Sets & Steps
Math Practice & Worked Solutions

Precalculus

Matrices, Transformations & Transition Models | 7-Topic Precalculus Bundle

7-topic bundle | 672 problems | Four activity formats

Included topics

The following pages list all 7 included resources in the suggested order, with a link to each product. Each resource retains its complete preview.

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

56 PDFs and 7 READMEs; 562 buyer PDF pages across the 7 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Sets & Steps
Math Practice & Worked Solutions

Precalculus

Included topics

7-topic bundle | 672 problems | Four activity formats

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7 included resources | Complete resource previews follow this guide.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Matrix Operations and Multiplication

Matrix Operations and Multiplication | APPC-U04-P14

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for constraint comparison and verdict, numeric and label judgments, constraint and uniqueness conclusion, compatibility verdict and repair, valid and invalid labels with reason and intermediate and final sizes.

8 PDFs and a README; 66 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Matrix Operations and Multiplication

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

State the dimensions of A and give a_{23}.

matrix =

4	0	7
−2	5	−1

Bank label: ______

Compute Ax and confirm the result's dimensions. A: see the labelled matrix A; domain: matrix-vector multiplication; x: $[4, 5]$.

A =

2	−1
0	3

Bank label: ______

Recover x and y from both product entries and explain why neither condition is redundant. B: see the labelled matrix B; domain: row vector times invertible-looking coefficient matrix; product: $[5, 1]$; row: $[x, y]$.

B =

1	2
1	−1

Bank label: ______

Use a row-column product to find the meal's total calories. calories per serving: $[80, 120, 50]$; domain: meal calorie total; foods: [fruit, grain, milk]; servings: $[2, 1, 3]$.

Bank label: ______

AP Precalculus

Matrix Operations and Multiplication

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compute AB and BA and identify the entry that disproves commutativity. A: see the labelled matrix A; B: see the labelled matrix B; domain: diagonal matrix and shear.

A =

2	0
0	1

B =

1	3
0	1

Determine all possible x and y. Explain why the two output entries do not give two independent constraints. B: see the labelled matrix B; domain: row vector product with repeated columns; product: $[5, 5]$; row: $[x, y]$.

B =

1	1
1	1

Compute AB and state its dimensions. A: see the labelled matrix A; B: see the labelled matrix B; domain: row by rectangular matrix.

A =

2	-1

B =

3	0	4
5	2	-2

Is uv defined, and what size matrix results? domain: column times row vectors; u size: 4×1; v size: 1×2.

AP Precalculus

Matrix Operations and Multiplication

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(\text{AB})_{14}=9$.

Find x so that the indicated row-column product is 6. column: $[3, 1, 4]$; domain: one selected dot product; product entry: 6; row: $[2, x, -1]$.

Your response: ________________

Incoming: The product is $\begin{pmatrix} 8 & 9 & 4 \\ 6 & 13 & 8 \end{pmatrix}$. Rows are Site A and Site B, columns are wood, metal, and fabric, and the shared sum runs over basic and advanced kits.

Multiply the machine-hours matrix by the energy-rate column. Interpret both outputs and explain the unit cancellation. domain: shift energy use; kilowatt hours per hour: $[6, 2, 5]$; machine columns: [press, lathe, dryer]; shift rows: Evening: $[5, 0, 3]$; Morning: $[2, 4, 1]$.

Your response: ________________

Incoming: BA = CA=$\begin{pmatrix} 2 & 0 \\ 4 & 0 \end{pmatrix}$, but $B \neq C$. Right cancellation fails because multiplying by A erases the second column, exactly where B and C differ.

Find $(\text{AB})_{23}$ by recognizing the selector row. A row 2: $[0, 1, 0]$; B column 3: $[-4, 7, 2]$; domain: standard basis row selection; target: $(\text{AB})_{2}3$.

Your response: ________________

Incoming: $AA^{\top}$ is 2×2 and $A^{\top}A$ is 3×3, so they cannot be equal as matrices.

