

Precalculus  
**Polar Coordinates, Graphs, Rates & Models | 8-Topic Precalculus Bundle**

8-topic bundle | 768 problems | Four activity formats

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**Included topics**

The following pages list all 8 included resources in the suggested order, with a link to each product. Each resource retains its complete preview.

**In every topic**

|                                             |          |
|---------------------------------------------|----------|
| Answer Bank Challenge                       | 24 tasks |
| Grid Rounds                                 | 24 tasks |
| Match & Move                                | 24 cards |
| Error Analysis & Strategy                   | 24 tasks |
| Quick Answer Key and Full Worked Solutions  | included |
| Teacher / Parent Use Guide and Terms of Use | included |

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**Product previews**

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

**64 PDFs and 8 READMEs; 974 buyer PDF pages across the 8 products.**

Preview pages are not included in the buyer page count. Check the list against resources you already own.

## Sets & Steps

Math Practice & Worked Solutions

## Precalculus

# Included topics

8-topic bundle | 768 problems | Four activity formats

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### 01 Equivalent Polar Coordinates

[View individual product and its full preview](#)

### 02 Polar and Rectangular Coordinate Conversion

[View individual product and its full preview](#)

### 03 Polar Graphs from Tables

[View individual product and its full preview](#)

### 04 Polar Symmetry and Curve Tracing

[View individual product and its full preview](#)

### 05 Features of Polar Equations

[View individual product and its full preview](#)

### 06 Polar Rates of Change and Radial Motion

[View individual product and its full preview](#)

### 07 Radius Graphs vs Polar Traces

[View individual product and its full preview](#)

### 08 Polar Models in Context

[View individual product and its full preview](#)

8 included resources | Complete resource previews follow this guide.

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AP Precalculus  
Equivalent Polar Coordinates

Equivalent Polar Coordinates | APPC-U03-P23

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
| 01 | Answer Bank Challenge      | 24 tasks     |
| 02 | Grid Rounds                | 24 tasks     |
| 03 | Match & Move               | 24 cards     |
| 04 | Error Analysis & Strategy  | 24 tasks     |
| 05 | Quick Answer Key           | 96 answers   |
| 06 | Full Worked Solutions      | 96 solutions |
| 07 | Teacher / Parent Use Guide | included     |
| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for diagnosis and corrected pair, same or different, distance and location, one pair, one polar pair and equivalence group.

8 PDFs and a README; 157 buyer pages.

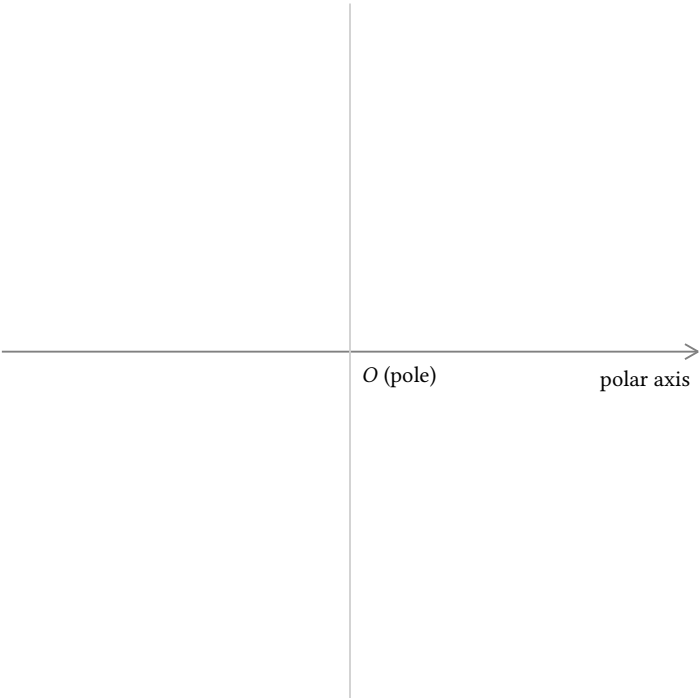
AP Precalculus

Equivalent Polar Coordinates

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the signed radius  $r$  that makes  $(r, 5\frac{\pi}{6})$  equivalent to  $(5, 11\frac{\pi}{6})$ .



| Condition or evidence   | Supplied value         |
|-------------------------|------------------------|
| Incomplete polar record | $(r, 5\frac{\pi}{6})$  |
| Known polar record      | $(5, 11\frac{\pi}{6})$ |

Bank label: \_\_\_\_\_

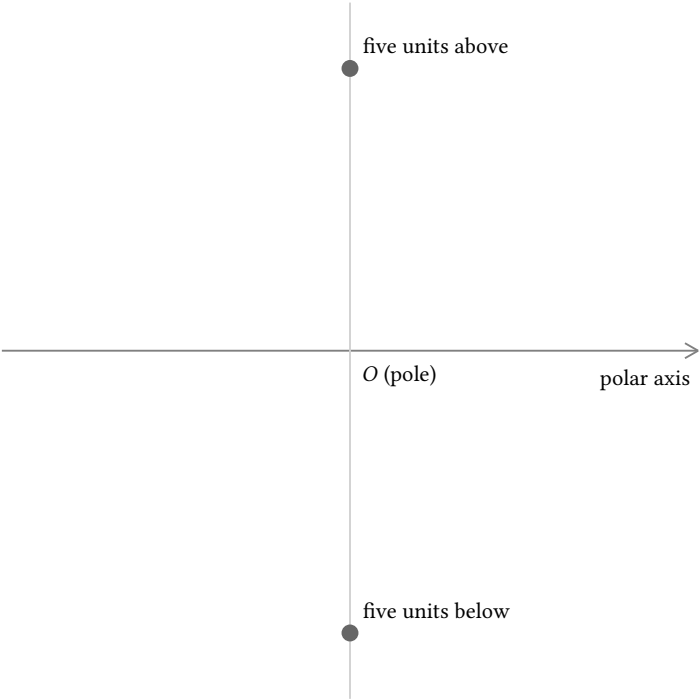
AP Precalculus

Equivalent Polar Coordinates

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A plot offers points five units above and five units below the pole. Which one represents  $(-5, \frac{\pi}{2})$ ?



| Condition or evidence | Supplied value        |
|-----------------------|-----------------------|
| Candidate points      |                       |
|                       | five units above      |
|                       | five units below      |
| Given polar record    | $(-5, \frac{\pi}{2})$ |

AP Precalculus

Equivalent Polar Coordinates

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

**Incoming:**  $\theta = -3\frac{\pi}{10}$

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An equivalent pair for  $(7, 4\frac{\pi}{3})$  is required to have  $\theta = \frac{\pi}{3} + 2k\pi$ . What must  $r$  be for every integer  $k$ ?

