 of $8 = 3$ because $(-2)^3 = -8$

Evaluating Logarithms and Domains

Full Worked Solutions - excerpt

Worked example

Problem. Combine the argument inequality with the value that makes the logarithmic denominator zero.

Given: domain: real x; payload: statement: 1/log_3(2-x)

Answer. $x < 2$ with x not equal to 1

Explanation and check.

The argument requires 2-x>0, while log_3(2-x)=0 at 2-x=1, so x=1 is excluded.

Worked example

Problem. Require both logarithms to exist and neither denominator factor to be zero.

Given: domain: real x; payload: statement: 1/(ln(x+2) ln(5-x))

Answer. $-2 < x < 5$, excluding $x = -1$ and $x = 4$

Explanation and check.

Argument positivity gives -2<x<5; ln(x+2)=0 at -1 and ln(5-x)=0 at 4.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential and Logarithmic Inverse Models

Exponential and Logarithmic Inverse Models | APPC-U02-P18

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for adjusted inverse change, baseline-adjusted inverse change, baseline-adjusted inverse rule, inverse change, inverse multiplicative-additive rule and inverse rule.

8 PDFs and a README; 73 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential and Logarithmic Inverse Models

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Recover cycle number from remaining carbon and retain the discrete operating condition.

Your complete response must include inverse and physical inputs.

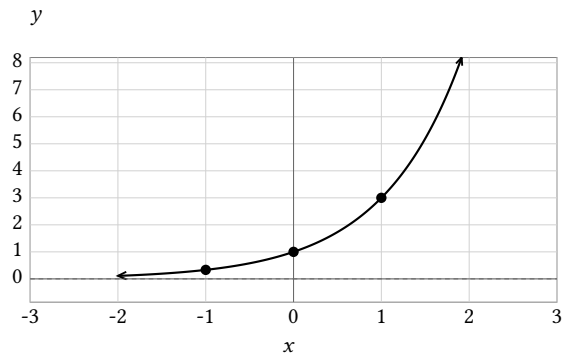
Use the condition whole-number cycles 0 through 10.

Use the model $Q(n) = 100(0.75)^n$.

Use cycles and grams as the stated units.

Bank label: _____

Reflect the marked graph across $y = x$.



-1	$\frac{1}{3}$
0	1
1	3

Bank label: _____

Undo the logarithm and both shifts.

Your complete response must include inverse.

Use the condition $x > -2$.

Consider the statement $g(x) = \log_5(x + 2) - 4$.

Bank label: _____

Recover treatment number from concentration and retain physical attainability.

Your complete response must include inverse and discrete condition.

Use the condition whole-number treatments 0 through 8.

Use the model $C(n) = 60(0.7)^n$.

Use treatments and milligrams per liter as the stated units.

Bank label: _____

AP Precalculus

Exponential and Logarithmic Inverse Models

Name: _____ **Date:** _____ **Class:** _____

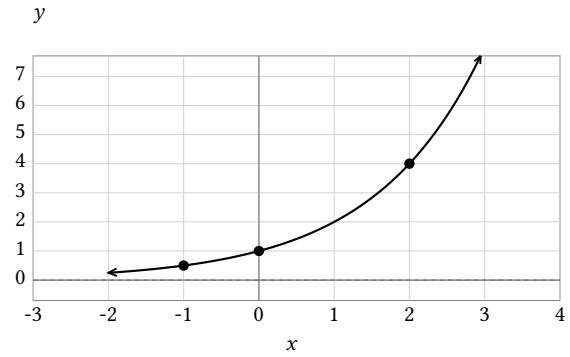
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Swap the coordinates to construct the inverse logarithm table and name its rule.

Given domain/context: exact table from $y = 2^x$

-1	0.5
0	1
1	2

Reflect the marked graph features across $y = x$.



-1	0.5
0	1
2	4

Undo the output shift and scale before taking a logarithm.

Your complete response must include inverse formula and domain.

Use the condition all real inputs.

Consider the statement $f(x) = 2 \cdot 3^x + 1$.

Construct the exponential inverse and state its range.

Your complete response must include inverse formula and exchanged sets.

Use the condition $x > 1$.

Consider the statement $g(x) = \log_2(x - 1) + 3$.

AP Precalculus

Exponential and Logarithmic Inverse Models

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Inverse points $(6, 0), (3, 1)$, and $(\frac{9}{4}, 2)$, with vertical asymptote $x = 2$ and domain $x > 2$.

Recover the scale coordinate from intensity and transfer both operating endpoints.

Your complete response must include inverse and attainable interval.

Use the condition $-2 \le s \le 3$ scale units.

Use the model $I(s) = 5 \cdot 10^{s}$.

Use scale units and watts as the stated units.

Your response: _____

Incoming: $s(I) = \log_{10}(\frac{I}{5})$, for $0.05 \le I \le 5000$ watts.

Translate the forward multiplier into the inverse relation.

Your complete response must include inverse change.

Use the condition exact base-4 process.

The forward rule states that input +1 multiplies output by 4.

Your response: _____

Incoming: $t(A) = 2\log_{4}(\frac{A}{20})$, for $20 \le A \le 1280$ square centimeters.

Recover hours from mold area during the finite observation window.

Your complete response must include inverse and domain.

Use the condition $0 \le h \le 15$ hours.

Use the model $A(h) = 4 \cdot 2^{\frac{h}{3}}$.

Use hours and square centimeters as the stated units.

Your response: _____

Incoming: $f^{-1}(x) = -2 + \log_{3}(\frac{8-x}{7})$, defined for $x < 8$.

