Sets & Steps
Math Practice & Worked Solutions

Precalculus

Trig Functions, Equations & Identities | 21-Topic Precalculus Bundle

21-topic bundle | 2,016 problems | Four activity formats

Included topics

The following pages list all 21 included resources in the suggested order, with a link to each product. Each resource retains its complete preview.

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

168 PDFs and 21 READMEs; 2,224 buyer PDF pages across the 21 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Math Practice & Worked Solutions

Precalculus

Included topics

21-topic bundle | 2,016 problems | Four activity formats

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01 **Periodic Functions, Radians, and the Unit Circle**

View individual product and its full preview

02 **Trig Ratios from Triangles and Coordinates**

View individual product and its full preview

03 **Exact Trig Values at Special Angles**

View individual product and its full preview

04 **Tangent Values and Domain**

View individual product and its full preview

05 **Trig Signs and Reference Angles**

View individual product and its full preview

06 **Sine and Cosine Parent Graphs**

View individual product and its full preview

07 **Sinusoidal Amplitude and Midline**

View individual product and its full preview

08 **Period and Frequency of Sinusoids**

View individual product and its full preview

09 **Phase Shifts of Sine and Cosine**

View individual product and its full preview

10 **Writing Sinusoidal Equations from Features**

View individual product and its full preview

11 **Sinusoidal Modeling**

View individual product and its full preview

21 included resources | Complete resource previews follow this guide.

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Precalculus

Included topics

21-topic bundle | 2,016 problems | Four activity formats

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12 **Interpreting Sinusoidal Transformations**

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13 **Tangent Parent Function Graphs**

View individual product and its full preview

14 **Tangent Function Transformations**

View individual product and its full preview

15 **Inverse Trigonometric Functions**

View individual product and its full preview

16 **Solving Trigonometric Equations**

View individual product and its full preview

17 **Trigonometric Inequalities**

View individual product and its full preview

18 **Secant and Cosecant Functions**

View individual product and its full preview

19 **Cotangent Functions**

View individual product and its full preview

20 **Reciprocal, Quotient, and Pythagorean Trig Identities**

View individual product and its full preview

21 **Angle Sum, Difference, and Double-Angle Identities**

View individual product and its full preview

21 included resources | Complete resource previews follow this guide.

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Math Practice & Worked Solutions

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Periodic Functions, Radians, and the Unit Circle | APPC-U03-COMB-P01-P02

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for directed radian measure, supported periodicity claim, recurrence rates and range comparison, exact degrees, exact radians and exact ordering.

8 PDFs and a README; 131 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Express $-2\frac{\pi}{9}$ radians as an exact directed angle in degrees.

Bank label: _____

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

On a circle of radius 4 m, a point rotates through $\frac{3}{8}$ radians. Find the directed arc displacement s and state whether the motion is clockwise or counterclockwise.	For the directed angle $13\frac{\pi}{6}$, describe its terminal ray and list every coterminal representative in the interval $(\pi, 5\pi]$. Respect the endpoint symbols.
An exact periodic graph has period 9. A cycle marker occurs at $t = 5$; the graph reaches a specified decreasing phase 4 time units after every marker. Is $t = 38$ that same phase? Give an equivalent cycle window beginning at the marker.	For the directed angle $15\frac{\pi}{8}$, describe its terminal ray and list every coterminal representative in the interval $(0, 4\pi)$. Respect the endpoint symbols.

AP Precalculus

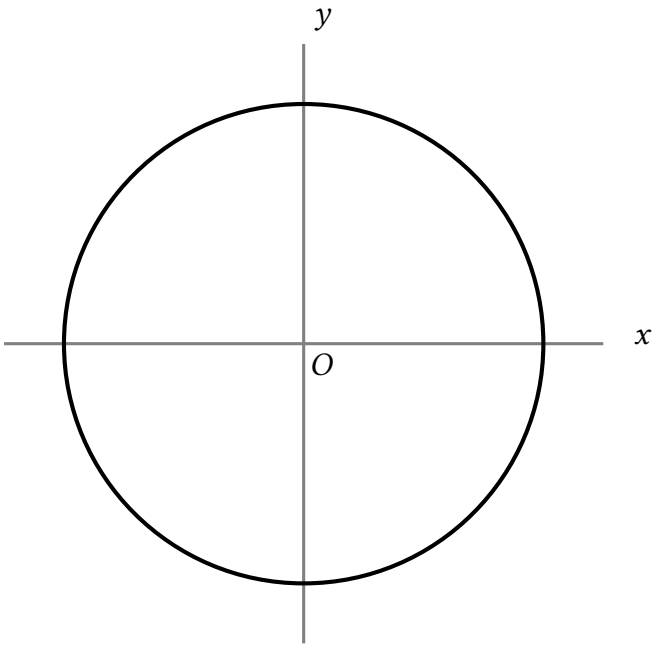
Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: 105°

Point $P = (\frac{8}{17}, y)$ lies on the unit circle and is below the x-axis. Find y and verify the point.



Your response: _____

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims that the smallest period is 7 time units for these exact repeating (sample index, value) pairs: $(0, 78)$; $(1, 71)$; $(2, 76)$; $(3, 73)$; $(4, 72)$; $(5, 73)$; $(6, 82)$; $(7, 78)$; $(8, 71)$; $(9, 76)$; $(10, 73)$; $(11, 72)$; $(12, 73)$; $(13, 82)$. Consecutive sample indices are 3 time units apart. Diagnose the claim, state the corrected smallest period, and cite one decisive comparison.

Identify the error or first invalid claim, give the corrected conclusion, and justify the correction.

sample index	value
0	78
1	71
2	76
3	73
4	72
5	73
6	82

sample index	value
7	78
8	71
9	76
10	73
11	72
12	73
13	82

A circle centered at O has radius 18 cm. A point moves counterclockwise along the circle through an arc of length $\frac{162}{5}$ cm. Find the directed angle in radians and explain how the sign records the sweep direction. Before solving, choose and justify the valid method. Method A: Use radius divided by arc length. Method B: Use signed arc length divided by radius.

Periodic Functions, Radians, and the Unit Circle

Full Worked Solutions - excerpt

Worked example

Problem. Convert the directed rotation 225° to an exact radian measure. Preserve its direction.

Answer. $5\frac{\pi}{4}$ radians

Explanation and check.

Multiply by $\frac{\pi}{180}^\circ$: $225(\frac{\pi}{180}) = 5\frac{\pi}{4}$. The sign is preserved.

Worked example

Problem. Convert the directed rotation -24° to an exact radian measure. Preserve its direction.

Answer. $-2\frac{\pi}{15}$ radians

Explanation and check.

Multiply by $\frac{\pi}{180}^\circ$: $-24(\frac{\pi}{180}) = -2\frac{\pi}{15}$. The sign is preserved.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Trig Ratios from Triangles and Coordinates

Trig Ratios from Triangles and Coordinates | APPC-U03-P03

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for exact coordinate ratios, radius and three ratios, selected exact ratios, scale, ray relation, and ratio behavior, domain and ratio comparison and ray directions and quotient behavior.

8 PDFs and a README; 112 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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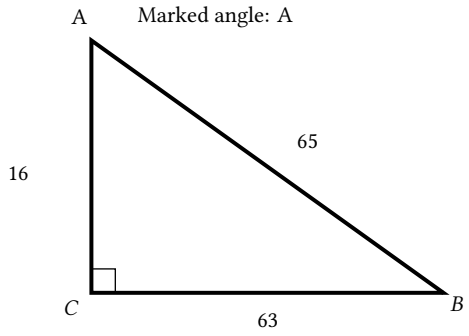
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

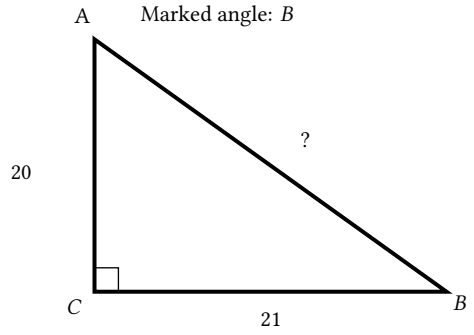
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

In right triangle ABC, $\angle C = 90^\circ$, AC = 16, BC = 63, and AB = 65. The marked angle is $\angle$A; the drawing may be rotated. Identify the opposite, adjacent, and hypotenuse sides relative to $\angle$A, then write its sine, cosine, and tangent.