Solve the card and move only to the card whose incoming header matches the entire requested response.

| Condition or evidence   | Supplied value                   |
|-------------------------|----------------------------------|
| Given angle form        | $\theta = \frac{\pi}{3} + 2k\pi$ |
| Incomplete polar record | $(r, \theta)$                    |
| Known polar record      | $(7, 4\frac{\pi}{3})$            |

Your response: \_\_\_\_\_

AP Precalculus

Equivalent Polar Coordinates

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For  $(-a, \frac{\pi}{6})$  with  $a = -5$ , evaluate the radius before interpreting the point. Correct the claim that the visible minus sign automatically reverses the ray. Justify the comparison by distinguishing signed radius, ordinary distance and actual location.

Compare the stated cases, methods, or representations and justify the conclusion using the supplied conditions.

| Condition or evidence    | Supplied value                                         |
|--------------------------|--------------------------------------------------------|
| Assumption               | $a = -5$                                               |
| Given polar record       | $(-a, \frac{\pi}{6})$                                  |
| Student claim to examine | The written minus sign always means a negative radius. |

## Equivalent Polar Coordinates

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** Fill in  $r$  so the table entry  $(r, 13\frac{\pi}{6})$  names the same point as  $(3, \frac{\pi}{6})$ .

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Candidate              | Reference            |
|------------------------|----------------------|
| $(r, 13\frac{\pi}{6})$ | $(3, \frac{\pi}{6})$ |

**Answer.**  $r = 3$

#### Explanation and check.

The angles differ by  $2\pi$ , so the radius retains both its magnitude and sign.

#### Worked example

**Problem.** Complete the row  $(r, 0)$  so it is equivalent to  $(-8, \pi)$ .

Solve the card and move only to the card whose incoming header matches the entire requested response.

| Condition or evidence | Supplied value |
|-----------------------|----------------|
| Incomplete record     | $(r, 0)$       |
| Known record          | $(-8, \pi)$    |

**Answer.**  $r = 8$

#### Explanation and check.

The negative radius reverses the  $\pi$  ray to the 0 ray at distance 8, so the second pair uses  $r = 8$ .

AP Precalculus  
Polar and Rectangular Coordinate Conversion

Polar and Rectangular Coordinate Conversion | APPC-U03-P24

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
| 01 | Answer Bank Challenge      | 24 tasks     |
| 02 | Grid Rounds                | 24 tasks     |
| 03 | Match & Move               | 24 cards     |
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| 06 | Full Worked Solutions      | 96 solutions |
| 07 | Teacher / Parent Use Guide | included     |
| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for polar pair plus check, polar coordinates, polar pair with quadrant check, rounded polar coordinates, rounded coordinates and positive-radius polar pair.

8 PDFs and a README; 154 buyer pages.

## AP Precalculus

Polar and Rectangular  
Coordinate Conversion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A point has rectangular form  $(3, y)$ , polar radius 5, and lies above the  $x$ -axis. Complete the missing coordinate.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence         | Supplied value |
|-------------------------------|----------------|
| Condition                     | $y > 0$        |
| Given polar radius            | 5              |
| Incomplete rectangular record | $(3, y)$       |

Bank label: \_\_\_\_\_

For  $(-5, 2)$ , find positive-radius polar coordinates to the nearest tenth of a degree. Explain the quadrant correction and check the signs of the two projections.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| x  | y |
|----|---|
| -5 | 2 |

| Condition or evidence   | Supplied value                    |
|-------------------------|-----------------------------------|
| Required angle interval | $0^\circ \leq \theta < 360^\circ$ |
| Precision               | nearest tenth of a degree         |

Bank label: \_\_\_\_\_

Convert the measured point  $(0.4, -6)$  to positive-radius polar form using the stated rounding. Preserve the quadrant near the negative  $y$ -axis.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| x   | y  |
|-----|----|
| 0.4 | -6 |

| Condition or evidence   | Supplied value                    |
|-------------------------|-----------------------------------|
| Required angle interval | $0^\circ \leq \theta < 360^\circ$ |
| Angle precision         | nearest tenth                     |
| Radius precision        | nearest hundredth                 |

Bank label: \_\_\_\_\_

A calculator returns a reference angle near  $20.6^\circ$  for  $(-8, -3)$ . Produce the correct polar angle, radius, and a sign check by reprojection.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence        | Supplied value                                                                                 |
|------------------------------|------------------------------------------------------------------------------------------------|
| Required angle interval      | $0^\circ \leq \theta < 360^\circ$                                                              |
| Given rectangular point      |                                                                                                |
|                              | −8                                                                                             |
|                              | −3                                                                                             |
| Precision                    | <b>Given angle <math>\theta</math></b> nearest tenth degree<br><b>Radius</b> nearest hundredth |
| Given calculator ratio angle | about $20.6^\circ$                                                                             |

Bank label: \_\_\_\_\_

AP Precalculus

Polar and Rectangular  
Coordinate Conversion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A positive-radius point has  $\theta = 3\frac{\pi}{2}$  and  $y = -4$ . Determine  $r$  and the missing  $x$ -coordinate.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

| Condition or evidence | Supplied value   |
|-----------------------|------------------|
| Given angle $\theta$  | $3\frac{\pi}{2}$ |
| Radius convention     | positive         |
| Given $x$ -coordinate | missing          |
| Given $y$ -coordinate | $-4$             |

AP Precalculus

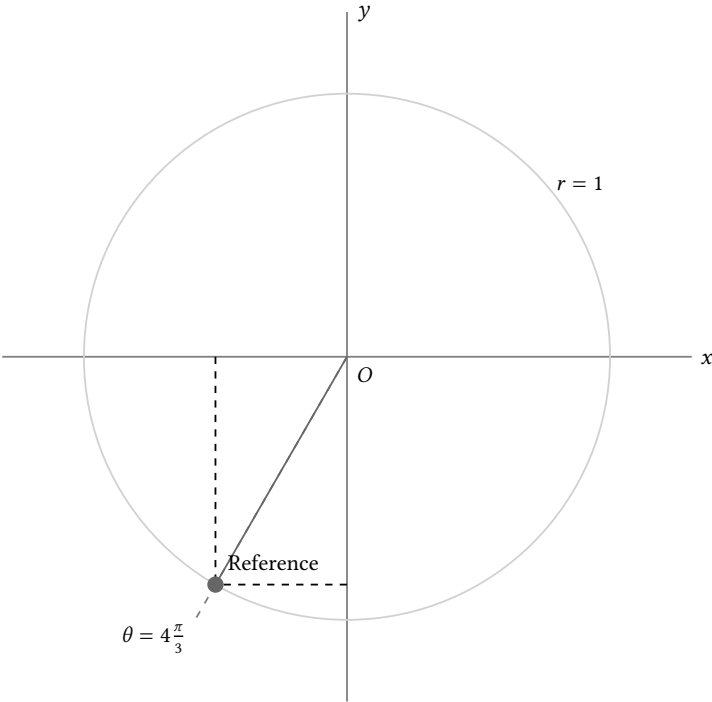
Polar and Rectangular  
Coordinate Conversion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming:  $(11, 3\frac{\pi}{2})$