Normalize above the baseline before taking a logarithm.

Your complete response must include inverse and domain.

Use the condition all real inputs.

Consider the statement $f(x) = 2 \cdot 6^{x} + 4$.

Your response: _____

Incoming: $t(M) = 2\log_{3}(\frac{M}{25})$, for $25 \le M \le 2025$ microbes.

Recover elapsed time from the covered area.

Your complete response must include inverse and attainable domain.

Use the condition $0 \le t \le 6$ hours.

Use the model $A(t) = 20 \cdot 4^{\frac{t}{2}}$.

Use hours and square centimeters as the stated units.

Your response: _____

Incoming: $g^{-1}(x) = 5 + 2^{4-x}$, with range $x > 5$.

Recover time from a count over the stated experiment.

Your complete response must include inverse and interval.

Use the condition $0 \le t \le 8$ hours.

Use the model $M(t) = 25 \cdot 3^{\frac{t}{2}}$.

Use hours and microbes as the stated units.

Your response: _____

AP Precalculus

Exponential and Logarithmic Inverse Models

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Test the proposed inverse and repair the shift. Identify the wrong shift in the proposed inverse and repair it. Verify your correction by composition.

Your complete response must include correction.

Use the condition real exponential inputs.

Let $f(x) = 2^{x+3}$.

The proposed inverse is $\log_2(x) + 3$.

Assess the formulas and the claimed inverse domain separately. Diagnose the claimed inverse domain separately from the two formulas.

Your complete response must include domain repair.

Use the condition real exponential inputs.

Evaluate the claim g has domain $x > 0$.

Let $f(x) = 2^x + 1$.

Let $g(x) = \log_2(x - 1)$.

Exponential and Logarithmic Inverse Models

Full Worked Solutions - excerpt

Worked example

Problem. Translate the forward multiplier into the inverse relation.

Your complete response must include inverse change.

Use the condition exact base-4 process.

The forward rule states that input $+1$ multiplies output by 4.

Answer. Multiplying an inverse input by 4 increases its recovered output by 1.

Explanation and check.

The former output ratio becomes the inverse input ratio.

Worked example

Problem. Normalize above the baseline before taking a logarithm.

Your complete response must include inverse and domain.

Use the condition all real inputs.

Consider the statement $f(x) = 2 \cdot 6^{x} + 4$.

Answer. $f^{-1}(x) = \log_{6}(\frac{x-4}{2})$, for $x > 4$.

Explanation and check.

Subtract 4 and divide by 2 to expose the positive power.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Logarithmic Function Features

Logarithmic Function Features | APPC-U02-P19

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for boundary and domain, domain and asymptote, domain and boundary side, domain and branch location, domain and graph boundary and domain and side.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Logarithmic Function Features

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the strict positivity of the affine argument to identify the boundary side. The input condition is real x. Report domain and asymptote. Use rule $y = \log_7(9 - 3x)$.

Bank label: _____

Identify where the real logarithmic branch begins. The input condition is real x. Report boundary and domain. Use rule $y = \log_5(x + 6)$.

Bank label: _____

Locate the excluded boundary and the graph's allowed side. The input condition is real x. Report domain and asymptote. Use rule $y = \log_3(x - 8)$.

Bank label: _____

Determine the vertical scale from the table's ratio-to-difference pattern.

Given model family: $y = a\log_5(x)$

x	y
1	0
5	2
25	4

Bank label: _____

AP Precalculus

Logarithmic Function Features

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

State which quantities have the constant ratio and why the outputs are equally spaced.

Given model: $y = \log_5(x - 2)$

x	y
3	0
7	1
27	2

Specify three exact anchor points and the boundary needed for a structured sketch. The input condition is $x > 0$. Report anchors, asymptote, and shape. Use axes unit linear scales. Use rule $y = \log_2(x)$.

Find both attained output extrema on the restricted interval. The input condition is $1 \le x \le 8$. Report attained extrema. Use rule $y = \log_2(x)$.

State the multiplicative input change paired with each unit output change.

Given model: $y = \log_{10}(x)$

x	y
0.1	-1
1	0
10	1

Use argument values $1, 3$, and $\frac{1}{3}$ to specify a structured sketch. The input condition is $x > 1$. Report exact graph features. Use axes unit linear scales. Use rule $y = 2\log_3(x - 1) + 4$.

Solve the argument inequality and identify the graph's side of the boundary. The input condition is real x. Report domain, asymptote, and side. Use rule y=ln(5-x).

AP Precalculus

Logarithmic Function Features

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

State both limiting outputs and note whether the vertical shift changes them. The input condition is $x < 3$. Report end behavior. Use rule $y = \log_2(3 - x) + 4$.

Your response: _____

Incoming: As x approaches 3 from the left, y approaches -infinity; as x heads to -infinity, y approaches infinity.

Use reciprocal e-based inputs to organize a precise sketch. The input condition is $x > 0$. Report exact points and shape. Use axes unit linear scales. Use rule $y = -\ln(x)$.

Your response: _____

Incoming: Plot $(\frac{1}{e}, 1), (1, 0)$, and $(e, -1)$; asymptote $x = 0$; the graph decreases and is concave up.

Track the outer negative sign at both ends of the domain. The input condition is $x > -3$. Report signed limiting behavior. Use rule $y = -\log$ base $\frac{1}{2}$ of $(x + 3)$.

Your response: _____

Incoming: The output tends to -infinity near -3 from the right and to infinity as x grows without bound.

Give the signs of unbounded behavior for a base below one. The input condition is $x > 1$. Report both end limits. Use rule $y = \log$ base $\frac{1}{4}$ of $(x - 1)$.