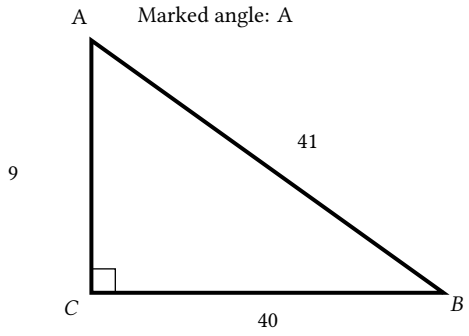
Bank label: _____

Right triangle ABC has $\angle C = 90^\circ$, AC = 20, BC = 21, and marked angle $\angle B$. First recover AB, then use the side roles relative to $\angle B$ to find sine, cosine, and tangent.



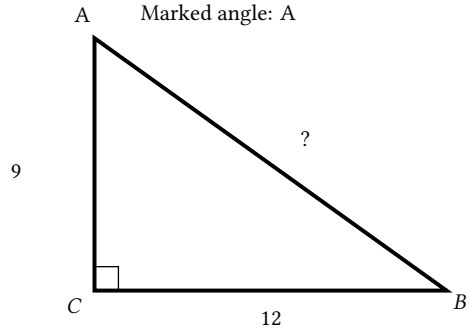
Bank label: _____

In right triangle ABC, $\angle C = 90^\circ$, AC = 9, BC = 40, and AB = 41. The marked angle is $\angle$A; the drawing may be rotated. Identify the opposite, adjacent, and hypotenuse sides relative to $\angle$A, then write its sine, cosine, and tangent.



Bank label: _____

Right triangle ABC has $\angle C = 90^\circ$, AC = 9, BC = 12, and marked angle $\angle$A. First recover AB, then use the side roles relative to $\angle$A to find sine, cosine, and tangent.



Bank label: _____

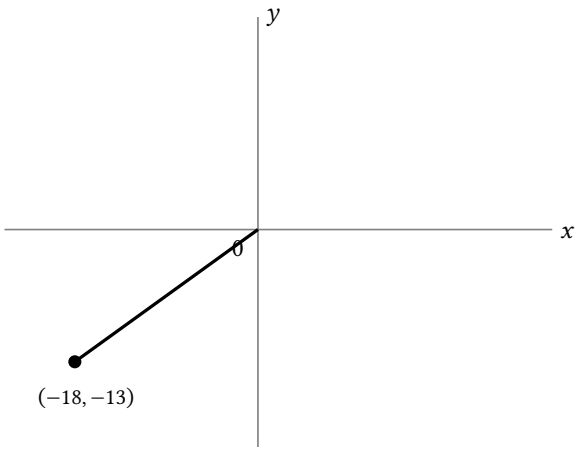
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A graphed ray starts at the origin and passes through $(-18, -13)$. Find its slope and interpret that quotient as $\tan\theta$ for the ray's directed angle.



AP Precalculus

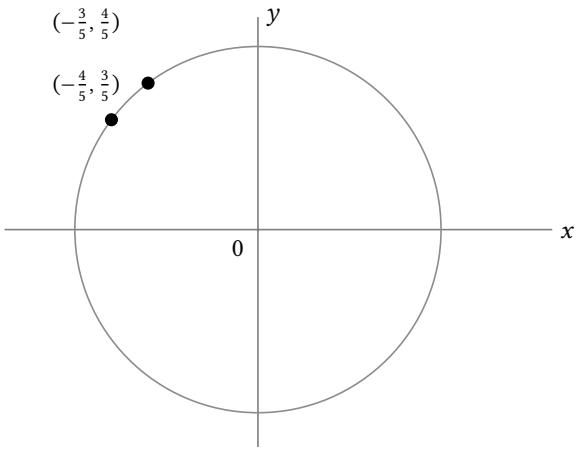
Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

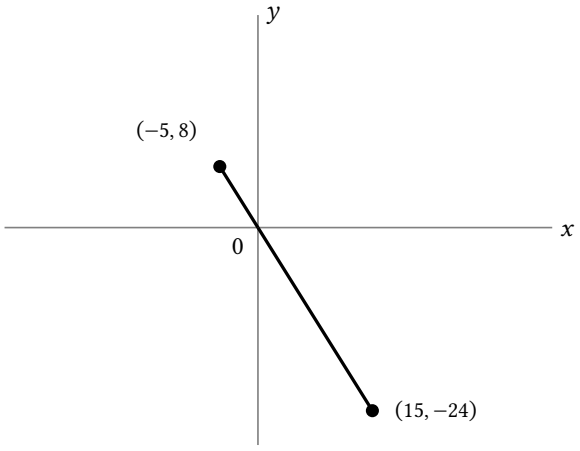
A unit-circle point moves along the shorter counterclockwise arc in the second quadrant from $\left(-\frac{3}{5}, \frac{4}{5}\right)$ to $\left(-\frac{4}{5}, \frac{3}{5}\right)$. Describe the endpoint changes in sine and cosine and compare the initial and final tangent ratios.



Your response: _____

Incoming: sin: $\frac{4}{5} \to \frac{3}{5}$; cos: $-\frac{3}{5} \to -\frac{4}{5}$; tan: $-\frac{4}{3} \to -\frac{3}{4}$

Rays OP and OQ pass through $P = (-5, 8)$ and $Q = (15, -24)$. Compare their directions, slopes, and tangent values. Explain what slope loses that sine and cosine would retain.



Your response: _____

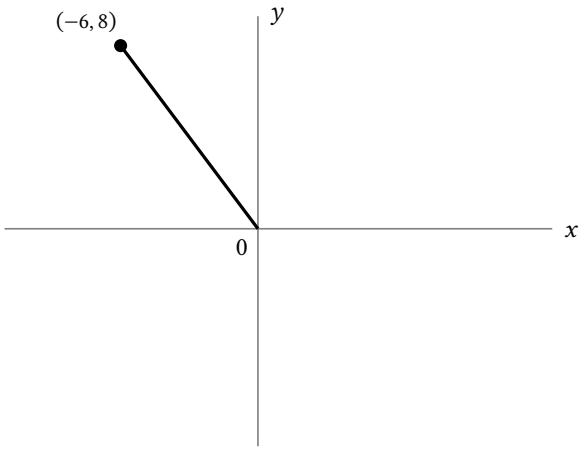
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For ray point $(-6, 8)$, a student assigns a negative radius because a coordinate is negative. Diagnose the error, state the correct r, and repair sine and cosine.



Trig Ratios from Triangles and Coordinates

Full Worked Solutions - excerpt

Worked example

Problem. The only information about a nonzero terminal ray is $\tan\theta = -\frac{7}{4}$. Describe the ray or rays that are determined, give representative point(s), and explain the scale or sign ambiguity that remains.

Answer. Two opposite rays, represented by $(4,-7)$ and $(-4,7)$; any positive scaling on either ray also works.

Explanation and check.

Tangent fixes $\frac{y}{x}$, so it fixes a line through the origin but not which of the two opposite rays or the radial scale.

Worked example

Problem. Can a real nonzero ray satisfy $\sin\theta = \frac{6}{5}$ under the coordinate definition? Decide and prove the conclusion using $|\text{coordinate}| \le r$.

Answer. No real ray satisfies the claim.

Explanation and check.

For every real point, the magnitude of the relevant coordinate is at most r, so $|\sin\theta| \le 1$. Here $\left|\frac{6}{5}\right| > 1$, a contradiction.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exact Trig Values at Special Angles

Exact Trig Values at Special Angles | APPC-U03-P04

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for correct equality and approximation, mode diagnosis and exact value, valid exact and approximate relations, exact equality groups and ordering, all permitted angle labels and completed exact row.

8 PDFs and a README; 128 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A special-angle table row has $(\sin\theta, \cos\theta) = (1, 0)$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate.

Required response

Angle	sin θ	cos θ
_____	1	0

Bank label: _____

On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos\theta = \frac{1}{2}$. Use the unit-circle line intersection and check both endpoint flags.