Convert  $(-9, 4\frac{\pi}{3})$  and explain why both coordinates become positive.



| $\cos \theta$  | $\sin \theta$         |
|----------------|-----------------------|
| $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ |

| Condition or evidence    | Supplied value         |
|--------------------------|------------------------|
| Given quadrant to verify | I                      |
| Given polar record       | $(-9, 4\frac{\pi}{3})$ |

Your response: \_\_\_\_\_

AP Precalculus

Polar and Rectangular  
Coordinate Conversion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student converts  $(4, \frac{\pi}{6})$  to  $(2, 2\sqrt{3})$ . Identify the swapped roles and repair the ordered pair.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

| Condition or evidence                 | Supplied value       |
|---------------------------------------|----------------------|
| Given polar record                    | $(4, \frac{\pi}{6})$ |
| Student rectangular result to examine | $(2, 2\sqrt{3})$     |

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# Polar and Rectangular Coordinate Conversion

## Full Worked Solutions - excerpt

### Worked example

**Problem.** A point has rectangular form  $(3, y)$ , polar radius 5, and lies above the  $x$ -axis. Complete the missing coordinate. Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence         | Supplied value |
|-------------------------------|----------------|
| Condition                     | $y > 0$        |
| Given polar radius            | 5              |
| Incomplete rectangular record | $(3, y)$       |

**Answer.**  $y = 4$

### Explanation and check.

From  $3^2 + y^2 = 5^2$ ,  $y^2 = 16$ . The upper-half-plane condition selects  $y = 4$  rather than  $-4$ .

### Worked example

**Problem.** A point has radius 10,  $y = -6$ , and lies left of the  $y$ -axis. Reconstruct  $x$ .

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence         | Supplied value |
|-------------------------------|----------------|
| Condition                     | $x < 0$        |
| Given polar radius            | 10             |
| Incomplete rectangular record | $(x, -6)$      |

**Answer.**  $x = -8$

### Explanation and check.

The norm equation gives  $x^2 + 36 = 100$ , so  $x = \pm 8$ . The left-half-plane condition selects  $x = -8$ .

AP Precalculus  
Polar Graphs from Tables

Polar Graphs from Tables | APPC-U03-P25

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
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| 07 | Teacher / Parent Use Guide | included     |
| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for two new rows and adequacy explanation, two radius values, ordered rows, directed partial trace, polar graph and directed trace.

8 PDFs and a README; 93 buyer pages.

## AP Precalculus

# Polar Graphs from Tables

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

|                                                                                                                                                                                                   |                                                                                                                                                                                                  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Complete the radius column for <math>r = 2</math> at <math>\theta = 0, \frac{\pi}{2}</math>, and <math>\pi</math>, preserving the given angle order.</p> <p>_____</p> <p>Bank label: _____</p> | <p>For <math>r = 1 + \sin \theta</math>, evaluate the outputs at <math>0, \frac{\pi}{2}</math>, and <math>\pi</math> and place them in the same order.</p> <p>_____</p> <p>Bank label: _____</p> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## AP Precalculus

Polar Graphs  
from Tables

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Plot the ordered table for  $r = 1$  and add an arrow showing the direction as  $\theta$  increases from 0 to  $2\pi$ .

Required response

|                  |   |
|------------------|---|
| 0                | 1 |
| $\frac{\pi}{2}$  | 1 |
| $\pi$            | 1 |
| $3\frac{\pi}{2}$ | 1 |
| $2\pi$           | 1 |

Sketch only the portion traced by  $r = 2$  for  $0 \leq \theta \leq \frac{\pi}{2}$ , marking both endpoint statuses.

The  $\theta - r$  graph is the horizontal line  $r = 3$  for one full turn. Which plane trace matches, and why is the line  $y = 3$  not the same graph?

The listed samples for  $r = \sin(2\theta)$  are all zero. Explain why they do not show the trace, then evaluate the proposed extra input  $\frac{\pi}{4}$ .

Required response

|                 |   |
|-----------------|---|
| 0               | 0 |
| $\frac{\pi}{2}$ | 0 |
| $\pi$           | 0 |

Use the ordered rows for  $r = \theta$  to construct the trace on  $[0, \frac{\pi}{2}]$  and describe its direction.

Required response

|                 |                 |
|-----------------|-----------------|
| 0               | 0               |
| $\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| $\frac{\pi}{2}$ | $\frac{\pi}{2}$ |

Evaluate  $r = 2 \cos \theta - 1$  at the four listed angles. Keep zero and negative outputs unchanged for later plotting. The supplied angles, in order, are  $\theta = 0, \frac{\pi}{3}, \pi, 3\frac{\pi}{2}$ .

## AP Precalculus

Polar Graphs  
from Tables

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

**Incoming:**  $(0, 1), (\frac{\pi}{4}, 0), (\frac{\pi}{2}, -1), (\pi, -1)$ Determine the plane arc traced by  $r = -1$  for  $0 \leq \theta \leq \frac{\pi}{2}$ .

Required response

|                 |    |
|-----------------|----|
| 0               | -1 |
| $\frac{\pi}{2}$ | -1 |

Your response: \_\_\_\_\_

**Incoming:** The upper arc from filled  $(2, 0)$  through  $(1, 1)$ , approaching but not including the pole.Complete the table for  $r = 3 \cos \theta$  at  $\frac{\pi}{6}, \frac{\pi}{2}$ , and  $5\frac{\pi}{6}$ .

Your response: \_\_\_\_\_

**Incoming:** An outward quadrant-III curve from  $(-1, 0)$  to  $(0, -\frac{3}{2})$ , with both endpoints filled.Determine the portion traced by  $r = -1$  for  $-\pi \leq \theta \leq \frac{\pi}{2}$ , including its direction.

Required response

|                  |    |
|------------------|----|
| $-\pi$           | -1 |
| $-\frac{\pi}{2}$ | -1 |
| 0                | -1 |
| $\frac{\pi}{2}$  | -1 |

Your response: \_\_\_\_\_

**Incoming:** A three-quarter unit-circle arc from  $(1, 0)$  through  $(0, 1), (-1, 0)$ , to  $(0, -1)$ .Sketch the windowed trace of  $r = \theta + \pi$  from  $\theta = -\pi$  to  $\theta = -\frac{\pi}{2}$ .