Your response: _____

Incoming: At x = 1 from the right, y tends to infinity; as x tends to infinity, y tends to -infinity.

Solve the argument inequality containing a fractional coefficient. The input condition is real x. Report domain and boundary. Use rule $y = \log_7(3 - \frac{x}{2})$.

Your response: _____

Incoming: The graph has asymptote x = 6 and domain $x < 6$.

Describe what happens near $x = 4$ and far to the left. The input condition is $x < 4$. Report allowed-side limits. Use rule $y = \ln(8 - 2x)$.

Your response: _____

AP Precalculus

Logarithmic Function Features

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Correct the claimed effect of the output translation. Identify why adding 6 to the output cannot move the vertical asymptote. Correct the claimed boundary. The input condition is real logarithm. Report repaired feature claim. Use claim Adding 6 to $\log_2(x-4)$ moves its vertical asymptote from $x=4$ to $x=10$..

Repair the inference by separating monotonicity from concavity. Locate the invalid implication from decreasing to concave down, then give both correct properties. The input condition is real logarithm. Report corrected curvature claim. Use claim The graph of log base $\frac{1}{3}$ of x is concave down because it is decreasing..

Repair the statement by distinguishing input position from possible outputs. Identify the confusion between the input side and the output set. Repair both statements. The input condition is real logarithm. Report corrected domain and range. Use claim Since ln(4-x) is drawn left of x=4, its range is y<4..

Explain why an asymptote cannot be used as an anchor point here. Diagnose why the proposed asymptote point is not on the graph and give a valid zero anchor. The input condition is real logarithm. Report correct boundary interpretation. Use claim The vertical asymptote $x=2$ supplies the point $(2,0)$ on $y=\log_3(x-2)$..

Logarithmic Function Features

Full Worked Solutions - excerpt

Worked example

Problem. Separate the sign of an input from the sign of a logarithm output.

Answer. The range is all real numbers; for example $\log_2(\frac{1}{2}) = -1$ even though the argument $\frac{1}{2}$ is positive.

Explanation and check.

Argument positivity is a domain condition, not a restriction to positive outputs.

Worked example

Problem. Solve the strict positivity condition for the entire argument. The input condition is real x. Report domain, asymptote, and branch side. Use rule $y = \ln(-3(x-4))$.

Answer. Vertical asymptote x = 4; domain $x < 4$, so the branch is on the left.

Explanation and check.

The condition $-3(x-4) > 0$ reverses to $x < 4$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Logarithmic Transformations and Applications

Logarithmic Transformations and Applications | APPC-U02-P20

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for asymptotes and formulas, formulas and asymptotes, ordered results, resulting rules and relation, two expressions and offset and two formulas and asymptotes.

8 PDFs and a README; 71 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Logarithmic Transformations and Applications

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Describe the two vertical transformations and the unchanged asymptote. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given: $x > 0$

Parent: $f(x) = \log_3(x)$; Rule: $g(x) = -\log_3(x) + 2$

Bank label: _____

Evaluate the model at the given trial count and classify the prediction. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given: $2 \le n \le 40$ trials

Input: 27; Model: $R(n) = 15 + 6 \cdot \log_3(n)$; Output units: successful responses

Bank label: _____

Identify the row that prevents fitting the requested logarithmic regression to the full table. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context/domain: raw predictor must be positive

0	3.1
1	4.0
3	5.2
9	6.4

Bank label: _____

Map the three points and state what happens to the asymptote. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given: $x > 0$

Axes: unit linear scales; Map: $(x, y) \to (x, 2y + 1)$; Parent asymptote: $x = 0$

Context/domain: $x > 0$

1	0
2	1
4	2

Bank label: _____

AP Precalculus

Logarithmic Transformations and Applications

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Describe the transformation from f to g and locate the new vertical asymptote.

Given: $x > 3$

Parent: $f(x) = \log_2(x)$; Rule: $g(x) = \log_2(x - 3) + 4$

Use the documented logarithmic fit to predict the held-out score and compute observed minus predicted.

Given: $x > 0$ hours

Documented fit: $y = 2 + 1.5 \cdot \ln(x)$; Held out row: $(4, 4.2)$; Output units: score points; Rounding: nearest hundredth

Context/domain: $x > 0$ hours

1	2.1
2	3.0
3	3.7
6	4.6

Construct a model that is zero at $x = 2$ and gains 3 whenever input doubles.

Map the three anchors and the vertical asymptote.

Given: $x > 2$

Axes: unit linear scales; Map: $(x, y) \to (x + 2, y - 1)$; Parent asymptote: $x = 0$

Context/domain: $x > 2$

1	0
2	1
0.5	−1

AP Precalculus

Logarithmic Transformations and Applications

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Determine whether the model supplies the requested prediction. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $q > 10$ minutes, calibrated for $12 \le q \le 50$

Input: 10; Model: $W(q) = 7+\ln(q-10)$; Output units: watts

Your response: _____

Incoming: A gives $\log_3(3(x-6))$ with asymptote 6; B gives $\log_3(3x-6)$ with asymptote 2.

Map the full defining set to the translated graph. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $x > 4$

Axes: unit linear scales; Map: $(x, y) \to (x+4, y-2)$; Parent asymptote: $x = 0$

Context/domain: $x > 4$

1	0
3	1
$\frac{1}{3}$	-1

Your response: _____

Incoming: k = 6 and $a = -4$, so $g(x) = -4\cdot\log_2(-x-3)+6$; the branch increases.