Bank label: _____

AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

The pair at $\frac{\pi}{3}$ is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. The target ray $\frac{\pi}{6}$ is related by reverse complementary transfer. Track coordinate exchange and any signs to infer its exact pair.

Audit this special-angle table row: $\theta = \frac{\pi}{3}, \sin\theta = \frac{1}{2}, \cos\theta = \frac{1}{2}$. Identify the earliest exact inconsistency and replace the row with the correct pair.

Required response

Angle	sin θ	cos θ
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{1}{2}$

AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A unit-circle table row for $\theta = 7\frac{\pi}{6}$ shows $\cos\theta$ missing and the other coordinate equal to $-\frac{1}{2}$. Complete the row exactly and use the angle's location to justify the square-root sign.

Required response

Angle	sin θ	cos θ
$7\frac{\pi}{6}$	$-\frac{1}{2}$	_____

Your response: _____

Incoming: $\sin\theta = -\frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}$

A special-angle table row has $(\sin\theta, \cos\theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate.

Required response

Angle	sin θ	cos θ
_____	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

Your response: _____

Incoming: $\theta = \frac{\pi}{6}$

On the finite one-turn window $(-\pi, \pi]$, locate every special-angle ray satisfying $\cos\theta = 1$. Use the unit-circle line intersection and check both endpoint flags.

Your response: _____

Incoming: $\theta = 0$

On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos\theta = -1$. Use the unit-circle line intersection and check both endpoint flags.

Your response: _____

AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A calculator in degree mode evaluates the radian expression $\cos(\frac{\pi}{3})$ and displays 0.9998. Diagnose the mode error, give the correct exact value, and explain why the display is not evidence of equality.

Exact Trig Values at Special Angles

Full Worked Solutions - excerpt

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{3}$ on the unit circle and evaluate the ordered pair $(\sin\theta, \cos\theta)$ exactly.

Answer. $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{3}$ has $x = \frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$. Therefore $(\sin\theta, \cos\theta) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$.

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{6}$ on the unit circle and evaluate the ordered pair $(\sin\theta, \cos\theta)$ exactly.

Answer. $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{6}$ has $x = \frac{\sqrt{3}}{2}$ and $y = \frac{1}{2}$. Therefore $(\sin\theta, \cos\theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Tangent Values and Domain

Tangent Values and Domain | APPC-U03-P05

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for classification and decisive evidence, exact slope and sign evidence, classification and earliest error, classification and evidence, exact ordering and earliest error and repair.

8 PDFs and a README; 146 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Give exact answers and all requested reasoning; record a bank letter. If bank entries give equivalent final responses for the same quantity and context, either label is valid and may be reused. A shared numerical value alone does not make entries interchangeable.

Order these exact scalar values from least to greatest, grouping ties and using no decimals: $\tan(2\frac{\pi}{3}), \sin(3\frac{\pi}{2}), \cos(5\frac{\pi}{6})$.

Solve the problem and show enough exact work for the requested result.

$\tan(2\frac{\pi}{3})$
$\sin(3\frac{\pi}{2})$
$\cos(5\frac{\pi}{6})$

Bank label: _____

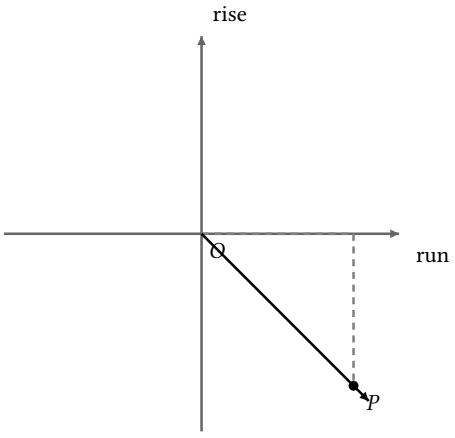
Given $\tan(\frac{\pi}{3}) = \sqrt{3}$, use "negate angle" to infer $\tan(-\frac{\pi}{3})$. Explain whether one or both coordinate signs change.

Bank label: _____

A terminal ray passes through $(-4, -4\sqrt{3})$. Evaluate its exact tangent from rise/run and explain why radius normalization is unnecessary.

Bank label: _____

A marked ray has signed run 12 and rise -12. Calculate its exact tangent and explain which geometric datum controls the sign.



Bank label: _____

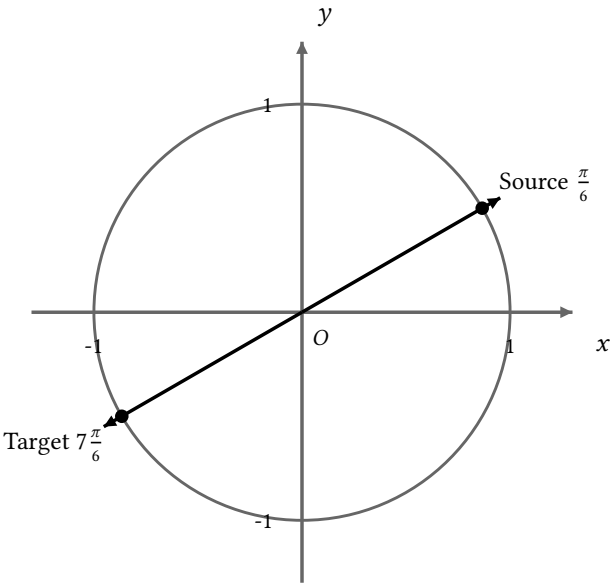
AP Precalculus

Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Given $\tan(\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$, use "add π" to infer $\tan(7\frac{\pi}{6})$. Explain whether one or both coordinate signs change.



AP Precalculus

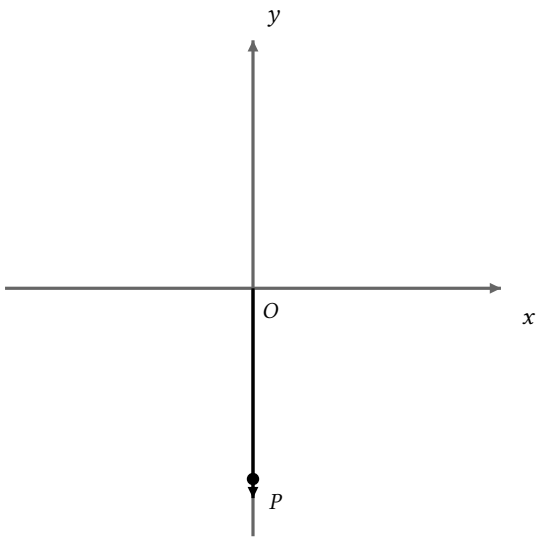
Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\tan(-5\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$

A ray is represented by the exact point $(0, -3)$. Classify its tangent as zero, nonzero, or undefined and cite numerator and denominator roles.



zero
nonzero
undefined

Your response: _____

AP Precalculus

Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the claim "$\tan(\pi)$ is undefined because sine is zero." Classify tangent correctly and identify the first quotient or domain error.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

denominator
permission
sign

A student claims: "Because the x-coordinate contains a radical, only a decimal estimate is possible." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Evidence case: an exact point $(2\sqrt{3}, 2)$. Decide whether it determines an exact tangent, only an estimate, or no domain conclusion. State the decisive evidence and any exact datum still needed.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Value status	x	y
exact	$2\sqrt{3}$	2

Tangent Values and Domain

Full Worked Solutions - excerpt

Worked example

Problem. At $\theta = \pi$, $(\sin\theta, \cos\theta) = (0, -1)$. Evaluate $\tan\theta$ as a signed coordinate quotient and simplify exactly.

Answer. $\tan(\pi) = 0$

Explanation and check.

Cosine -1 is the denominator. The quotient $\frac{0}{-1}$ simplifies to 0.

Worked example

Problem. Given $\tan(3\frac{\pi}{4}) = -1$, use "add π" to infer $\tan(7\frac{\pi}{4})$. Explain whether one or both coordinate signs change.

Answer. $\tan(7\frac{\pi}{4}) = -1$

Explanation and check.

Adding π takes the opposite ray, so both x and y change sign. The quotient $\frac{y}{x}$ is unchanged, giving $\tan(7\frac{\pi}{4}) = -1$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Trig Signs and Reference Angles

Trig Signs and Reference Angles | APPC-U03-P06

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for three signs with coordinate evidence, all allowed angles, terminal location and reference angle, correct boundary classification, possible location or inconsistency and all consistent pairings or contradiction.