State the dimensions of c and give c_{31}.

matrix =

-4
7
2

Your response: ________________

Incoming: The product is $\begin{pmatrix} 0 \end{pmatrix}$.

Multiply the unit-count matrix by the mass column and interpret the two outputs. domain: factory package mass; factory rows: East: $[5, 2]$; West: $[1, 6]$; mass per unit: $[3, 8]$; product columns: [box, crate].

Your response: ________________

Incoming: Morning uses 25 kWh and Evening uses 45 kWh. Each dot product sums machine-hours times kWh per machine-hour over the shared machine labels.

Find $(\text{AB})_{12}$. A row 1: $[5, 0]$; B column 2: $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(\text{AB})_{1}2$.

Your response: ________________

AP Precalculus

Matrix Operations and Multiplication

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Compute both orders and use them to assess the scalar-style claim that a zero product forces one matrix factor to be zero.

A =

1	−1
1	−1

B =

1	1
1	1

The inner labels match and the numeric dimensions conform. Does quantity-in-kilograms times dollars-per-item yield a valid total cost? State a minimal unit repair.

Matrix Operations and Multiplication

Full Worked Solutions - excerpt

Worked example

Problem. Compute the row-by-column matrix product. column: $[7, -2, 5]$; domain: zero row times a column; row: $[0, 0, 0]$.

Answer. The product is (0).

Explanation and check.

All three row-column contributions are zero.

Worked example

Problem. Find $(AB)_{12}$. A row $1:$ $[5, 0]$; B column $2:$ $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(AB)_{1}2$.

Answer. $(AB)_{12} = -10$.

Explanation and check.

The dot product is $5(-2) + 0 \cdot 3$, so only the first contribution remains.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Determinants and Area

Determinants and Area | APPC-U04-P15

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for dot product, determinant, area, singular set and dependence explanation, singular set, determinant and conclusion, empty real solution set with reason and all parameter values and dependence.

8 PDFs and a README; 55 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Determinants and Area

Name: ______ **Date:** ______ **Class:** ______

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the lower-triangular structure to evaluate the determinant without an unnecessary cross-product calculation.

Required response

-3	0
4	2

Bank label: ______

Use the triangular structure to find the determinant of the matrix.

Required response

7	0
-2	-3

Bank label: ______

Evaluate the determinant.

Required response

7	0
0	1

Bank label: ______

Read the determinant from the triangular form.

Required response

1	5
0	3

Bank label: ______

Find the determinant of the coordinate-swap matrix.

Required response

0	1
1	0

Bank label: ______

Find the determinant after the row swap.

Apply this change: interchange row 1 and row 2. The original determinant is 13.

Bank label: ______

Without reconstructing the entries, find the determinant after the columns are interchanged.

Apply this change: interchange the two columns. The original determinant is -5.

Bank label: ______

Find the determinant after the stated row scaling.

Apply this change: multiply row 2 by $\frac{1}{5}$. The original determinant is 20.

Bank label: ______

AP Precalculus

Determinants and Area

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Find the signed and ordinary areas of the spanned parallelogram.

The first column is $(0, 1)$. The second column is $(-3, 0)$.

Find the signed and ordinary areas of the column parallelogram.

The first column is $(2, 0)$. The second column is $(0, 5)$.

Compute the signed and ordinary areas and classify the column order.

The first column is $(-2, 0)$. The second column is $(0, -7)$.

The second column is a90-degree counterclockwise rotation of the first. Use that structure to determine the signed and ordinary areas of their parallelogram.

The first column is $(3, -1)$. The second column is $(1, 3)$.

Find the determinant and ordinary area, then state the exact column relation.

The first column is $(-1, -2)$. The second column is $(-3, -6)$.

Find the parallelogram's ordinary area and classify the ordered-column orientation.

The first column is $(4, 1)$. The second column is $(1, 3)$. The vertices are $(0, 0)$, $(4, 1)$, $(1, 3)$, $(5, 4)$.

AP Precalculus

Determinants and Area

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The image area is 15 square units, and orientation is preserved.

Verify that the matrices have the same determinant, then show from their columns that they do not define the same map.