Predict the held-out mass and report its residual with units. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $x > 0$ days

Documented fit: $y = 12 + 2.5\ln(x)$; Held out row: $(3, 14.5)$; Output units: grams; Rounding: nearest tenth

Context/domain: $x > 0$ days

1	12.1
2	13.8
5	15.9
9	17.4

Your response: _____

Incoming: Prediction about $5.8g$; residual about $0.3g$, so the model underpredicts.

Write a logarithmic rule satisfying the zero and fivefold-change conditions. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $x > 0$

Input ratio: 5; Normalization: use base 5; Output increment: 2; Zero input: 3

Your response: _____

AP Precalculus

Logarithmic Transformations and Applications

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Decide whether the requested prediction exists and explain why. Student work: No real prediction exists at $x = 10$ because the logarithm argument is 0; the model requires $x \geq 10$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

error hunt

Given: $x > 10$ seconds, calibrated for $12 \leq x \leq 100$

Input: 10; Model: $V(x) = 8 + 3 \cdot \log_2(x - 10)$; Output units: meters per second

---

Decide whether a fitted residual can be computed for the held-out row. Student work/claim: The displayed fitted equation is an exact identity for every input, including values outside the observed data window. Locate the unsupported exact-fit/extrapolation step, then state the properly qualified model and justify the qualification from the displayed data.

error hunt

Given: $x > 0$ hours

Documented fit: $y = 6 + 1.4\ln(x)$; Held out row: $(0, 5.5)$; Output units: points

Context/domain: $x > 0$ hours

1	6.1
2	7.0
5	8.3

Logarithmic Transformations and Applications

Full Worked Solutions - excerpt

Worked example

Problem. Build a log curve through $(6, 0)$ whose output drops 4 whenever input triples.

Answer. A suitable rule is $y = -4 \cdot \log_3\left(\frac{x}{6}\right)$.

Explanation and check.

The normalized argument is 1 at $x = 6$; tripling x raises the logarithm by 1, so scale -4 creates the stated drop.

Worked example

Problem. Construct a model that is zero at $x = 2$ and gains 3 whenever input doubles.

Answer. One normalized model is $y = 3 \cdot \log_2\left(\frac{x}{2}\right)$.

Explanation and check.

The ratio $\frac{x}{2}$ anchors the zero, and $\log_2$ gains 1 per doubling, so the output scale is 3.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Log Properties and Change of Base

Log Properties and Change of Base | APPC-U02-P21

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for value to nearest thousandth, correction and denominator issue, valid subset and failing branch, correct interval, correct form and domain and source domain and extra condensed intervals.

8 PDFs and a README; 60 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Log Properties and Change of Base

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Rewrite the logarithm using natural logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_2(7)$. Bank label: _____	Combine the two terms and carry along the original domain. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_5(u) + \log_5(v)$. Bank label: _____
Separate the positive factors into individual base-five logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_5(7rs)$. Bank label: _____	Convert the expression to a quotient of common logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_5(11)$. Bank label: _____
Combine the difference into one base-eight logarithm and retain both source restrictions. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_8(p) - \log_8(q)$. Bank label: _____	Expand the product into separate logarithms of its constant and variable factors. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\ln(11at)$. Bank label: _____
Represent the value using only natural logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_6(13)$. Bank label: _____	Correct the proposed power-law rewrite so it is equivalent on the full domain of the original expression. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The candidate expression is $2\ln(x)$. Start with $\ln(x^2)$. Bank label: _____

AP Precalculus

Log Properties and Change of Base

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Expand the logarithm completely, keeping the positive constant inside its own logarithm. Start with $\log_2(3xy)$.	Condense to one logarithm and retain the domain of the displayed expression. Start with $\log_3(x) + \log_3(y)$.
Decide whether the claim is an identity, and justify the decision with one exact counterexample if it is false. Evaluate the claim $\ln(u + v) = \ln(u) + \ln(v)$.	Identify the first invalid line and state what valid law could replace the mistaken idea. The student writes: $\ln(a + b) = \ln(a) + \ln(b) = \ln(ab)$.
Write the expression as a sum and difference of natural logarithms with no powers inside them. Start with $\ln(a^2 \cdot \frac{\sqrt{b}}{c})$.	Produce an equivalent single logarithm and report the original restrictions. Start with $2\ln(a) - \ln(b)$.

AP Precalculus

Log Properties and Change of Base

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Expand the quotient and simplify the logarithm of the numerical factor. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Start with $\log_6(\frac{x}{36yz})$.

Your response: _____

Incoming: Yes; the equality holds for every allowed a and x.

Combine the shifted logarithms and derive the allowed x-values from the separate terms. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Start with $\log_7(x+2) + \log_7(x-4)$.

Your response: _____

Incoming: $k = \frac{1}{2}$

Express the function using a base-three logarithm and simplify the coefficient. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use logarithms with base 3.

Start with $\log_9(y)$.

Your response: _____

Incoming: 1.661

Pull the exponent in front without losing inputs less than the center. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Start with $\log_7((z-1)^2)$.

Your response: _____

Incoming: $6\log_5|2x-1|$, with $x \neq \frac{1}{2}$

Test the proposed quotient rule and disprove it exactly if it is not an identity. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Evaluate the claim $\log_4(\frac{u}{v}) = \frac{\log_4(u)}{\log_4(v)}$.

Your response: _____

Incoming: $\log_3(m^4 \cdot \frac{\sqrt[3]{p}}{n^2})$, with $m > 0, n > 0, p > 0$

Find the scaling constant that makes the two logarithmic functions identical. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the identity $K\log_3(x) = \log_{81}(x)$.