8 PDFs and a README; 107 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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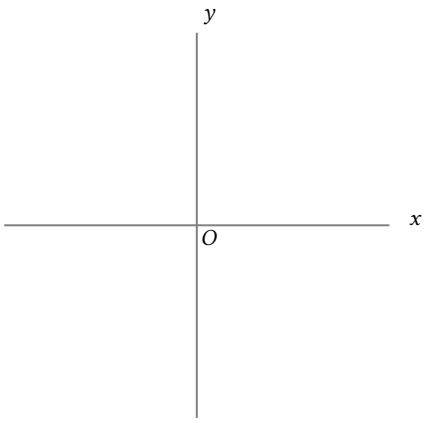
AP Precalculus

Trig Signs and Reference Angles

Name: _____ **Date:** _____ **Class:** _____

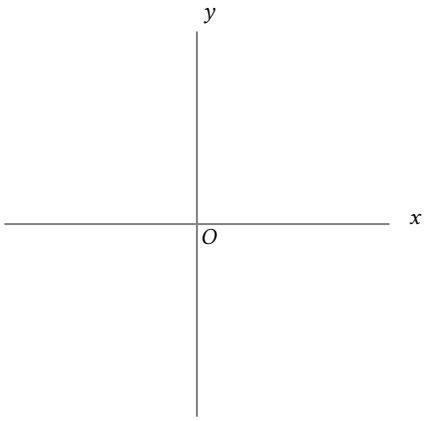
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Intersect the constraints $\sin\theta > 0, \cos\theta \neq 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.



Bank label: _____

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(2\pi - \alpha)$ using $\cos\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.



Bank label: _____

Intersect the constraints $\cos\theta < 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Bank label: _____

Construct every angle in $[0, 2\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant IV. Respect both endpoint flags and include all admitted turns.

Bank label: _____

AP Precalculus

Trig Signs and Reference Angles

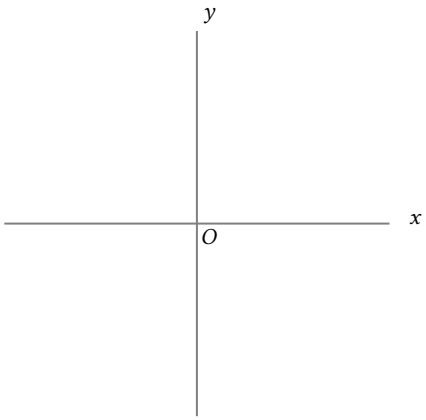
Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For quadrant IV coordinate box with $x > 0, y < 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.

Given evidence	Sine	Cosine	Tangent
$x > 0, y < 0$	_____	_____	_____

For positive y-axis ray with $x = 0, y > 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.



Reduce the standard-position rotation $\theta = 7\frac{\pi}{6}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

Reduce the standard-position rotation $\theta = 11\frac{\pi}{4}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

AP Precalculus

Trig Signs and Reference Angles

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\theta = 2\frac{\pi}{3}$

For negative x-axis ray with $x < 0, y = 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.

Given evidence	Sine	Cosine	Tangent
$x < 0, y = 0$	_____	_____	_____

Your response: _____

Incoming: $\theta = 7\frac{\pi}{6}$

Intersect the constraints $\sin\theta = 0, \cos\theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Your response: _____

Incoming: $5\frac{\pi}{6}$

Intersect the constraints $\sin\theta > 0, \cos\theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Your response: _____

Incoming: quadrant I

Construct every angle in $[0, 4\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant III. Respect both endpoint flags and include all admitted turns.

Your response: _____

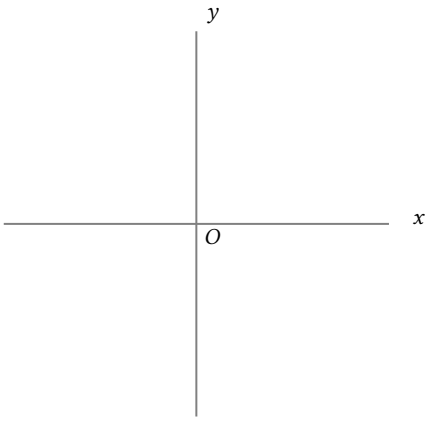
AP Precalculus

Trig Signs and Reference Angles

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(3\pi - \alpha)$ using $\cos\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α. First select and justify the valid method. Method A: Reduce $3\pi - \alpha$ to $\pi - \alpha$ and use the reflected x-coordinate. Method B: Cancel 3π as though it were an integer multiple of a full turn.



Trig Signs and Reference Angles

Full Worked Solutions - excerpt

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\sin(3\pi - \alpha)$ using $\sin\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.

Answer. $\sin(3\pi - \alpha) = \sin\alpha$

Explanation and check.

The reference magnitude is $\sin\alpha$; reduce to $\pi - \alpha$ in quadrant II. Therefore $\sin(3\pi - \alpha) = \sin\alpha$.

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\tan(2\pi - \alpha)$ using $\tan\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.

Answer. $\tan(2\pi - \alpha) = -\tan\alpha$

Explanation and check.

The reference magnitude is $\tan\alpha$; quadrant IV has opposite coordinate signs. Therefore $\tan(2\pi - \alpha) = -\tan\alpha$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sine and Cosine Parent Graphs

Sine and Cosine Parent Graphs | APPC-U03-P07

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for direction, sign, turning, and bending description, exact average rate and interpretation, missing projection and graph identity, parent graph and projection explanation, projection error and correction and missing anchors and verified curve.

8 PDFs and a README; 191 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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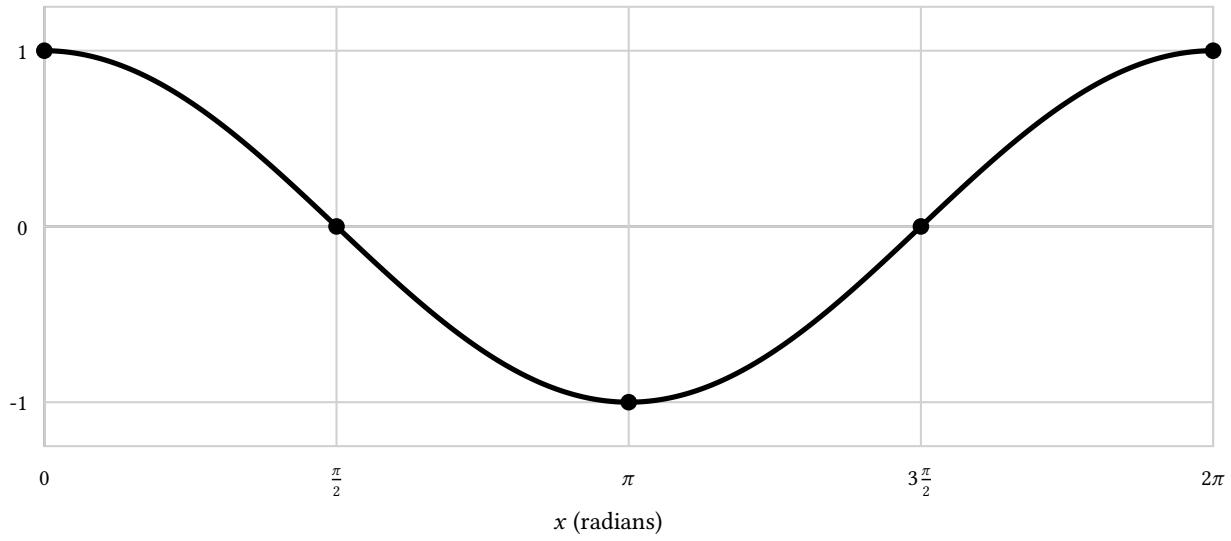
AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

From the labeled parent graph $y = \cos x$ on the closed window $[0, 2\pi]$, identify its zeros. List event coordinates rather than confusing x-locations with output values, and include boundary events.