Required response

|                   |                 |
|-------------------|-----------------|
| $-\pi$            | 0               |
| $-3\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| $-\frac{\pi}{2}$  | $\frac{\pi}{2}$ |

Your response: \_\_\_\_\_

## AP Precalculus

# Polar Graphs from Tables

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student replaces the negative output of  $r = \cos \theta$  by its magnitude. Repair the  $\theta = \pi$  row and state where that signed-radius point plots.

Required response

|                 |   |
|-----------------|---|
| 0               | 1 |
| $\frac{\pi}{2}$ | 0 |
| $\pi$           | 1 |

A student submits the horizontal  $\theta - r$  graph as the plane trace of  $r = 2$ . Explain the coordinate-system confusion and replace it with the correct polar graph.

## Polar Graphs from Tables

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** Evaluate  $r = 2 + \sin \theta - \cos \theta$  at the four quadrantal angles.

**Answer.**  $(0, 1), (\frac{\pi}{2}, 3), (\pi, 3), (3\frac{\pi}{2}, 1)$

#### Explanation and check.

Insert the four sine-cosine pairs  $(0, 1), (1, 0), (0, -1)$ , and  $(-1, 0)$  into the rule.

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#### Worked example

**Problem.** Compute  $r = 2 \sin \theta + \cos(2\theta)$  at the four quadrantal angles.

**Answer.**  $(0, 1), (\frac{\pi}{2}, 1), (\pi, 1), (3\frac{\pi}{2}, -3)$

#### Explanation and check.

Substitution gives  $0 + 1, 2 - 1, 0 + 1$ , and  $-2 - 1$ , respectively.

AP Precalculus

Polar Symmetry and Curve Tracing

Polar Symmetry and Curve Tracing | APPC-U03-P26

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
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| 04 | Error Analysis & Strategy  | 24 tasks     |
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| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for sufficient interval and retrace explanation, symmetry conclusion, once-through subinterval and retrace explanation, pole and nonpole multiplicities, pole multiplicity and nonpole verdict and partial/full/retraced judgment.

8 PDFs and a README; 156 buyer pages.

AP Precalculus

Polar Symmetry and Curve Tracing

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

On  $r = 2 \sin(3\theta)$ , show that  $\theta = \frac{\pi}{6}$  and  $\theta = 7\frac{\pi}{6}$  represent the same petal tip and give its Cartesian coordinates.  
Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Given $\theta$   | Given $r$ |
|------------------|-----------|
| $\frac{\pi}{6}$  | 2         |
| $7\frac{\pi}{6}$ | -2        |

| Condition or evidence | Supplied value                    |
|-----------------------|-----------------------------------|
| Original polar rule   | $r = 2 \sin(3\theta)$             |
| Given target point    | distance 2 on ray $\frac{\pi}{6}$ |

Bank label: \_\_\_\_\_

How many times does  $r = 2 + 2 \cos \theta$  visit the pole on  $0 \leq \theta < 2\pi$ ? Distinguish that count from the closed-curve endpoint convention.  
Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence          | Supplied value          |
|--------------------------------|-------------------------|
| Original polar rule            | $r = 2 + 2 \cos \theta$ |
| Given equation for zero radius | $\cos \theta = -1$      |

Bank label: \_\_\_\_\_

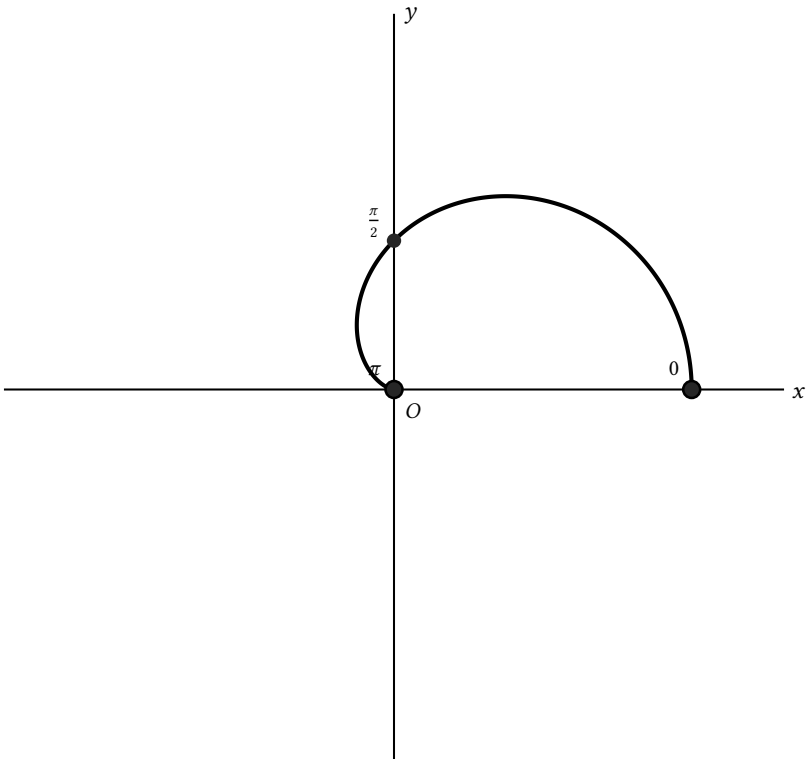
AP Precalculus

Polar Symmetry and Curve Tracing

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For  $r = 1 + \cos \theta$  on  $0 \leq \theta \leq \pi$ , describe the direction of travel through the three labeled anchors as  $\theta$  increases.



| Given parameter | Given point or location |
|-----------------|-------------------------|
| 0               | (2, 0)                  |
| $\frac{\pi}{2}$ | (0, 1)                  |
| $\pi$           | pole                    |

| Condition or evidence | Supplied value        |
|-----------------------|-----------------------|
| Original polar rule   | $r = 1 + \cos \theta$ |