Your response: _____

AP Precalculus

Log Properties and Change of Base

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Evaluate the identity claim and, if needed, refute it using powers of 2. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

Evaluate the claim $\log_2(uv) = \log_2(u)\log_2(v)$.

Use base 2 as the change-of-base reference to find the exact value. Student work: $-\frac{3}{1}$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

The reference logarithm base is 2.

Start with $\log_{\frac{1}{4}}(8)$.

Log Properties and Change of Base

Full Worked Solutions - excerpt

Worked example

Problem. Expand the logarithm completely, keeping the positive constant inside its own logarithm.

Start with $\log_2(3xy)$.

Answer. $\log_2(3) + \log_2(x) + \log_2(y)$

Explanation and check.

The argument is a product of the three positive factors $3, x$, and y, so the product law separates them.

Worked example

Problem. Decide whether the claim is an identity, and justify the decision with one exact counterexample if it is false.

Evaluate the claim $\ln(u + v) = \ln(u) + \ln(v)$.

Answer. False; $u = v = 1$ gives $\ln(2)$ on the left and 0 on the right.

Explanation and check.

Logarithms convert products, not sums, into sums; the positive pair $1, 1$ directly contradicts the claim.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Solving Exponential Equations

Solving Exponential Equations | APPC-U02-P22

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for real solution set, time-domain conclusion, time or impossibility, solution set, all real solutions and finite none or all.

8 PDFs and a README; 64 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Solving Exponential Equations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Rewrite the reciprocal as a power of 3 and equate exponents. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $3^{x-2} = \frac{1}{27}$. Bank label: _____	Solve the exponential equation exactly using logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $3^x = 10$. Bank label: _____
Use the indicated positive substitution and recover every admissible x-value. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $4^x - 5 \cdot 2^x + 4 = 0$; substitution: $u = 2^x$. Bank label: _____	Find the unique real input where the two exponential expressions are equal. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $2 \cdot 3^x = 6 \cdot 2^x$. Bank label: _____
Rewrite the reciprocal and the left side as powers of 2. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $4^{x-1} = \frac{1}{8}$. Bank label: _____	Give the exact solution using natural logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $7^x = 13$. Bank label: _____
Factor after substitution and invert both positive values of u. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $9^x - 5 \cdot 3^x + 4 = 0$; substitution: $u = 3^x$. Bank label: _____	Divide to obtain one base-two power and solve the crossover exactly. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $2 \cdot 4^x = 16 \cdot 2^x$. Bank label: _____

AP Precalculus

Solving Exponential Equations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Choose a so that $x = 2$ is a solution, then decide whether that equation has any other real solution. Context/domain: $a > 0$ and real x. equation: $a \cdot 3^x = 45$; required solution: $x = 2$.	Identify the first invalid logarithm step and solve the original equation correctly. Context/domain: real x. equation: $2^x + 3 = 11$; student work: $[\ln(2^x) + \ln(3) = \ln(11), x = \frac{\ln(11)}{\ln(2)+\ln(3)}]$.
Solve without dividing by an expression that could be zero, and classify the solution set. Context/domain: real x. equation: $5 \cdot 2^x = 7 \cdot 2^x$.	Choose k from the prescribed solution, then inspect whether $x = 3$ remains the only solution. Context/domain: $k > 0$ and real x. equation: $k \cdot 2^x = 4 \cdot 2^{x+1}$; required solution: $x = 3$.
Find the first point at which the student turns an inadmissible substitution root into an x-value. Context/domain: real x. equation: $4^x + 3 \cdot 2^x - 4 = 0$; student work: $[u = 2^x, (u-1)(u+4) = 0, x = 0$ or $x = 2]$.	Solve the level equation conceptually and determine whether its time belongs to the model's operating domain. Context/domain: $t \geq 0$ years. model: $A(t) = 200(1.1)^t$; target: 150 units.

AP Precalculus

Solving Exponential Equations

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Solve exactly without approximating the irrational exponent. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $11^x = 2$.

Your response: _____

Incoming: $x \approx 1.771$, uniquely on $[-2, 3]$

Use the positive substitution and invert both factored roots. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $16^x - 5 \cdot 4^x + 4 = 0$; substitution: $u = 4^x$.

Your response: _____

Incoming: n = 4

Express every factor as a power of 3 and solve. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $3 \cdot 9^x = 81 \cdot 3^x$.

Your response: _____

Incoming: x = 4

Use the unique exponent that produces 1 for base 5, then solve the resulting algebraic equation. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $5^{x^2-3x} = 1$.

Your response: _____

Incoming: x = $\frac{2}{3}$

Isolate the positive exponential quantity and state the exact real solution. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $7 \cdot 3^{2x} - 1 = 0$.

Your response: _____

Incoming: x = 1 or $x = \frac{\ln(3)}{\ln(2)}$

Form one exponential ratio and solve for the crossover input. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $3 \cdot 5^x = 12 \cdot 2^x$.

Your response: _____

AP Precalculus

Solving Exponential Equations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Determine whether the declining model reaches the higher target after $t = 0$. Student work: No nonnegative time; 360 would require extending the growth model backward. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: $t \geq 0$ weeks. model: $A(t) = 300(0.9)^t$; target: 360 units.

Apply the repeated-exponential substitution and verify both recovered exponents. Student work: $x = 1$ or $x = 4$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: real x. equation: $9^x - 30 \cdot 3^x + 81 = 0$; substitution: $u = 3^x$.

Choose a to make $x = 3$ a solution and check whether the solution is unique. Student work: $a = 3$; strict increase makes $x = 3$ not unique Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: $a > 0$. equation: $a \cdot 4^{x-1} = 48$; required solution: $x = 3$.