Required response

2	0
0	2

1	3
0	4

Your response: ______________

START

Find every real t for which the matrix is singular and explain what happens to its columns then.

Required response

t	2
3	t

Your response: ______________

Incoming: It is singular for every real t because the second column is always twice the first, and the determinant $2t - 2t$ is identically zero.

Verify the common determinant and give a column-based reason the maps differ.

Required response

2	0
0	1

1	1
-1	1

Your response: ______________

Incoming: The determinant is $2(\frac{3}{4}) - 1(-1) = \frac{5}{2}$. Therefore the original area is $\frac{35}{\frac{5}{2}} = 14$.

Find the image area and state whether orientation is preserved.

The determinant is $\frac{1}{4}$. The original area is 20.

Your response: ______________

Incoming: The determinant is $k^2 + 6$, which is positive for every real k. Therefore there are no real singular parameter values.

Find the square's original area and its image area.

The determinant is 2. The square has side length 3.

Your response: ______________

Incoming: The original area is 9 and the image area is 18 square units.

Find all real t that make the matrix singular and explain why the off-diagonal 1 does not change the condition.

Required response

t	1
0	t

Your response: ______________

AP Precalculus

Determinants and Area

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Correct the area statement while preserving the sign's geometric information.

The determinant is -7. The student claims: The parallelogram area is -7 square units..

Repair the statement by identifying the actual column relationship and its area consequence.

Use the matrix $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$. The student claims: The determinant is zero, so both columns must be zero..

Repair the claim by stating what equality of determinants does guarantee and one fact showing the maps differ.

Use matrix A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix B = $\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$. The student claims: Both determinants are 4, so A and B are the same map..

Assess the student's claim using the displayed columns. Then replace it with a correct geometric statement.

The proposed first and second columns, respectively, are $(2, -3)$, $(-4, 6)$. The student claims: If $a2 \times 2$ determinant is zero, its two columns must be equal..

Determinants and Area

Full Worked Solutions - excerpt

Worked example

Problem. Give two distinct diagonal matrices with determinant 9 and verify them.

Answer. For example, $\begin{pmatrix} 1 & 0 \\ 0 & 9 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ both have determinant 9, but their diagonal scalings are different.

Explanation and check.

Choose two different factor pairs whose product is 9.

Worked example

Problem. Correct the area statement while preserving the sign's geometric information.

The determinant is -7. The student claims: The parallelogram area is -7 square units..

Answer. The ordinary area is 7 square units, and the negative determinant indicates reversed orientation.

Explanation and check.

Take an absolute value for area but not for the orientation classification.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Matrices and Linear Systems

Inverse Matrices and Linear Systems | APPC-U04-P16

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for values in example and valid general conclusion, conclusion and valid multiplication side, corrected table and verdict, valid and invalid cases, invertible and singular lists and decision and determinant evidence.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Inverse Matrices and Linear Systems

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Find the inverse by undoing the two independent coordinate scalings.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

2	0
0	5

Bank label: ______

Treat the two right-hand sides as columns of one table and use the same inverse once to solve both systems.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Coefficient matrix

2	1
1	1

Right-hand-side column 1

5
3

Right-hand-side column 2

1
−1

Supplied inverse

1	−1
−1	2

Bank label: ______

Use the bottom-right identity entry to find q.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

4	0
0	−2

$\frac{1}{4}$	0
0	q

Bank label: ______

Find the inverse of the three-factor expression ABA^{-1} and verify the result without commuting A and B.

Bank label: ______

AP Precalculus

Inverse Matrices and Linear Systems

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Use the supplied inverse rather than elimination, then substitute the result into both original equations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Coefficient matrix

3	1
1	1

Right-hand-side column

8
4

Supplied inverse

$\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{1}{2}$	$\frac{3}{2}$

Compute the exact inverse, preserving the sign of the determinant throughout the scalar division.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

1	3
2	4

Test the candidate and determine whether a row-column transposition caused the failure.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Proposed inverse

3	-2
-1	1

Given matrix

1	1
2	3

Find the inverse with a single common denominator and verify one diagonal product entry.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

4	3
1	2

AP Precalculus

Inverse Matrices and Linear Systems

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The determinant is $k^2 + 1$, which is positive for every real k. Every matrix in the family is invertible.