x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$3\frac{\pi}{2}$	0
2π	1

Bank label: _____

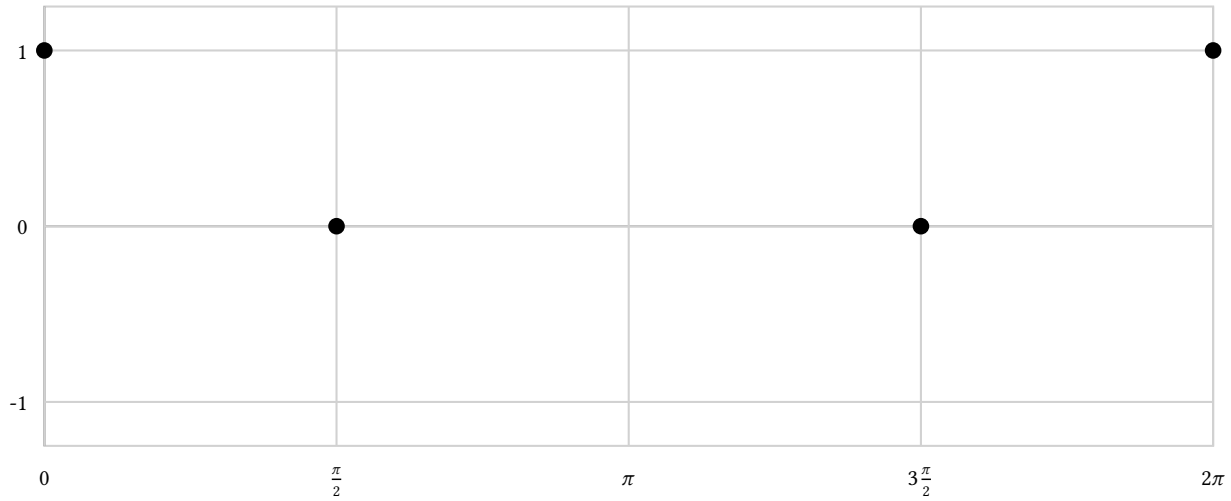
AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For the stated parent $y = \cos x$ on $[0, 2\pi]$, restore every question-mark quarter-turn anchor from the partial data, then sketch one smooth continuous parent curve with exact endpoints.



x	**y**
0	1
$\frac{\pi}{2}$	0
π	_____
$3\frac{\pi}{2}$	0
2π	1

AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $y = \cos x$ on $[0, \frac{\pi}{2}]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Required response

x	y
0	1
$\frac{\pi}{2}$	0

Your response: _____

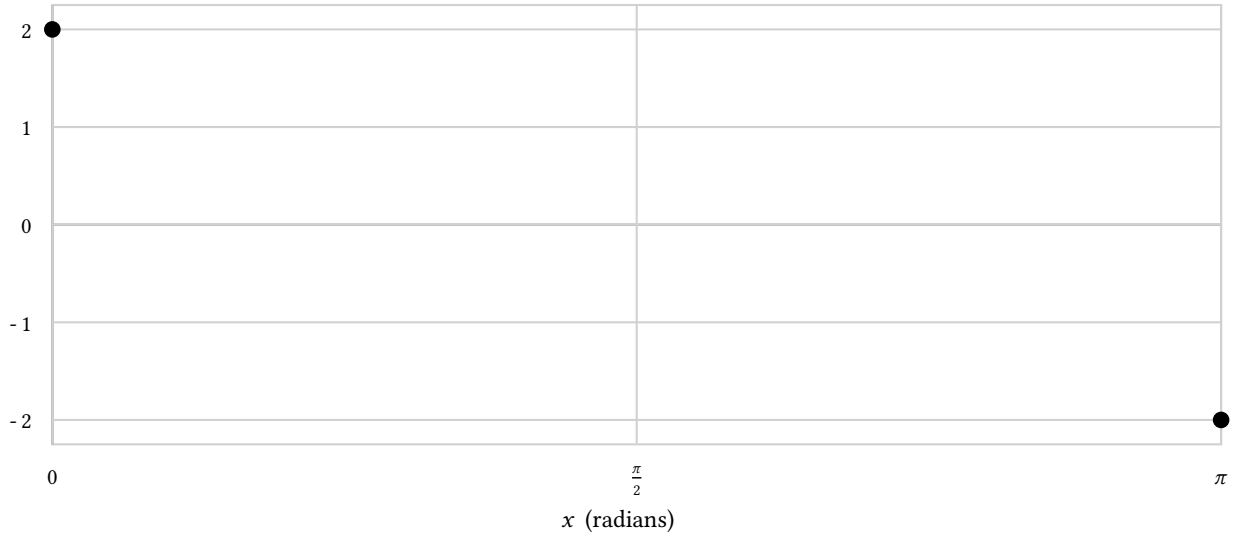
AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the proposed $y = \cos x$ sketch using value at zero, quarter-turn anchors, range, cycle joins, and smoothness. Identify the first structural error and give the smallest sufficient repair.



x	y
0	2
π	-2

Sine and Cosine Parent Graphs

Full Worked Solutions - excerpt

Worked example

Problem. For $y = \cos x$ on $[\frac{\pi}{2}, 2\pi]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Answer. average rate = $\frac{2}{3\pi}$; output rises overall

Explanation and check.

Compute $\frac{(1)-(0)}{(2\pi)-(\frac{\pi}{2})} = \frac{2}{3\pi}$. Thus output rises overall.

Worked example

Problem. For $y = \cos x$ on $[3\frac{\pi}{2}, 2\pi]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Answer. average rate = $\frac{2}{\pi}$; output rises overall

Explanation and check.

Compute $\frac{(1)-(0)}{(2\pi)-(3\frac{\pi}{2})} = \frac{2}{\pi}$. Thus output rises overall.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sinusoidal Amplitude and Midline

Sinusoidal Amplitude and Midline | APPC-U03-P08

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for exact transformed local range, feature comparison and invariant, global pointwise relation and proof, vertical transformation equation, first error and minimal repair and invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

For $y = -4\cos x + 1$, complete the table by applying the vertical output map to each listed parent output. Use the sign of the scale rather than reading a decorative graph height.

Required response

parent output	transformed output
0	_____
1	_____

Bank label: _____

Under $y = 3\sin x + 2$, a transformed output $Y = \frac{23}{4}$ is requested. Reverse the output map to find the required parent output, then decide whether that output is available from the sin parent.

Required response

required parent output	transformed output
_____	$\frac{23}{4}$

Bank label: _____

For $y = 2\cos x + 5$, complete the table by applying the vertical output map to each listed parent output. Use the sign of the scale rather than reading a decorative graph height.

Required response

parent output	transformed output
-1	_____
0	_____
1	_____

Bank label: _____

A complete cycle has minimum 2 and maximum 6. For the supplied parent $p(x) = \sin x$, the labeled anchor has parent output -1 and transformed output 6. Recover amplitude, midline, and every coefficient sign supported by this evidence.

Required response

feature	output	model output	parent output
global minimum	2	_____	_____
global maximum	6	_____	_____
anchor $x = 3\frac{\pi}{2}$	_____	6	-1

Bank label: _____

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $y = (-\frac{1}{2})\cos x + 2$, the horizontal rule is fixed as the cos parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	On the stated x-window $[0, \frac{\pi}{2}]$, the sin parent has certified local output range $[0, 1]$. Under $y = 2\sin x + 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.
For $y = -3\sin x - 2$, the horizontal rule is fixed as the sin parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	On the stated x-window $[-\frac{\pi}{2}, 0]$, the sin parent has certified local output range $[-1, 0]$. Under $y = 4\sin x + 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.
For $y = 4\sin x - 3$, the horizontal rule is fixed as the sin parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	Compare $y = 3\cos x - 1$ and $y = -3\cos x - 1$. State amplitudes, midlines, full-cycle ranges, whether the ranges overlap, relative vertical orientation, and the horizontal invariant.

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

On the stated x-window $[\frac{\pi}{3}, \pi]$, the cos parent has certified local output range $[-1, \frac{1}{2}]$. Under $y = -2\cos x + 2$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.

Your response: _____

Incoming: $[1, 4]$; order reversed under the negative scale

Starting from $p(x) = \cos x$, perform this vertical-only task: Translate parent outputs by $\frac{3}{2}$, scale the result by 2, and finally translate by -5. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Your response: _____

Incoming: $y = 2(\cos x + \frac{3}{2}) - 5$

On the stated x-window $[0, \frac{\pi}{2})$, the cos parent has certified local output range $(0, 1]$. Under $y = 3\cos x - 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.