Evaluate the x-independent exponential and decide whether the equation is an identity or contradiction. Student work: No x works because the left side is fixed at 171, not 169. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: real x. equation: $7 \cdot 5^{0x+2} - 4 = 170$.

Solving Exponential Equations

Full Worked Solutions - excerpt

Worked example

Problem. Use base 3, solve the resulting quadratic, and retain both real roots. Context/domain: real x. equation: $9^{x^2} = 3^{5x+6}$.

Answer. x = $\frac{5-\sqrt{73}}{4}$ or $x = \frac{5+\sqrt{73}}{4}$

Explanation and check.

Equating exponents gives $2x^2 - 5x - 6 = 0$, whose discriminant is 73.

Worked example

Problem. Express every factor as a power of 3 and solve. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $3 \cdot 9^x = 81 \cdot 3^x$.

Answer. x = 3 from $2x + 1 = x + 4$

Explanation and check.

The sides become 3^{2x+1} and 3^{x+4}, whose exponents agree only at $x = 3$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Solving Logarithmic Equations

Solving Logarithmic Equations | APPC-U02-P23

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for complete solution set, solution interval, solution intervals, solution set, correct root and method error and correct solution set and domain error.

8 PDFs and a README; 94 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Solving Logarithmic Equations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the intensity ratio represented by the logarithmic scale target. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Your complete response must include positive intensity ratio. Use the condition $\frac{I}{I_0} > 0$. Use the model $L = 10\log(\frac{I}{I_0})$. The target is $L = 30$ decibels. Bank label: _____	Solve the quadratic in the logarithmic quantity and exponentiate each real u-root. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Your complete response must include positive x-solutions. Use the condition $x > 0$. Solve the equation $(\log_2(x))^2 - 3\log_2(x) + 2 = 0$. Use the substitution $u = \log_2(x)$. Bank label: _____
Solve by exponentiating and verify the logarithm's original argument condition. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Your complete response must include real solution set. Use the condition real x. Solve the equation $\log_2(x-1) = 3$. Bank label: _____	Recover the concentration represented by the given pH target. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Your complete response must include positive hydrogen-ion concentration. Use the condition $[H+] > 0$ mol/L. Use the model pH=-log($[H+]$). The target is pH=6.5. Bank label: _____

AP Precalculus

Solving Logarithmic Equations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Check both algebraic candidates in the two original logarithm arguments.

Your complete response must include valid solution set and rejection reason.

Use the condition real x.

Solve the equation $\log_2(x-3)+\log_2(x-5)=3$.

The student proposes the candidate values 17.

Condense the logs, solve the quadratic, and enforce both starting restrictions.

Your complete response must include valid roots.

Use the condition real x.

Solve the equation $\log_2(x-1)+\log_2(x+1)=3$.

Exponentiate, solve the quadratic, and test both roots in the original argument.

Your complete response must include all real roots.

Use the condition real x.

Solve the equation $\log_3(x^2-5)=2$.

Use the supplied maximum fact to determine whether the curves ever meet.

Your complete response must include solution set.

Use the condition $x>0$.

Solve the equation $\ln(x)=x$.

Use the supplied evidence: For $x>0, \ln(x)-x$ has maximum -1 at $x=1$.

Recover the intensity ratio, then decide whether the calibrated model supports that target.

Your complete response must include ratio and calibration status.

Use the condition $1 \le \frac{I}{I_0} \le 10^8$ calibration range.

Use the model $L=10\log(\frac{I}{I_0})$.

The target is 100 decibels.

Explain why exponentiating to obtain 9 does not validate the student's answer in the real logarithm setting.

Your complete response must include correct conclusion.

Use the condition real x.

Solve the equation $\log_{-3}(x)=2$.

The student reports $x=9$.

AP Precalculus

Solving Logarithmic Equations

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: x = $2 + \sqrt{3}$ only

Factor the quadratic in u and invert the natural logarithm for both roots. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include positive x-solutions.

Use the condition $x > 0$.

Solve the equation $(\ln(x))^2 - 5\ln(x) + 6 = 0$.

Use the substitution $u = \ln(x)$.

Your response: _____

Incoming: x = 10

Exponentiate the equation and verify positivity of the original argument. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include real root.

Use the condition real x.

Solve the equation $\log_7(2x - 5) = 2$.

Your response: _____

Incoming: x = e^2 or $x = e^3$

Use both roots of the difference of squares and exponentiate each one. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include positive x-solutions.

Use the condition $x > 0$.

Solve the equation $(\log_4(x))^2 - 1 = 0$.

Use the substitution $u = \log_4(x)$.

Your response: _____

Incoming: x = e is the only intersection

Determine the input count that reaches the stated logarithmic capacity target. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include positive device count.

Use the condition $n > 0$ devices.

Use the model $C(n) = 12\log_2(n)$.

The target is $C = 48$ units.

Your response: _____

Incoming: $x \approx 2.763$, the sole intersection for $x > 1$

Intersect the argument restrictions and then equate the arguments. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include valid root.

Use the condition real x.

Solve the equation $\ln(2 - x) = \ln(x + 5)$.

Your response: _____

Incoming: x = $2 + \sqrt{10}$ only

Condense the difference and retain the common positivity restriction. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include valid real root.

Use the condition real x.

Solve the equation $\ln(x + 2) - \ln(x - 4) = \ln(3)$.

Your response: _____

AP Precalculus

Solving Logarithmic Equations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use injectivity and check the result against both affine positivity conditions. Student work: $x = \frac{9}{3}$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Your complete response must include valid rational root.