Square the matrix to determine whether using the original matrix as its own inverse is valid.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	0
2	-1

1	0
2	-1

Your response: ______________

START

Verify algebraically whether the proposed matrix is an inverse for every real k, rather than checking sample values.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	-1
1-k	k

k	1
$k-1$	1

Your response: ______________

Incoming: The determinant is $8k - 8 = 8(k-1)$. Only $k = 1$ is singular; $k = -2, -1, 0, 2, 3$ give invertible matrices.

Use the bottom-right identity constraint to find q.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	2
2	5

5	-2
-2	q

Your response: ______________

Incoming: The inverse is $[[\frac{1}{3}, -\frac{5}{6}], [0, \frac{1}{2}]]$.

Use A’s inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Your response: ______________

AP Precalculus

Inverse Matrices and Linear Systems

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “The proposed matrix cannot be an inverse because its off-diagonal entries are nonzero.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Multiply the matrix by the proposed inverse and decide whether the proposal satisfies the defining identity relation.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

1	-1
-1	2

2	1
1	1

Recompute the product instead of trusting the student’s table. Decide whether the error lies in the candidate or in the verification arithmetic.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

$\frac{1}{3}$	$-\frac{1}{3}$
$\frac{1}{3}$	$\frac{2}{3}$

2	1
-1	1

1	0
1	1

Inverse Matrices and Linear Systems

Full Worked Solutions - excerpt

Worked example

Problem. Find the inverse of the three-factor expression AB A^{-1} and verify the result without commuting A and B.

Answer. The inverse is A B^{-1} A^{-1}. Multiplication gives A B A^{-1} A B^{-1} A^{-1} =I after adjacent cancellations.

Explanation and check.

Reverse all three factors and invert each one; the inverse of A inverse is A itself.

Worked example

Problem. Use A's inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Answer. Multiplying both sides on the right by A^{-1} gives $B = C$.

Explanation and check.

The common factor is on the right, so its inverse must also be placed on the right to cancel it.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Matrices for Linear Transformations

Matrices for Linear Transformations | APPC-U04-P17

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for consistency, uniqueness, and two example matrices, determination and nonlinear compatible rule, determination and two compatible rules, classification and decisive test, classification and matrix and classification.

8 PDFs and a README; 60 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Construct the matrix and use it to map $(2, -1)$ as a check. $T(e_1) = (0,\ 2)$ $T(e_2) = (3,\ 0)$ Bank label: ______	Project the point with the displayed matrix. Use the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Vectors are column vectors. The given point is $(-3,\ 5)$. Bank label: ______
Determine the complete image of the unit disk under the diagonal projection, including exact segment endpoints. Use the matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Vectors are column vectors. Use the disk with center $(0,\ 0)$ and radius 1. Bank label: ______	Solve the two component systems to recover the matrix from these nonbasis correspondences. Assume T is linear. The input-output observations are $(1,\ 2) \mapsto (5,\ 1)$; $(2,\ -1) \mapsto (1,\ 8)$. Bank label: ______

AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the projection matrix by tracking both basis vectors onto the line $y = x$. The map acts by orthogonally project onto the line $y = x$.	Normalize each scaled-axis correspondence to obtain the two basis images. Assume T is linear. The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(3,\ 6)$.
Classify the rule using the origin test and distinguish affine from linear behavior. Use the rule $F(x, y) = (x + 2, y - 1)$.	Do the axis-only samples establish linearity of an unspecified plane map? Exhibit contrasting compatible rules. The input-output observations are $(0,\ 0)\mapsto(0,\ 0)$; $(1,\ 0)\mapsto(1,\ 0)$; $(2,\ 0)\mapsto(2,\ 0)$.
Construct the matrix and explain geometrically why its image is one-dimensional. $T(e_1) = (2,\ 1)$ $T(e_2) = (-4,\ -2)$	Map the side generators and determine the image area and orientation change. Use the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Vectors are column vectors. The two side vectors are $u = (1,\ 0)$ and $v = (0,\ 2)$.

AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Because $(1,1)+(1,-1)=(2,0)$, outputs require $(2+k,4)=(5,4)$, so $k=3$. Then $T(e1)=(\frac{5}{2},2), T(e2)=(-\frac{1}{2},2)$, and the matrix is $[[\frac{5}{2},-\frac{1}{2}],[2,2]]$.

Construct the exact reflection matrix using the double-angle form.

The map acts by reflect across the line through the origin making a30-degree angle with the x-axis.

Your response: ______________

Incoming: The image generators are $(2,0)$ and $(1,2)$. Their determinant has magnitude 4, so the image area is 4.

Use the two directed basis images to assemble the standard matrix.

$T(e_1)=(0,\ -2)$

$T(e_2)=(3,\ 0)$

Your response: ______________

Incoming: Under a linearity assumption they determine matrix $[[2,-1],[1,3]]$. Without that assumption they do not prove the map is linear, because nonlinear rules can share the same two values.

Map both points and verify that their known scalar relation survives the transformation.

Use the matrix $A=\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$. Vectors are column vectors.

The given points are $P=(2,\ 1)$; $Q=(-6,\ -3)$.

The relation is $Q=-3P$.

Your response: ______________

Incoming: Every member is linear, with matrix $[[a,b],[c,d]]$.

Normalize the two scaled basis correspondences and construct the standard matrix.

Assume T is linear.

The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(-6,\ -9)$.

Your response: ______________

Incoming: The matrix is $[[0,1],[-1,0]]$.

Complete the image table for all three input rows.

Use the matrix $A=\begin{pmatrix}-1 & 2\\ 3 & 1\end{pmatrix}$. Vectors are column vectors.

The given points are $(0,\ 1)$, $(2,\ 0)$, $(-1,\ -2)$.

Your response: ______________

Incoming: The first rows give matrix $[[1,3],[2,4]]$, which sends $(1,2)$ to $(7,10)$, not $(8,10)$. No linear map fits all rows.

Recover $T(e2)$ from the image of $e1+e2$ and construct the matrix.

Assume T is linear.

The input-output observations are $(1,\ 0)\mapsto(-1,\ 4)$; $(1,\ 1)\mapsto(2,\ 2)$.

Your response: ______________

AP Precalculus

Matrices for Linear Transformations

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The student read the rows as basis images. Correct both images and explain the relevant matrix convention.

Use the matrix $A = \begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix}$. Vectors are column vectors.

Evaluate these student claims: $T(e1) = (2,\, 5)$; $T(e2) = (-1,\, 3)$.

A student claims: " $A90^\circ$ rotation produces a different region because every boundary point moves." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Describe the disk's image and verify the claim with one boundary point.

Use the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Vectors are column vectors.

Use the disk with center $(0,\, 0)$ and radius 2.

Evaluate this student claim: $A90^\circ$ rotation produces a different region because every boundary point moves..

Matrices for Linear Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Write the matrix for a half-turn about the origin.

The map acts by rotate 180 degrees about the origin.

Answer. The matrix is $[[-1, 0], [0, -1]]$.

Explanation and check.

Both basis vectors reverse direction under a half-turn.

Worked example

Problem. Classify the rule and give a matrix representation if one exists.

Use the rule $F(x, y) = (2x - y, 3y)$.

Answer. The rule is linear with matrix $[[2, -1], [0, 3]]$.

Explanation and check.

Each output coordinate is a constant linear combination of the inputs with no added constant or nonlinear term.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Composing Matrix Transformations

Composing Matrix Transformations | APPC-U04-P18

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for projection and product, second matrix and product, all valid ordered pairs and verification principle, all valid pairs, all sequences and exclusions and all valid sequences and exclusion.

8 PDFs and a README; 156 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Composing Matrix Transformations

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Compute both orders and decide whether these maps commute.