\_\_\_\_\_

\_\_\_\_\_

AP Precalculus

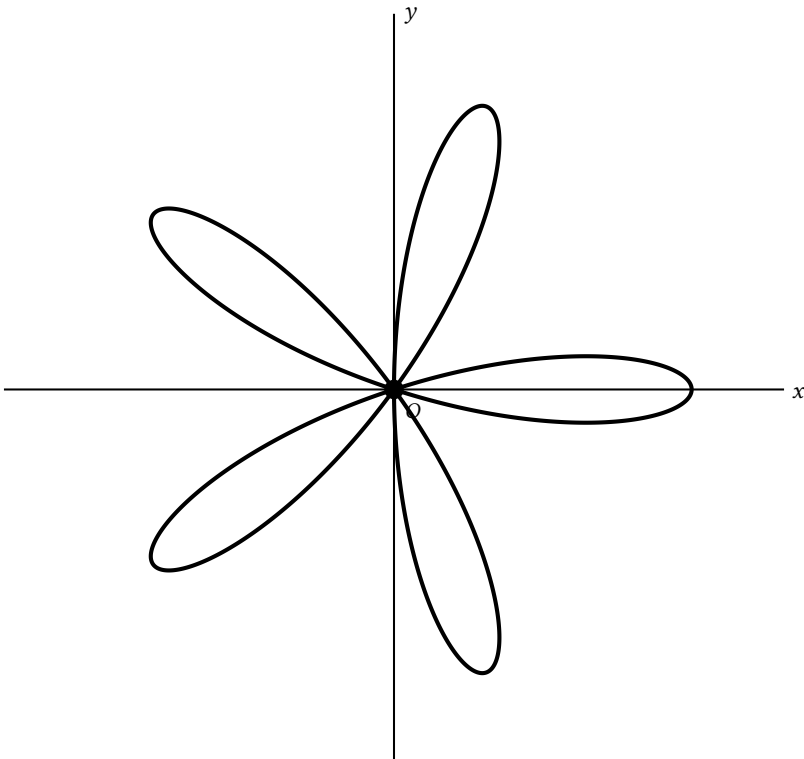
Polar Symmetry and Curve Tracing

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Determine whether  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$  is sufficient to trace every petal of  $r = \cos(5\theta)$  once. Justify with a half-turn correspondence rather than a memorized rule.



| Condition or evidence | Supplied value                 |
|-----------------------|--------------------------------|
| Half turn relation    | $r(\theta + \pi) = -r(\theta)$ |
| Interval length       | $\pi$                          |
| Original polar rule   | $r = \cos(5\theta)$            |

Your response: \_\_\_\_\_

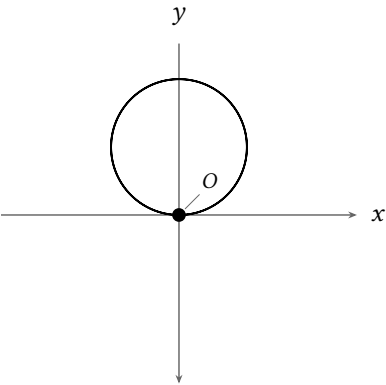
AP Precalculus

Polar Symmetry and Curve Tracing

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student says  $r = \sin \theta$  needs all of  $[0, 2\pi]$  for one trace because  $\sin \theta$  has period  $2\pi$ . Repair the claim using signed-radius equivalence.



| Condition or evidence    | Supplied value                                                                      |
|--------------------------|-------------------------------------------------------------------------------------|
| Half turn relation       | $r(\theta + \pi) = -r(\theta)$                                                      |
| Original polar rule      | $r = \sin \theta$                                                                   |
| Student claim to examine | A full $2\pi$ parameter interval is required because the radius has period $2\pi$ . |

\_\_\_\_\_

\_\_\_\_\_

## Polar Symmetry and Curve Tracing

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** Which parameters repeat the closing point of  $r = 4$  on the closed interval  $0 \leq \theta \leq 2\pi$ ?

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence        | Supplied value |
|------------------------------|----------------|
| Given common-point reference | $(4, 0)$       |
| Original polar rule          | $r = 4$        |

**Answer.**  $\theta = 0$  and  $\theta = 2\pi$  both give  $(4, 0)$ .

#### Explanation and check.

The radius is 4 at both coterminal endpoint angles, so the two polar pairs locate the same point.

#### Worked example

**Problem.** A polar table contains  $(\theta, r) = (\frac{\pi}{3}, 2)$  and  $(4\frac{\pi}{3}, -2)$ . Do the rows locate the same point? Give the coordinate.

Solve the card and move only to the card whose incoming header matches the entire requested response.

| $\theta$         | Given r |
|------------------|---------|
| $\frac{\pi}{3}$  | 2       |
| $4\frac{\pi}{3}$ | -2      |

**Answer.** Yes; both locate  $(1, \sqrt{3})$ .

#### Explanation and check.

The angles differ by  $\pi$  and the radii are opposites, so the polar pairs are equivalent. Converting the first gives  $(2 \cos(\frac{\pi}{3}), 2 \sin(\frac{\pi}{3})) = (1, \sqrt{3})$ .

AP Precalculus

Features of Polar Equations

Features of Polar Equations | APPC-U03-P27

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
| 01 | Answer Bank Challenge      | 24 tasks     |
| 02 | Grid Rounds                | 24 tasks     |
| 03 | Match & Move               | 24 cards     |
| 04 | Error Analysis & Strategy  | 24 tasks     |
| 05 | Quick Answer Key           | 96 answers   |
| 06 | Full Worked Solutions      | 96 solutions |
| 07 | Teacher / Parent Use Guide | included     |
| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for minimum distance and locations, distance and parameter, ordered exact parameter list, closest point and distance, distance and location and distinct coordinate set.

8 PDFs and a README; 83 buyer pages.

## AP Precalculus

# Features of Polar Equations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

|                                                                                                                                                                                                               |                                                                                                                                                                                                                 |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>In the family <math>r = a + \cos \theta</math>, choose <math>a</math> so that the curve reaches the pole at <math>\theta = \pi</math>.</p><br><br><br><br><br><br><br><br><br><br><p>Bank label: _____</p> | <p>For <math>r = a + 2 \sin \theta</math> with <math>a \geq 2</math>, determine <math>a</math> if the greatest distance from the pole is 5.</p><br><br><br><br><br><br><br><br><br><br><p>Bank label: _____</p> |
| <p>Determine <math>a</math> in <math>r = a + \sin \theta</math> if every radius is negative and the closest distance to the pole is 2.</p><br><br><br><br><br><br><br><br><br><br><p>Bank label: _____</p>    | <p>At what parameter does <math>r = 5\theta</math> reach the pole on <math>-1 \leq \theta \leq 1</math>?</p><br><br><br><br><br><br><br><br><br><br><p>Bank label: _____</p>                                    |

## AP Precalculus

# Features of Polar Equations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Find every  $\theta$  in  $0 \leq \theta \leq \pi$  for which  $r = 2 \sin \theta$  is zero, and name the plane point reached.



**AP Precalculus****Features of  
Polar Equations****Name:** \_\_\_\_\_ **Date:** \_\_\_\_\_ **Class:** \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

---

A table row gives  $(\theta, r) = (\frac{\pi}{4}, -7)$ , and a student reports a distance of  $-7$ . Correct both the distance and the plotted direction.

---

A student finds seven zero parameters for  $r = \sin(3\theta)$  on  $0 \leq \theta \leq 2\pi$  and reports seven different poles. Correct the feature count.

---

---

## Features of Polar Equations

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** Choose  $a$  in  $r = 2 + a \cos \theta$  so that  $\theta = 0$  is a pole parameter. Verify the resulting zero.

**Answer.**  $-2$

#### Explanation and check.

At  $\theta = 0$  the radius is  $2 + a$ . Setting it to zero gives  $a = -2$ , and substitution yields  $r(0) = 2 - 2 = 0$ .