Your response: _____

Incoming: $(-1, 2]$; order preserved under the positive scale

A complete cycle has minimum -9 and maximum -1. For the supplied parent $p(x) = \cos x$, the labeled anchor has parent output 1 and transformed output -1. Recover amplitude, midline, and every coefficient sign supported by this evidence.

Required response

feature	output	model output	parent output
global minimum	-9	_____	_____
global maximum	-1	_____	_____
anchor $x = 0$	_____	-1	1

Your response: _____

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the student's vertical-feature inference using full-cycle extrema, the affine output map, operation order, and visible-window limits as relevant. Identify the first mathematical error and give the smallest sufficient repair. output map: $Y = -2p + 3$; restricted parent range: $[0, 1]$; student range: $[1, 5]$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

A student claims: "Using the first two rows gives $a = 1$ and $d = 3$, so $Y = \cos x + 3$ fits all three rows." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Fit $Y = ap + d$ to the first two distinct parent-output rows in the supplied cos table, then audit the third row. State whether one vertical-only sinusoid fits all observations.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

parent output	target output
-1	-2
0	1
1	4

Sinusoidal Amplitude and Midline

Full Worked Solutions - excerpt

Worked example

Problem. Starting from $p(x) = \cos x$, perform this vertical-only task: First multiply every cos output by -3, then translate outputs by -1. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Answer. y = $-3\cos x - 1$

Explanation and check.

Apply the actions to the output p in order. $Y = -3p - 1$; hence $y = -3\cos x - 1$.

Worked example

Problem. Starting from $p(x) = \cos x$, perform this vertical-only task: Reflect the graph $y = \cos x$ across the horizontal line $y = 4$. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Answer. y = - $\cos x + 8$

Explanation and check.

Apply the actions to the output p in order. $Y = 2(4) - p = -p + 8$; hence $y = -\cos x + 8$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Period and Frequency of Sinusoids

Period and Frequency of Sinusoids | APPC-U03-P09

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for period and ordinary frequency, normalized frequencies and comparison, possible b values and verification, complete traversals, residual cycle, ending phase, first error and repair and supported candidates and needed evidence.

8 PDFs and a README; 75 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

For $y = \cos((-\frac{\pi}{4})x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: _____	Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 4π; input is measured in radian units. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units before using $\lvert b\rvert = \frac{2\pi}{T}$. Bank label: _____
A sinusoid has period $\frac{4}{3}$ seconds. An inclusive observation window lasts 5 seconds and begins $\frac{1}{2}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal. Bank label: _____	For $y = \cos((4)x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: _____
Construct $y = \sin(bx)$ or $y = \cos(bx)$ with frequency $\frac{1}{8}$ cycles per hour; input is measured in hours. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units if needed before using $\lvert b\rvert = \frac{2\pi}{T}$. Bank label: _____	A sinusoid has period 8 seconds. An inclusive observation window lasts 17 seconds and begins $\frac{7}{8}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal. Bank label: _____
For $y = \cos((3\frac{\pi}{2})x)$, the argument is in radians and x is measured in hours. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: _____	Construct $y = \sin(bx)$ or $y = \cos(bx)$ with frequency 2 cycles per second; input is measured in seconds. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units if needed before using $\lvert b\rvert = \frac{2\pi}{T}$. Bank label: _____

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

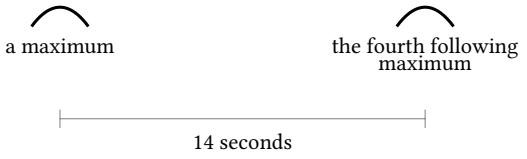
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Map the listed parent-angle anchors u to the graph of $y = \cos(-3x)$. Complete the transformed x-inputs using $u = bx$, retain each parent output, and state whether left-to-right anchor order is preserved or reversed.

Required response

parent angle	parent output	transformed input
0	1	_____
$\frac{\pi}{2}$	0	_____
π	-1	_____
$3\frac{\pi}{2}$	0	_____
2π	1	_____

A sinusoidal graph labels a maximum and the fourth following maximum 14 seconds apart. The graph explicitly certifies the stated events as consecutive or gives the exact number skipped, and crossing directions are shown. Determine the smallest period and justify the cycle fraction used.



Map the listed parent-angle anchors u to the graph of $y = \cos(\frac{1}{3}x)$. Complete the transformed x-inputs using $u = bx$, retain each parent output, and state whether left-to-right anchor order is preserved or reversed.

Required response

parent angle	parent output	transformed input
0	1	_____
$\frac{\pi}{2}$	0	_____
π	-1	_____
$3\frac{\pi}{2}$	0	_____

Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 360; input is measured in degrees of physical input. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units before using $|b| = \frac{2\pi}{T}$.

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A sinusoid has period 3 seconds. An inclusive observation window lasts 8 seconds and begins 0 of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal.

Your response: _____

Incoming: 2 complete cycles; residual $\frac{2}{3}$ cycle; ending phase $\frac{2}{3}$ cycle

Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 5; input is measured in minutes. The parent phase at the origin is fixed. Respect the stated sign information (positive) and list every supported b, converting units before using $|b| = \frac{2\pi}{T}$.

Your response: _____

Incoming: b = $2\frac{\pi}{5}$

For $y = \cos((\frac{1}{2})x)$, the argument is in radians and x is measured in radian input. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Your response: _____

Incoming: period 4π; frequency $\frac{1}{4\pi}$ cycles per input unit

A sinusoid has period 2 seconds. An inclusive observation window lasts 6 seconds and begins $\frac{1}{4}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal.

Your response: _____

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The supplied cos sample table is finite. Audit the listed candidate rules against every row and decide whether the period is identified. Explain any aliasing and name one minimal additional sample or bound that would resolve or narrow it. Measure the trigonometric arguments in radians. Compare the candidate rules $\cos(2\pi t), \cos(4\pi t)$. Work with sinusoidal recurrence with fixed origin and phase. No additional constraint is supplied. Only the displayed finite samples are given. The supplied samples are $(\text{output } 1;\ t0), (\text{output } -\frac{1}{2};\ t\frac{1}{3}), (\text{output } -\frac{1}{2};\ t\frac{2}{3}), (\text{output } 1;\ t1)$.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

output	t
1	0
$-\frac{1}{2}$	$\frac{1}{3}$
$-\frac{1}{2}$	$\frac{2}{3}$
1	1

Audit the proposed horizontal-scaling explanation. Check nonconstancy, full-cycle phase change, landmark direction, sampling limits, and input units as relevant. Identify the first error and give the smallest sufficient repair. Work with sinusoidal recurrence with fixed origin and phase. Use $y = \sin(\pi\frac{t}{30})$. A student claims: frequency is $\frac{\pi}{30}$ cycles/s Measure t in seconds.

Period and Frequency of Sinusoids

Full Worked Solutions - excerpt

Worked example

Problem. For $y = \sin((\pi)x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Answer. period 2; frequency $\frac{1}{2}$ cycles per input unit

Explanation and check.

For a nonconstant parent, $T = \frac{2\pi}{|b|} = 2$. Then ordinary frequency is $\frac{1}{T} = \frac{1}{2}$; b instead measures radians of phase per input unit.

Worked example

Problem. For $y = \sin((6)x)$, the argument is in radians and x is measured in days. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Answer. period $\frac{\pi}{3}$; frequency $\frac{3}{\pi}$ cycles per input unit

Explanation and check.