Use the condition real x.

Solve the equation $\log_7(3x + 2) = \log_7(11 - x)$.

Factor in u and remember that both negative logarithmic values produce positive inputs. Student work: $x = e^{+1}$ or $x = e^{-5}$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Your complete response must include positive x-values.

Use the condition $x > 0$.

Solve the equation $(\ln(x))^2 + 6\ln(x) + 5 = 0$.

Use the substitution $u = \ln(x)$.

Solving Logarithmic Equations

Full Worked Solutions - excerpt

Worked example

Problem. Solve the condensed quadratic and screen its roots against each source logarithm.

Your complete response must include valid solution set.

Use the condition real x.

Solve the equation $\log_5(x) + \log_5(x + 4) = 1$.

Answer. x = 1; discard $x = -5$

Explanation and check.

The source requires $x > 0$. Since $x(x + 4) = 5$ has roots 1 and -5, only 1 remains.

Worked example

Problem. Equate the same-base arguments, then check positivity at each resulting root.

Your complete response must include all valid roots.

Use the condition real x.

Solve the equation $\log_3(x^2 + 2) = \log_3(5x - 4)$.

Answer. x = 2 or $x = 3$; both arguments are positive

Explanation and check.

The equation $x^2 + 2 = 5x - 4$ factors as $(x - 2)(x - 3) = 0$, and both roots make $5x - 4$ positive.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential and Logarithmic Inequalities

Exponential and Logarithmic Inequalities | APPC-U02-P24

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for case intervals, solution by k region, solution intervals by base region, solutions by m sign, solutions by p sign and case solutions.

8 PDFs and a README; 69 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exponential and Logarithmic Inequalities

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Combine the logarithm domain with the transformed inequality. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: real x or the stated contextual schedule.

Solve the inequality $\log_2(x-1) > 3$.

Bank label: _____

Solve the quadratic inequality in u and map its positive interval back to x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: real x or the stated contextual schedule.

Solve the inequality $4^x - 5 \cdot 2^x + 4 < 0$.

The substitution is $u = 2^x > 0$.

Bank label: _____

Isolate the positive exponential and solve the strict threshold. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: real x or the stated contextual schedule.

Solve the inequality $2^x + 1 > 9$.

Bank label: _____

Translate the base-seven comparison directly into exponent order. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: real x.

Solve the inequality $7^{x+2} \le 7^5$.

Bank label: _____

AP Precalculus

Exponential and Logarithmic Inequalities

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Use the intersection as a boundary and identify where the exponential is at or below the horizontal line.

Context: real x or the stated contextual schedule.

the horizontal axis measures input; the vertical axis measures output; the displayed window has $-2 \le x \le 4$ and $0 \le y \le 25$.

The comparison is $e^x \le e^2$.

Compare the curves $(y = e^x, y = e^2)$.

The intersection is at $x = 2$; this input is included.

Separate increasing and decreasing valid bases before comparing exponents.

Context: real x or the stated contextual schedule.

Solve the inequality $a^x > a^2$.

Assume $a > 0$ and $a \neq 1$.

Check the order direction for the reciprocal base.

Context: real x or the stated contextual schedule.

Solve the inequality $(\frac{1}{2})^x > (\frac{1}{2})^3$.

Evaluate this student answer: $x > 3$.

Separate positive, negative, and zero scale factors before dividing.

Context: real x or the stated contextual schedule.

Solve the inequality $k \cdot 2^x \le 8k$.

Assume real k.

First reduce the two positivity conditions, then compare the arguments and intersect the sign solution.

Context: real x or the stated contextual schedule.

Solve the inequality $\ln(x^2 - 4) > \ln(3x)$.

Partition the parameter by monotonicity and retain the positive argument domain in both cases.

Context: real x or the stated contextual schedule.

Solve the inequality $\log_a(x) > 1$.

Assume $a > 0$ and $a \neq 1$.

AP Precalculus

Exponential and Logarithmic Inequalities

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Combine positivity with the nonpositive logarithm condition. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x.

Solve the inequality $\ln(x+5) \le 0$.

Your response: _____

Incoming: Week 4

Compare exponents of the increasing natural base. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x.

Solve the inequality $e^{2x-1} > e^{3}$.

Your response: _____

Incoming: $[2, 8]$ within the displayed domain

Remove the factor 2 before using the base-five boundary. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x.

Solve the inequality $2 \cdot 5^{x} + 3 > 13$.

Your response: _____

Incoming: $\log_{\frac{1}{2}}(3) < x < -1$

Separate coefficient signs before division. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x and real s.

Solve the inequality $s \cdot 2^{x} < 4s$.

Your response: _____

AP Precalculus

Exponential and Logarithmic Inequalities

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use the crossing and graph order on either side. Student work: $(-1, \infty)$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Context: real x.

Use standard Cartesian axes; the horizontal axis measures input; the vertical axis measures function value.

The comparison is $e^x > 1$.

Compare the curves $(y = e^x, y = 1)$.

The intersection is at $x = 0$; this input is excluded.

The displayed window has $-3 \le x \le 3$ and $0 \le y \le 8$.

Use strict positivity of the exponential to classify the threshold. Student work: $x = 0$ is a solution because both sides of $2 - 5^x \ge 2$ are then nonnegative. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Exponential and Logarithmic Inequalities

Full Worked Solutions - excerpt

Worked example

Problem. Use strict positivity of the exponential to classify the threshold. Student work: $x = 0$ is a solution because both sides of $2 - 5^x \geq 2$ are then nonnegative. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Answer. No real solution

Explanation and check.