---

#### Worked example

**Problem.** For  $k > 0$ , the curve  $r = k(1 - \cos \theta)$  has farthest distance 10. Determine  $k$ .

**Answer.**  $k = 5$ .

#### Explanation and check.

The factor  $1 - \cos \theta$  is largest at  $\theta = \pi$ , where it equals 2. Thus the farthest distance is  $2k$ , and  $2k = 10$  gives  $k = 5$ .

AP Precalculus  
Polar Rates of Change and Radial Motion

Polar Rates of Change and Radial Motion | APPC-U03-P28

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
| 01 | Answer Bank Challenge      | 24 tasks     |
| 02 | Grid Rounds                | 24 tasks     |
| 03 | Match & Move               | 24 cards     |
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| 08 | Terms of Use & License     | included     |

**Topic focus**  
Across 96 tasks this topic asks for supported or undetermined with alternative, estimated radius, exact signed rate and interpretation, local closest or farthest, local feature and inward and outward intervals.  
**8 PDFs and a README; 108 buyer pages.**

## AP Precalculus

# Polar Rates of Change and Radial Motion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

|                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>The average signed-radius rate from <math>\theta = 0</math> to <math>\theta = 2</math> is 3 units per radian, and <math>r(0) = -1</math>. Recover <math>r(2)</math>.</p> <p>_____</p> <p>Bank label: _____</p>                                                  | <p>The average radial rate over <math>2 \leq \theta \leq 4</math> is <math>-3</math>, and <math>r(4) = 1</math>. Use the secant model to estimate <math>r(3)</math>.</p> <p>_____</p> <p>Bank label: _____</p>          |
| <p>The average radial rate from <math>\theta = \frac{1}{2}</math> to <math>\theta = \theta_2</math> is <math>-4</math>. If <math>r(\frac{1}{2}) = 6</math> and <math>r(\theta_2) = 2</math>, find <math>\theta_2</math>.</p> <p>_____</p> <p>Bank label: _____</p> | <p>The average signed-radius rate from <math>30^\circ</math> to <math>90^\circ</math> is 2 units per radian, and <math>r(30^\circ) = 1</math>. Find <math>r(90^\circ)</math>.</p> <p>_____</p> <p>Bank label: _____</p> |

## AP Precalculus

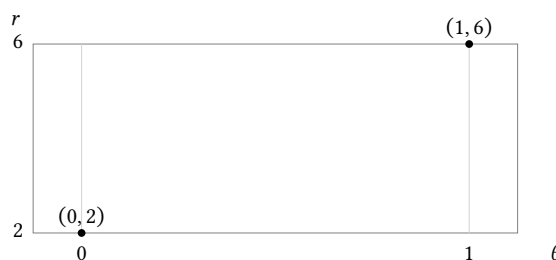
# Polar Rates of Change and Radial Motion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Throughout an angle interval,  $r$  is negative and increases from  $-7$  toward  $-2$ . Is the point moving inward or outward relative to the pole?

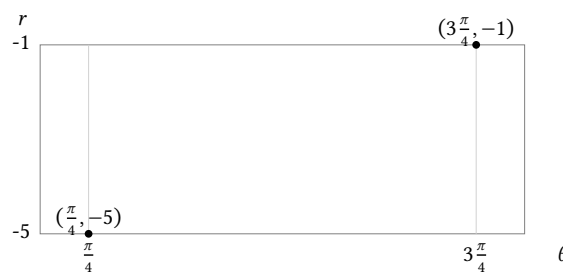
Using the average rate between the two table rows, estimate  $r$  at  $\theta = \frac{1}{2}$ . Is the result necessarily exact?



|   |   |
|---|---|
| 0 | 2 |
| 1 | 6 |

Near  $\theta = a$ , a radius graph stays positive and has a local minimum  $r = 2$ . What distance feature does the polar point have at  $a$ ?

Compute the average signed-radius rate from  $\theta = \frac{\pi}{4}$ , where  $r = -5$ , to  $\theta = 3\frac{\pi}{4}$ , where  $r = -1$ .



|                  |    |
|------------------|----|
| $\frac{\pi}{4}$  | -5 |
| $3\frac{\pi}{4}$ | -1 |

## AP Precalculus

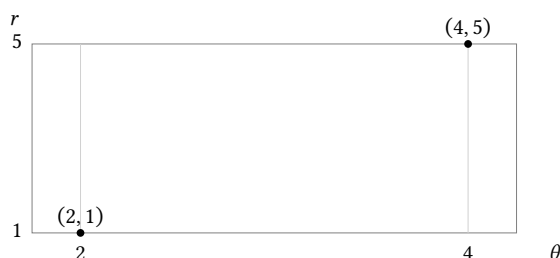
Polar Rates of Change  
and Radial Motion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

**Incoming:**  $r(90^\circ) \approx 4$ .

A table gives  $r(2) = 1$  and  $r(4) = 5$ . Use the average rate to estimate  $r(3)$ , and say whether the estimate must be exact.



|   |   |
|---|---|
| 2 | 1 |
| 4 | 5 |

Your response: \_\_\_\_\_

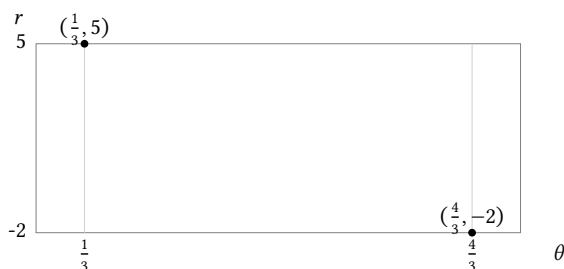
**Incoming:** Signed-radius rate 2; distance rate  $\frac{4}{3}$  radial units per radian.

For  $r = 2^\theta$ , find the average radial rate from  $\theta = 0$  to  $\theta = 1$ .

Your response: \_\_\_\_\_

**Incoming:** Both rates are  $-3$  radial units per radian.

Find the average radial rate from  $r(\frac{1}{3}) = 5$  to  $r(\frac{4}{3}) = -2$ .

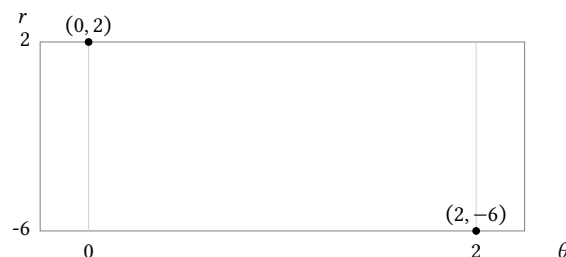


|               |    |
|---------------|----|
| $\frac{1}{3}$ | 5  |
| $\frac{4}{3}$ | -2 |

Your response: \_\_\_\_\_

**Incoming:** The estimate is 10 and the actual radius lies in  $[9.5, 10.5]$ .

A diagram labels  $r = 2$  at  $\theta = 0$  and  $r = -6$  at  $\theta = 2$ . Find the average radial and distance rates.



|   |    |
|---|----|
| 0 | 2  |
| 2 | -6 |

Your response: \_\_\_\_\_

AP Precalculus

Polar Rates of Change  
and Radial Motion

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student computes  $\frac{5-2}{90-30} = \frac{1}{20}$  and labels it radial units per radian. Repair the interpretation.