For a nonconstant parent, $T = \frac{2\pi}{|b|} = \frac{\pi}{3}$. Then ordinary frequency is $\frac{1}{T} = \frac{3}{\pi}$; b instead measures radians of phase per input unit.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Phase Shifts of Sine and Cosine

Phase Shifts of Sine and Cosine | APPC-U03-P10

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for equivalence and canonical representatives, first error and minimal repair, new phase shift and event verification, shifted anchors and window qualification, phase classes or consistency conclusion and phase class and canonical representative.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \cos x$ horizontally by $h = 3\frac{\pi}{2}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** -1	_____
x $-\frac{\pi}{2}$ **y** 0	_____
x 0 **y** 1	_____
x $\frac{\pi}{2}$ **y** 0	_____

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: upward midline; x: 1); period: 4; signed amplitude: 1

Required response

event	x
upward midline	1

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (event: minimum; x: 7); period: 10; signed amplitude: 1

Required response

event	x
minimum	7

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: downward midline; x: 5); period: 8; signed amplitude: 1

Required response

event	x
downward midline	5

Bank label: _____

AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \sin x$ horizontally by $h = 13\frac{\pi}{4}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** 0	_____
x 0 **y** 0	_____
x π **y** 0	_____
x 2π **y** 0	_____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (direction: not specified; level: midline; x: 3); period: 8; signed amplitude: 1

Required response

direction	level	x
not specified	midline	3

Translate every listed landmark of $y = \cos x$ horizontally by $h = -4\frac{\pi}{3}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** -1	_____
x $-\frac{\pi}{2}$ **y** 0	_____
x 0 **y** 1	_____
x $\frac{\pi}{2}$ **y** 0	_____

For y=-cos(2π(x-h)/10), amplitude sign and period are fixed. A labeled maximum occurs at x=7. Find h modulo the period and give the representative in [0,10).

AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: First issue: a positive-sine peak occurs a quarter period after h, so $h = 0$; repair/defense: subtract $\frac{T}{4}$ from the peak location.

Rewrite the radian argument of y=cos((7/3)x-14π/9) in the form b(x-h). State h in x-units and distinguish it from the angular constant -14π/9.

Your response: _____

Incoming: not equivalent; representatives $\frac{9}{2}$ and 2 differ modulo 5

Two sine rules have the same signed amplitude and period 7, with shifts h_1=−2 and h_2=12. Decide whether they define the same graph. Reduce each shift modulo the period; a half-period difference does not count as equivalent when the amplitude sign is fixed.

Your response: _____

Incoming: $h \equiv \frac{3}{2} \pmod 5$; canonical $h = \frac{3}{2}$

Rewrite the radian argument of y=sin((-4/3)x+10π/9) in the form b(x-h). State h in x-units and distinguish it from the angular constant 10π/9.

Your response: _____

Incoming: equivalent; difference 12 = 1 period(s); common representative 7

For y=sin(2π(x-h)/(5/2)), amplitude sign and period are fixed. A labeled minimum occurs at x=19/8. Find h modulo the period and give the representative in [0,5/2).

Your response: _____

Incoming: argument $\frac{7}{3}(x - 2\frac{\pi}{3})$; $h = 2\frac{\pi}{3}$

Rewrite the radian argument of y=sin((-9/4)x-27π/8) in the form b(x-h). State h in x-units and distinguish it from the angular constant -27π/8.

Your response: _____

Incoming: argument $10(x - 3\frac{\pi}{10})$; $h = 3\frac{\pi}{10}$

Rewrite $y = 2\cos(2\pi\frac{x-\frac{3}{2}}{9})$ using the sin reference while holding period and midline fixed. Track the signed coefficient and explain the quarter-cycle phase convention; for advanced cases give both same-sign and opposite-sign target representatives.

Your response: _____

AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the proposed phase-shift claim. Normalize the argument, use the named event and signed family, and reduce representatives modulo the fixed period. Identify the first error or defend a valid noncanonical shift.

Given: domain: phase analysis with amplitude and period fixed; rule: $y = \sin(2x - \pi)$; student claim: shift right π

A student applies the horizontal shift to output values instead of translating the input landmarks with the stated sign. Diagnose the first mathematical error and state the correction. Then complete the original task: Translate every listed landmark of $y = \sin x$ horizontally by $h = \frac{\pi}{2}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x 0 **y** 0	_____
x $\frac{\pi}{2}$ **y** 1	_____
x π **y** 0	_____

Phase Shifts of Sine and Cosine

Full Worked Solutions - excerpt

Worked example

Problem. Rewrite the radian argument of y=cos((-7/2)x+21π/4) in the form b(x-h). State h in x-units and distinguish it from the angular constant 21π/4.

Answer. argument $-\frac{7}{2}(x-3\frac{\pi}{2})$; $h=3\frac{\pi}{2}$

Explanation and check.

Solve $-\frac{7}{2}(x-h)=(-\frac{7}{2})x+21\frac{\pi}{4}$. Since $h=-\frac{c}{b}, h=3\frac{\pi}{2}$; the inside constant itself is not the displacement unless $b=-1$.

Worked example

Problem. Rewrite the radian argument of y=cos(10x-3π) in the form b(x-h). State h in x-units and distinguish it from the angular constant -3π.

Answer. argument $10(x-3\frac{\pi}{10})$; $h=3\frac{\pi}{10}$

Explanation and check.

Solve $10(x-h)=10x-3\pi$. Since $h=-\frac{c}{b}, h=3\frac{\pi}{10}$; the inside constant itself is not the displacement unless $b=-1$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Writing Sinusoidal Equations from Features

Writing Sinusoidal Equations from Features | APPC-U03-P11

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for existence decision and first contradiction, equivalence decision and decisive normalized evidence, one compatible rule and sufficiency explanation, one complete rule and parameter verification, one example rule or impossibility proof and one sine rule with directional check.

8 PDFs and a README; 152 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$-\frac{5}{2}$
next global maximum	1	$\frac{5}{2}$

Bank label: _____

Determine which sinusoidal parameters are forced by the partial exact features and which remain free. Demonstrate any nonuniqueness by checking the two explicit candidate constructions, or replace either if it fails.

The first proposed rule is $y = 3\cos(2\pi\frac{x+\frac{5}{2}}{6})$.

The second proposed rule is $y = 3\cos(2\pi\frac{x+\frac{5}{2}}{3})$.

The global range is $[-3, 3]$.

The two stated maxima are not specified to be consecutive.

Maxima occur at $x = -\frac{5}{2}$ and $x = \frac{7}{2}$.

Bank label: _____

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$\frac{3}{2}$
next global maximum	3	$\frac{21}{2}$

Bank label: _____

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 10.

The claimed range is $[-4, 4]$.

The reference input is $x = -\frac{3}{2}$.

The first proposed rule is $y = 4\sin(2\pi\frac{x+\frac{3}{2}}{10})$.

The second proposed rule is $y = 4\cos(2\pi\frac{x-1}{10})$.

Bank label: _____

AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

An exact partial graph is supplied with its global range, labeled events and directions, and the stipulation that the curve continues sinusoidally. Construct one compatible cosine rule and explain why clipping does or does not hide needed information.

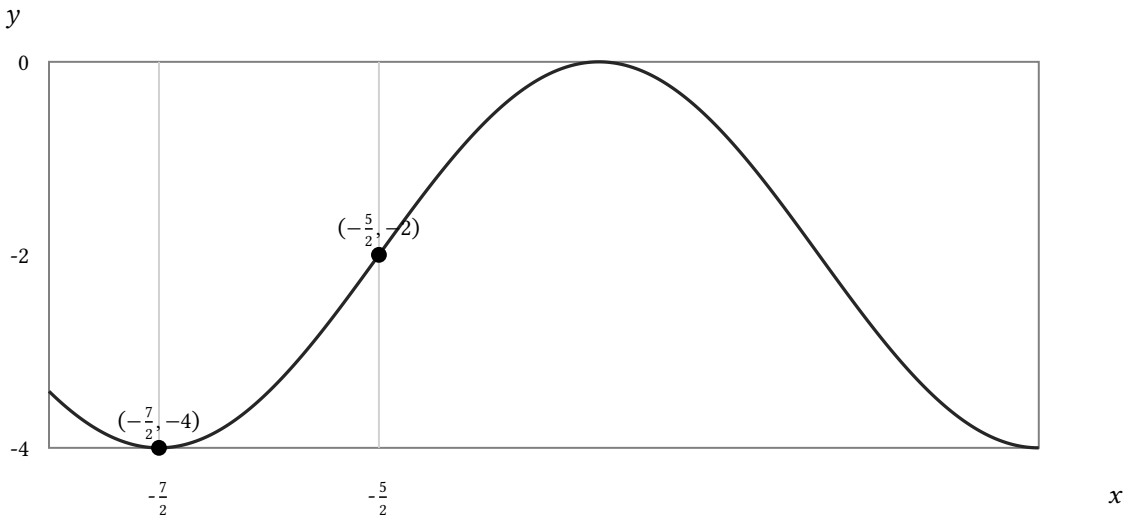
The curve continues sinusoidally beyond the displayed interval.