Use the reflection matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Use the scaling matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.

Bank label: ______

AP Precalculus

Composing Matrix Transformations

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the ordered composite and verify it stage by stage. First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The original input vector is $(1, 2)$.	Classify all allowed two-stage sequences that produce reflection across $y = x$. The allowed transformations are exactly one 90-degree counterclockwise rotation and one coordinate-axis reflection. The target matrix is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

AP Precalculus

Composing Matrix Transformations

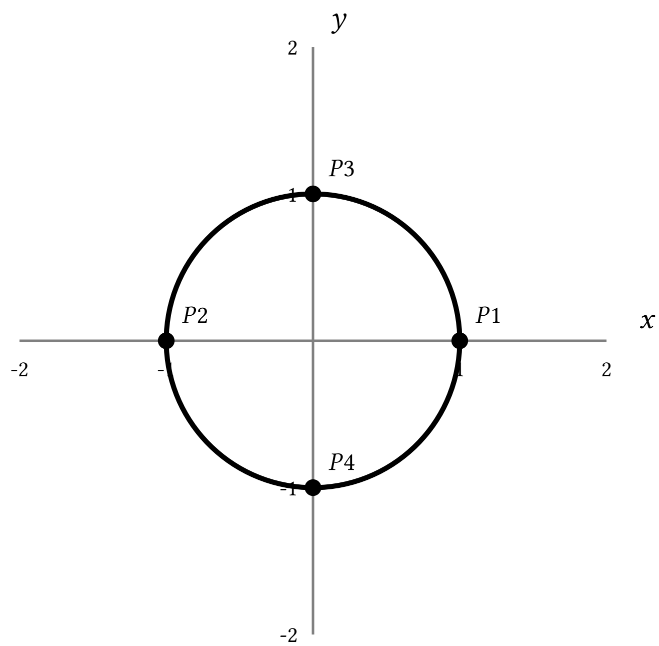
Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The first two stages form a half-turn; the final product is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, reflection across the y-axis.

Track the four key boundary points and state the major-axis direction after each stage.

The boundary points are $(1,0)$, $(-1,0)$, $(0,1)$, $(0,-1)$. First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.



Your response: ______________

AP Precalculus

Composing Matrix Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student multiplies the two matrices in the written transformation order and claims the composite is the first matrix times the second. Identify the order error, construct the correct composite, and verify it on the test vector.

First apply $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Then apply $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. The original input vector is $(2, 1)$.

Composing Matrix Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare both orders and identify their common net action.

Use matrix A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix B = $\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}$.

Answer. Both products equal -I, so either order is a half-turn with no change in length.

Explanation and check.

Scalar matrices commute, and the scale factors multiply to -1.

Worked example

Problem. Complete the table and explain why the final column returns to the original data.

First apply $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Use the points A: $(1,2)$; B: $(-2,1)$; C: $(0,-3)$. Then apply $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

Answer. Intermediate: $A=(5,2), B=(0,1), C=(-6,-3)$. Final: $A=(1,2), B=(-2,1), C=(0,-3)$, the original points.

Explanation and check.

The second shear subtracts the same vertical-coordinate multiple that the first added.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Transformations and Preimages

Inverse Transformations and Preimages | APPC-U04-P19

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for inverse existence and two inverse images, conclusion and example matrices, qualified conclusion, existence and uniqueness qualifications, all preimages and all inputs and geometric set.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Inverse Transformations and Preimages

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Recover the input from the mixed-sign image and verify it with the forward map.

The image point is $(-2, 3)$.

Required response

Supplied inverse matrix

1	-1
0	1

Forward matrix

1	1
0	1

Bank label: ______

Identify the inverse map without recomputing an adjugate.

Required response

0	1
1	0

Bank label: ______

Apply the inverse turn to recover every labeled original vertex.

The image vertex A' is $(0, 1)$.

The image vertex B' is $(-2, 1)$.

The image vertex C' is $(-2, 3)$.

The image vertex D' is $(0, 3)$.