Required response

|     |   |
|-----|---|
| 30° | 2 |
| 90° | 5 |

## Polar Rates of Change and Radial Motion

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** A signed radius rises from 2 to 12 at average rate 5. What is the angular interval width?

**Answer.**  $\theta_2 - \theta_1 = 2$  radians.

#### Explanation and check.

The radius change is 10. Dividing change by rate gives a width of 2 radians.

---

#### Worked example

**Problem.** For  $r = 2^\theta$ , find the average radial rate from  $\theta = 0$  to  $\theta = 1$ .

**Answer.** 1 radial unit per radian.

#### Explanation and check.

The radius changes from 1 to 2 over one radian, so the secant quotient is 1.

AP Precalculus

Radius Graphs vs Polar Traces

Radius Graphs vs Polar Traces | APPC-U03-P29

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
| 01 | Answer Bank Challenge      | 24 tasks     |
| 02 | Grid Rounds                | 24 tasks     |
| 03 | Match & Move               | 24 cards     |
| 04 | Error Analysis & Strategy  | 24 tasks     |
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| 07 | Teacher / Parent Use Guide | included     |
| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for graph sets, qualification and least point repeat, change descriptions, opposite trends, qualification about least period and two graph descriptions.

8 PDFs and a README; 167 buyer pages.

## AP Precalculus

# Radius Graphs vs Polar Traces

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

For  $r = \sin(3\theta)$ , determine the scalar period and the least positive shift that repeats the same parameterized polar point for every generic  $\theta$ . Explain why they differ.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence            | Supplied value                 |
|----------------------------------|--------------------------------|
| Given parameter range or context | all real $\theta$              |
| Given half-turn relation         | $r(\theta + \pi) = -r(\theta)$ |
| Original rule                    | $r = \sin(3\theta)$            |

Bank label: \_\_\_\_\_

A table gives  $r = 1, 2, 1$  at  $\theta = 0, \frac{\pi}{2}, \pi$ . State the plotted samples in the scalar and polar views.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| $\theta$        | Given r |
|-----------------|---------|
| 0               | 1       |
| $\frac{\pi}{2}$ | 2       |
| $\pi$           | 1       |

| Condition or evidence            | Supplied value       |
|----------------------------------|----------------------|
| Given parameter range or context | three sampled angles |

Bank label: \_\_\_\_\_

AP Precalculus

Radius Graphs vs  
Polar Traces

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A complete scalar radius graph has highest value 5 and lowest value  $-7$ . Repair the claim that its top point must correspond to the farthest polar point.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

| Condition or evidence            | Supplied value                                                  |
|----------------------------------|-----------------------------------------------------------------|
| Given parameter range or context | complete scalar graph                                           |
| Claim to examine                 | The top of the scalar graph is always the farthest polar point. |
| Marked values                    |                                                                 |
|                                  | <div><div><math>-7</math></div></div>                           |
|                                  | <div><div>5</div></div>                                         |

AP Precalculus

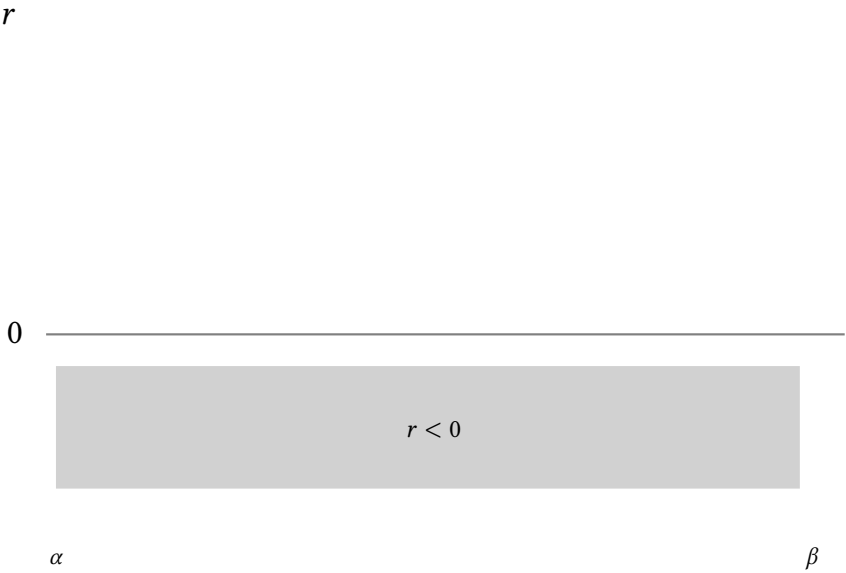
Radius Graphs vs  
Polar Traces

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

**Incoming:** The scalar value decreases, but the distance increases from 1 to 3.

A scalar  $\theta - r$  graph is negative only on  $\alpha < \theta < \beta$  and is zero at both endpoints. Translate this entire feature into a statement about the polar trace.



| Condition or evidence            | Supplied value                                |
|----------------------------------|-----------------------------------------------|
| Boundary values                  | $r(\alpha) = r(\beta) = 0$                    |
| Condition                        | $r(\theta) < 0$ for $\alpha < \theta < \beta$ |
| Given parameter range or context | interval $(\alpha, \beta)$                    |

Your response: \_\_\_\_\_

## AP Precalculus

# Radius Graphs vs Polar Traces

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student says every negative value on  $a\theta - r$  graph becomes a polar point below the  $x$ -axis. Use  $r = -2$  at  $\theta = 3\frac{\pi}{2}$  to repair the claim.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

| Condition or evidence            | Supplied value                                   |
|----------------------------------|--------------------------------------------------|
| Given parameter range or context | $\theta = 3\frac{\pi}{2}$                        |
| Polar pair                       |                                                  |
|                                  | <div>-2</div>                                    |
|                                  | <div><math>3\frac{\pi}{2}</math></div>           |
| Student claim to examine         | negative $r$ means the plotted point has $y < 0$ |

A student claims: “Because the scalar radius repeats after  $\pi$ , the same polar point also repeats after  $\pi$ .” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: The scalar rule  $\cos(2\theta)$  has period  $\pi$ . Does shifting  $\theta$  by  $\pi$  repeat the same polar point?