The inequality would require $-5^x \geq 0$, impossible because $5^x > 0$.

Worked example

Problem. Reverse when removing the negative factor, then intersect with strict argument positivity.

Context: real x.

Solve the inequality $-\log_2(x^2 - 4) \geq -1$.

Answer. $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

Explanation and check.

The conditions are $x^2 > 4$ and $x^2 - 4 \leq 2$, so $4 < x^2 \leq 6$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Linearizing Exponential Data

Linearizing Exponential Data | APPC-U02-P25

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for corrected line, least integer threshold, ratio and observed value, slope evolution, tail behavior and values and explanation.

8 PDFs and a README; 73 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Linearizing Exponential Data

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Linearize the decay model in base 3. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use $y = 9(\frac{1}{3})^x$. Use the transformation $Y = \log_3(y)$.

Bank label: _____

Read the original output at the marked height $x = 2$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use these axis scales: scale logarithmic y; linear x; x input; y original y on base-10 logarithmic scale. The marked point has $x2$; y label 100. The labeled levels are $1, 10, 100, 1000$.

Bank label: _____

Use the fitted table to predict the original output at $x = 4$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context/domain: data

x	fitted Y=ln(y)
0	0
2	1
4	2

Bank label: _____

Place the decay observations on a base-2 transformed table. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use logarithm base 2. The raw observations are $(0, 8), (1, 4)$, and $(2, 2)$; only these observations are asserted. Construct the transformed output column yourself.

Context/domain: data

x	raw y
0	8
1	4
2	2

Bank label: _____

AP Precalculus

Linearizing Exponential Data

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Diagnose the parameter error and state the correct factor. The given line is $Y = 1 + 0.3x$. A student claims: The raw growth factor is 0.3. Use the transformation $Y = \log_{10}(y)$.

Write the transformed line and identify its intercept and slope. Use $y = 4(2^x)$. Use the transformation $Y = \log_2(y)$.

Correct the student's reading of the plotted point. Use these axis scales: scale linear-linear transformed plot; x input; $yY = \log_2(y)$. The point are $3, 5$. A student reads $y = 5$.

Express the semi-log line exactly and interpret the two coefficients. Use $y = 5(100^x)$. Use the transformation $Y = \log_{10}(y)$.

Compare the initial values and one-input growth factors. Use these axis scales: scale linear-linear transformed plot; x input unit; $yY = \log_{10}(y)$. The lines are (model P; rule $Y = 1 + 0.4x$), (model Q; rule $Y = 2 + 0.4x$).

Map all three raw observations to the base-4 plot.

Context/domain: data

0	2
1	8
3	128

AP Precalculus

Linearizing Exponential Data

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Recover the positive raw y-value represented by $Y = -2$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use these axis scales: scale linear in Y; x input; yY =$\log_{10}(y)$. The point are $-1, -2$.

Your response: _____

Incoming: Y-values $\log_2(16)$=4, $\log_2(20)$, $\log_2(28)$; their increments are unequal.

Predict y at $x = 2$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The fitted relation is $Y = 1 + 0.5x$ where Y =$\log_{10}(y)$. Evaluate at input 2. The observations cover x in $0, 4$.

Your response: _____

Incoming: x = 1 and common $y = 27$

State the original output at the plotted point $(3, 5)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use these axis scales: scale linear in Y; x stage; yY =$\log_2(y)$. The point are $3, 5$.

Your response: _____

Incoming: Y = 4.5 and $y = 16\sqrt{2}$

Convert the fitted height at $x = 1$ to original units. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The fitted relation is $Y = 3$-x where Y =$\log_2(y)$. Evaluate at input 1. The observations cover x in $0, 3$.

Your response: _____

Incoming: The slope is initially negative but approaches 0 as Y approaches $\log_3(9)$=2.

What multiplicative change is encoded by moving one labeled vertical interval from 1 to 10? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: _____

Incoming: y = $100(\frac{1}{\sqrt{5}})^x$

Translate the half-unit rise into a ratio of original outputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use these axis scales: scale linear in Y; x input; yY =$\log_4(y)$. The vertical displacement is 0.5.

Your response: _____

AP Precalculus

Linearizing Exponential Data

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Judge the student's claim that equal-looking log increments prove $y = 10(1.5^x)$ exactly. Student work/claim: The displayed fitted equation is an exact identity for every input, including values outside the observed data window. Locate the unsupported exact-fit/extrapolation step, then state the properly qualified model and justify the qualification from the displayed data.

Context/domain: data

x	raw y	Y=log_base(y)
0	10.0	1.0
1	15.1	1.179
2	22.4	1.35
3	34.0	1.531

Construct every admissible transformed row and diagnose the remaining observation. Student work: $(0, 1.079)$ and $(3, 2.079)$ are admissible; $x = 1$ can be plotted because $y = 0$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Context/domain: data

x	raw y	Y=log_base(y)
0	12	1.079
1	0	_____
3	120	2.079

Linearizing Exponential Data

Full Worked Solutions - excerpt

Worked example

Problem. Write the transformed line and identify its intercept and slope. Use $y = 4(2^x)$. Use the transformation $Y = \log_2(y)$.

Answer. Y = $2 + x$; intercept 2 and slope 1

Explanation and check.

$\log_2(4) = 2$ and $\log_2(2) = 1$, so logarithms turn the product into $2 + x$.

Worked example

Problem. What multiplicative change is encoded by moving one labeled vertical interval from 1 to 10? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Answer. a factor of 10

Explanation and check.

Consecutive decade levels differ by a ratio of 10 even though their numerical differences are not constant.