Required response

Forward matrix

0	-1
1	0

Bank label: ______

Find every input mapped to the target and describe the fiber.

The target point is $(3, -6)$.

Required response

Forward matrix

2	-1
-4	2

Bank label: ______

AP Precalculus

Inverse Transformations and Preimages

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recover the original rectangle and state its width and height.

Required response

2	0
0	$\frac{1}{2}$

-4	-1
4	-1
4	1
-4	1

Determine whether the target has any preimage.

The target point is $(4, 1)$.

Required response

Forward matrix

1	0
0	0

Decide separately whether the map is determined and whether it has an inverse.

Use $T(e_1) = (1, 2)$.

Use $T(e_2) = (1, 2)$.

Assume that T is linear.

Explain why these four checks are sufficient for mutual inverses.

Assume that F and G are linear.

The supplied checks are $G(F(e_1)) = e_1$; $G(F(e_2)) = e_2$; $F(G(e_1)) = e_1$; $F(G(e_2)) = e_2$.

AP Precalculus

Inverse Transformations and Preimages

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The preimage vertices are $(2, 1), (-1, 0), (0, -3)$.

Use the supplied inverse to reconstruct the labeled triangle.

Required response

Forward matrix

2	1
1	1

Image vertices (one coordinate pair per row)

0	0
2	1
2	2

Supplied inverse matrix

1	-1
-1	2

Your response: ______________________

Incoming: Map (X, Y) to $(X, Y + 2X)$, using [[1, 0], [2, 1]].

Find when the target lies on the image line and then parameterize its fiber.

The target point is $(k, 9)$.

Required response

Forward matrix

1	2
3	6

Your response: ______________________

Incoming: Reachability requires k = 3. Then $x + 2y = 3$, so the preimages are $(3 - 2t, t)$.

Compute both compositions on their proper domains and classify the relationship.

Let $F(x, y) = (x, 0)$ from R^2 to the x-axis.

Let $G(t, 0) = (t, 0)$ from the x-axis to R^2.

Your response: ______________________

Incoming: The preimage is $x^2 + y^2 \geq 1$ with $y > 0$. The circle is included, while the x-axis is excluded.

Prove the candidate is an inverse and explain exactly where the nonzero condition is needed.

Required response

1	a
0	b

1	$-\frac{a}{b}$
0	$\frac{1}{b}$

Your response: ______________________

AP Precalculus

Inverse Transformations and Preimages

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: "A shear matrix cannot be invertible unless it is diagonal." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Verify the inverse-function claim on all vectors by checking both compositions.

Required response

1	1
0	1

1	−1
0	1

Repair the claim that one recovered input proves a global inverse function.

Use the map $F(x, y) = (x, 0)$.

The student proposes the preimage $(3, 0)$.

The target point is $(3, 0)$.

Inverse Transformations and Preimages

Full Worked Solutions - excerpt

Worked example

Problem. Prove the product-inverse rule by checking recovery in both directions and naming the undoing order.

Answer. Undo A first and B second. Composing this undoing map before or after the original product cancels adjacent inverse pairs and gives the identity in both directions.

Explanation and check.

A composite inverse reverses the original factor order so that each action meets its own undoing action.

Worked example

Problem. Prove that the inverse function of B is the original matrix function A.
Assume that $B = A^{-1}$.

Answer. AB = BA=I, so A satisfies both defining equations for B^{-1}. Therefore $(A^{-1})^{-1} = A$.

Explanation and check.