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

| Condition or evidence            | Supplied value                                                                                  |
|----------------------------------|-------------------------------------------------------------------------------------------------|
| Given parameter range or context | all real $\theta$                                                                               |
| Original rule                    | $r = \cos(2\theta)$                                                                             |
| Given scalar period              | $\pi$                                                                                           |
| Student claim to examine         | Because the scalar radius repeats after $\pi$ , the same polar point also repeats after $\pi$ . |

## Radius Graphs vs Polar Traces

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** On an interval,  $r$  rises from  $-6$  to  $-1$ . Compare the scalar graph's change with the polar point's distance change. Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence            | Supplied value                  |
|----------------------------------|---------------------------------|
| Given parameter range or context | an interval with $r < 0$        |
| Scalar behavior                  | $r$ increases from $-6$ to $-1$ |

**Answer.** The scalar graph increases, while the polar point moves inward because its distance decreases from  $6$  to  $1$ .

#### Explanation and check.

Scalar order uses signed outputs; polar distance uses absolute radius. Those measures move in opposite directions in this negative regime.

#### Worked example

**Problem.** For constant radius  $r = -4$  while  $\theta$  varies, compare scalar output change, polar distance change, and polar position change.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition or evidence            | Supplied value                   |
|----------------------------------|----------------------------------|
| Given parameter range or context | $\theta$ varies over an interval |
| Radius                           | constant $-4$                    |

**Answer.** Scalar output and distance are both constant, but the polar position changes around the circle on opposite rays.

#### Explanation and check.

Neither  $r$  nor  $|r|$  changes. The angle still changes the direction, so the point is not stationary in the plane.

AP Precalculus  
Polar Models in Context

Polar Models in Context | APPC-U03-P30

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
| 01 | Answer Bank Challenge      | 24 tasks     |
| 02 | Grid Rounds                | 24 tasks     |
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| 04 | Error Analysis & Strategy  | 24 tasks     |
| 05 | Quick Answer Key           | 96 answers   |
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| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for feasibility decision, two model choices and noninterchangeability, supported model, admissibility decision, feasibility and repair and decision.

8 PDFs and a README; 56 buyer pages.

## AP Precalculus

Polar Models  
in Context

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

A rotating sensor obeys  $r = 2 + \cos \theta$  and  $\theta = \pi t$ , with  $t$  in seconds. Write its radius as a function of time.

Bank label: \_\_\_\_\_

Held-out radii are 4.1 and 2.2 at  $\theta = \frac{\pi}{3}$  and  $2\frac{\pi}{3}$ . Model A predicts 4.0 and 2.0; Model B predicts 3.6 and 2.7. With tolerance 0.25 at each angle, which model is supported?

Required response

|                  |     |
|------------------|-----|
| $\frac{\pi}{3}$  | 4.1 |
| $2\frac{\pi}{3}$ | 2.2 |

Bank label: \_\_\_\_\_

Engineers measure the maximum audible distance from a siren separately for each compass bearing. Choose a model that directly supports drawing the warning boundary.

Bank label: \_\_\_\_\_

A radius graph has maximum 9 at  $\theta = \pi$  and minimum 3 at  $\theta = 0$ . Select the compatible model from  $r = 6 - 3 \cos \theta$ ,  $r = 6 + 3 \cos \theta$ , and  $r = 3 + 6 \sin \theta$ .

Bank label: \_\_\_\_\_

## AP Precalculus

Polar Models  
in Context

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

|                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                              |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>A boundary circle centered at <math>(3, 0)</math> with radius 3 satisfies <math>r^2 = 6r \cos \theta</math>. Find its nonzero radial model and the angles where it gives a forward boundary point.</p><br><br><br><br><br><br><br><br><br><br>                                                                     | <p>An accepted model <math>r = 5 - \sin \theta</math> is explicitly valid only for <math>0 \leq \theta \leq \pi</math>. A report requests the modeled location at <math>\theta = 7\frac{\pi}{6}</math>. What should be reported?</p><br><br><br><br><br><br><br><br><br><br> |
| <p>Models A and B have equal held-out RMSE 0.18. In two safety-critical directions A's residuals are 0.08 and 0.11, while B's are 0.32 and 0.29; tolerance there is 0.15. Which model is validated for the safety use?</p><br><br><br><br><br><br><br><br><br><br>                                                    | <p>A signed radial signal is <math>r = 5 \cos \theta</math>, and <math>\theta = \pi \frac{t-2}{4}</math> so that the polar axis is reached at <math>t = 2</math>s. Express <math>r</math> as a function of <math>t</math>.</p><br><br><br><br><br><br><br><br><br><br>       |
| <p>A radius graph has maxima at 0 and <math>\pi</math> and minima at <math>\frac{\pi}{2}</math> and <math>3\frac{\pi}{2}</math>. Which candidate fits: <math>r = 5 + 2 \cos \theta</math>, <math>r = 5 + 2 \cos(2\theta)</math>, or <math>r = 5 + 2 \sin(2\theta)</math>?</p><br><br><br><br><br><br><br><br><br><br> | <p>Radar gives the storm edge's distance from a station for every bearing at one instant. Choose a representation for drawing the current edge, then decide whether the same data can predict when the edge reaches a town.</p><br><br><br><br><br><br><br><br><br><br>      |



## AP Precalculus

**Polar Models  
in Context**

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A device claims that  $d(\theta) = 2 \cos \theta$  is its nonnegative distance to an obstacle for every  $0 \leq \theta \leq 2\pi$ . Is the model physically admissible as stated?

An unlabeled polar loop is used to claim that a vehicle moved more slowly where the curve bends sharply. Diagnose the claim.

A student claims: “Model  $B$  has the smaller mean absolute error because it exactly matches the last observation more closely.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Held-out radii are 5.0, 4.2, 3.1, 4.1. Models  $A$  and  $B$  predict (4.9, 4.0, 3.2, 4.3) and (5.4, 4.1, 2.7, 4.0). Which has the smaller mean absolute radial error?

A scanner can measure at most 10 m, but a proposed all-bearing model is  $r = 7 + 4 \cos \theta$ . Is the model within the hardware range?

## Polar Models in Context

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** A rotating sensor obeys  $r = 2 + \cos \theta$  and  $\theta = \pi t$ , with  $t$  in seconds. Write its radius as a function of time.

**Answer.**  $r(t) = 2 + \cos(\pi t)$

#### Explanation and check.

Substitute the supplied angle law  $\theta = \pi t$  into the radial rule.

---

#### Worked example

**Problem.** A circular safety boundary has model  $r = 6$ . What boundary point corresponds to  $\theta = \pi$ ?

**Answer.**  $(-6, 0)$

#### Explanation and check.

At angle  $\pi$ , cosine is  $-1$  and sine is  $0$ , so  $(r \cos \theta, r \sin \theta) = (-6, 0)$ .