The two-sided inverse equations are symmetric in A and B.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Transition Matrices and Long-Term Behavior

Transition Matrices and Long-Term Behavior | APPC-U04-P20

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for matrix family and parameter interval, terminal states and decision, switch counts, terminal states, and decision, complete set, fixed states for t=0 and t>0 and all possible priors.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

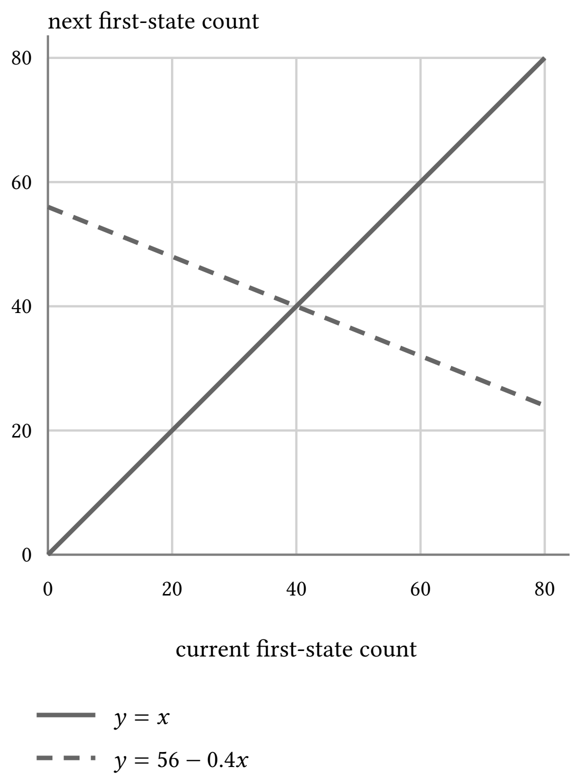
AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the graph equations to find the fixed distribution.



0.3	0.7
0.7	0.3

Bank label: ______

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Compare subscribed counts after two months and recommend the policy for that horizon. The components are ordered subscribed, then paused. The horizon in cycles is 2.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

100
0

Policy A

0.8	0.3
0.2	0.7

Policy B

0.6	0.5
0.4	0.5

Compute tomorrow's proportions and identify both contributions to the online state. The components are ordered online, then offline.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

0.7
0.3

Transition matrix

0.85	0.4
0.15	0.6

Classify all fixed distributions as t ranges from 0 through 0.5. The conserved total is 90. The allowed parameter interval is $0 \leq t \leq 0.5$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Transition matrix

1-t	$2t$
t	$1 - 2t$

Determine the two unknown first-row probabilities from the observations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Matrix pattern

p	q
1-p	1-q

Observation 1: initial state

50
50

Observation 1: later state

55
45

Observation 2: initial state

80
20

Observation 2: later state

70
30

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[80, 20]$

Convert the shares to counts and calculate the next state. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Initial proportions

0.25
0.75

Transition matrix

0.5	0.1
0.5	0.9

Your response: ______________

Incoming: $[28, 72]$

Find the equilibrium counts. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Transition matrix

0.9	0.3
0.1	0.7

Your response: ______________

Incoming: $[[0, 0], [1, 1]]$

Translate the conditional branches into a transition matrix. The components are ordered A, then B. The conditional A-to-B probability is 0.4; the B-to-A probability is 0.15.

Your response: ______________

Incoming: $[16, 64]$

Find the fixed distribution with total 120.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

0.7	0.3
0.3	0.7

Your response: ______________

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: "Because every displayed row rounds to $(50.0, 50.0)$, the state sequence converges to $(50, 50)$." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: A rounded table displays $(50.0, 50.0)$ repeatedly. Decide whether that proves convergence.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

49.96
50.04

0	1
1	0

Find the state after three periods using the supplied power, and explain why multiplying the one-period matrix by 3 would be invalid.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

40
60

0.8	0.3
0.2	0.7

0.65	0.525
0.35	0.475

Transition Matrices and Long-Term Behavior

Full Worked Solutions - excerpt

Worked example

Problem. Translate the conditional branches into a transition matrix. The components are ordered A, then B. The conditional A-to-B probability is 0.4; the B-to-A probability is 0.15.

Answer. $[[0.6, 0.15], [0.4, 0.85]]$

Explanation and check.

Each tree root is a current-state source, so its two destination branches become one matrix column.

Worked example

Problem. Encode the deterministic switching rule. The components are ordered left, then right. The transition rule is: every member switches states each cycle.

Answer. $[[0, 1], [1, 0]]$

Explanation and check.

Each source sends probability one to the opposite destination and zero to itself.