The global range is $[-4, 0]$.

The labeled minimum is $(-\frac{7}{2}, -4)$.

The labeled upward midline is $(-\frac{5}{2}, -2)$.

The displayed x-interval is $[-4, \frac{1}{2}]$.



event	x	y
minimum	$-\frac{7}{2}$	-4
upward midline	$-\frac{5}{2}$	-2

AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

event	value	x
global maximum	0	$\frac{21}{2}$
next global maximum	0	$\frac{57}{2}$
global minimum	-6	not specified

Your response: _____

AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the student's multi-parameter reconstruction against every exact feature. Identify the earliest failed parameter relation, retain valid deductions, give a corrected complete rule, and verify the repaired model.

Preserve the valid deductions when repairing the model.

The supplied features are: range $[0, 6]$, period 4, upward midline at $x = 1$.

The student proposes $y = 3 + 3\sin(2\pi\frac{x-1}{8})$.

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 14.

The claimed range is $[-3, 1]$.

The reference input is $x = \frac{11}{2}$.

The first proposed rule is $y = -1 + 2\sin(2\pi\frac{x-\frac{11}{2}}{14})$.

The second proposed rule is $y = -1 + 2\sin(2\pi\frac{x-9}{14})$.

Writing Sinusoidal Equations from Features

Full Worked Solutions - excerpt

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = 1$.

The global range is $[-2, 2]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-2+2}{2} = 0$, not 1. Therefore the feature set is inconsistent.

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = -1$.

The global range is $[-4, 0]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-4+0}{2} = -2$, not -1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-4+0}{2} = -2$, not -1. Therefore the feature set is inconsistent.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sinusoidal Modeling

Sinusoidal Modeling | APPC-U03-P12

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for family decision with evidentiary limit, model recommendation or insufficiency conclusion, invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result, invalid assumption and actionable repair, plausible fitted rule and uncertainty statement and signed-residual diagnosis and revision recommendation.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Sinusoidal Modeling

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 3\sin(2\pi\frac{t+\frac{5}{2}}{8})$. The event level is $y = 10$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[-\frac{31}{6}, \frac{27}{2}]$.

Bank label: _____

The contextual extrema are exact and the cycle is stipulated stationary only on the feasible time domain. Construct one sinusoidal model in the stated time unit, explain which events determine a full or half cycle, and retain the domain restriction. The feasible time domain is $[8, 80]$. Time is measured in days.

Required response

event	time	value
minimum	8	11
next minimum	26	11

Bank label: _____

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 20 + 3\cos(2\pi\frac{t+3}{8})$. The response is measured in units. The sample time window is $[-11, 13]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = -3$. Time is measured in hours.

Bank label: _____

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 3\sin(2\pi\frac{t-\frac{11}{2}}{14})$. The event level is $y = 12$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[\frac{5}{6}, \frac{67}{2}]$.

Bank label: _____

AP Precalculus

Sinusoidal Modeling

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compare the fitted sinusoids for the stated use case. Weigh observed-window error, physical range, prediction horizon, and divergence. If the evidence cannot distinguish them for the intended decision, recommend a specific validation measurement. The intended decision is at $t = 34$. The observed time window is $[0, 13]$. The response tolerance is 1 unit.

Required response

future prediction	model	observed window RMSE	period
21	A	0.43	11
26	B	0.45	14

The dataset and fully specified candidate regression outputs are supplied. Select every usable model in the stated angle mode. Verify at least one observed or held-out landmark and do not infer software behavior that is not shown. Use radians mode on the calculator.

Required response

formula	label
$y = 3 + 2\cos(2\pi\frac{t-10}{8})$	A
$y = 3 + 2\sin(2\pi\frac{t-8}{8})$	B

t	y
10	5
14	1
18	5

Use the signed residual definition shown. Assess both the pattern across phases and the held-out observation; a small total error does not excuse a systematic pattern. Recommend retention, revision, or investigation. The absolute tolerance is 0.75. At held-out time $t = 5$, the signed residual is 0.11.

Required response

observed minus predicted	t
0.11	0
-0.11	1
0.0	2
0.11	3
-0.11	4

Use several measurements rather than a single high or low value to estimate a stationary sinusoid. Give a plausible rule, identify the approximate baseline, amplitude, period, and phase, and state what the finite noisy sampling cannot establish exactly. The measured response is in units. Time is measured in hours.

Required response

measurement	time
24.2	$\frac{13}{2}$
19.8	$\frac{23}{2}$
20.2	$\frac{43}{2}$
23.8	$\frac{53}{2}$

AP Precalculus

Sinusoidal Modeling

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 23 + 4\cos(2\pi\frac{t-4}{12})$. The response is measured in units. The sample time window is $[-8, 28]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = 13$. Time is measured in hours.

Your response: _____

Incoming: Prediction 8: 23 units; interpolation inside the sampled window.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 4\sin(2\pi\frac{t-\frac{5}{2}}{8})$. The event level is $y = 12$. Find the second occurrence of the midline level in the window, counting either direction. Time is measured in hours. Use the time window $[\frac{5}{2}, \frac{21}{2}]$.

Your response: _____

Incoming: Event case 6: $t = \frac{13}{2}$ (downward).

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 9. Initially the point is initial vertical velocity zero, then downward. Use seconds as the time unit for t. The operating window is $\frac{1}{2} \le t \le \frac{85}{2}$. The radius is 6. The starting position is top. The starting time is $t = \frac{1}{2}$. One uniform rotation takes 14 seconds.

Your response: _____

Incoming: $y = 9 + 6\cos(2\pi\frac{t-\frac{1}{2}}{14})$

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Treat negative depth as physically valid. A fitted water-depth sinusoid has range $[-2, 8]$ meters.

Your response: _____

Incoming: Water depth in $[-2, 8]$ meters includes negative depth. Restrict use to nonnegative depths or refit with a nonnegative constrained model.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 6\sin(2\pi\frac{t-\frac{1}{2}}{14})$. The event level is $y = 10$. Find the all upward midline crossings in the window. Time is measured in hours. Use the time window $[\frac{1}{2}, \frac{57}{2}]$.

Your response: _____

Incoming: Event case 4: $t = \frac{1}{2}, \frac{29}{2}, \frac{57}{2}$.

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 7. Initially the point is initial vertical velocity zero, then upward. Use seconds as the time unit for t. The operating window is $-\frac{5}{2} \le t \le \frac{43}{2}$. The radius is 3. The starting position is bottom. The starting time is $t = -\frac{5}{2}$. One uniform rotation takes 8 seconds.

Your response: _____

AP Precalculus

Sinusoidal Modeling

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Combine times without a common clock origin. Tidal observations are in hours after midnight on different dates.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Repair only the phase of a stationary sinusoid. A sensor baseline rises steadily while residuals alternate.

Sinusoidal Modeling

Full Worked Solutions - excerpt

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Use $T = 3$ with t measured in seconds. The fitted period is 3 minutes but t is entered in seconds.

Answer. Assumption repair 3: convert the period to 180 seconds, or convert every input to minutes.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to convert the period to 180 seconds, or convert every input to minutes.

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Extrapolate the fitted cycle indefinitely. Seasonal data cover one year during an unusual shutdown.

Answer. Assumption repair 4: limit claims to the observed window and gather ordinary-year validation data.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to limit claims to the observed window and gather ordinary-year validation data.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Interpreting Sinusoidal Transformations

Interpreting Sinusoidal Transformations | APPC-U03-P13

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for changed and invariant physical features, relevant common-scale comparison, first misleading claim and corrected quantitative statement, phase event and initial-value qualification, contextual attained range and boundary status and parameter meanings and derived features.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes and $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 230 + 40\cos(2\pi\frac{t-90}{300})$; output unit before: centimeters. Bank label: _____	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 11 + 3\cos(2\pi\frac{t-3}{16})$; valid window: $0, 64$. Bank label: _____
Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 210 + 40\cos(2\pi\frac{t-30}{360})$; output unit before: centimeters. Bank label: _____	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 9 + 5\cos(2\pi\frac{t-4}{8})$; valid window: $0, 32$. Bank label: _____

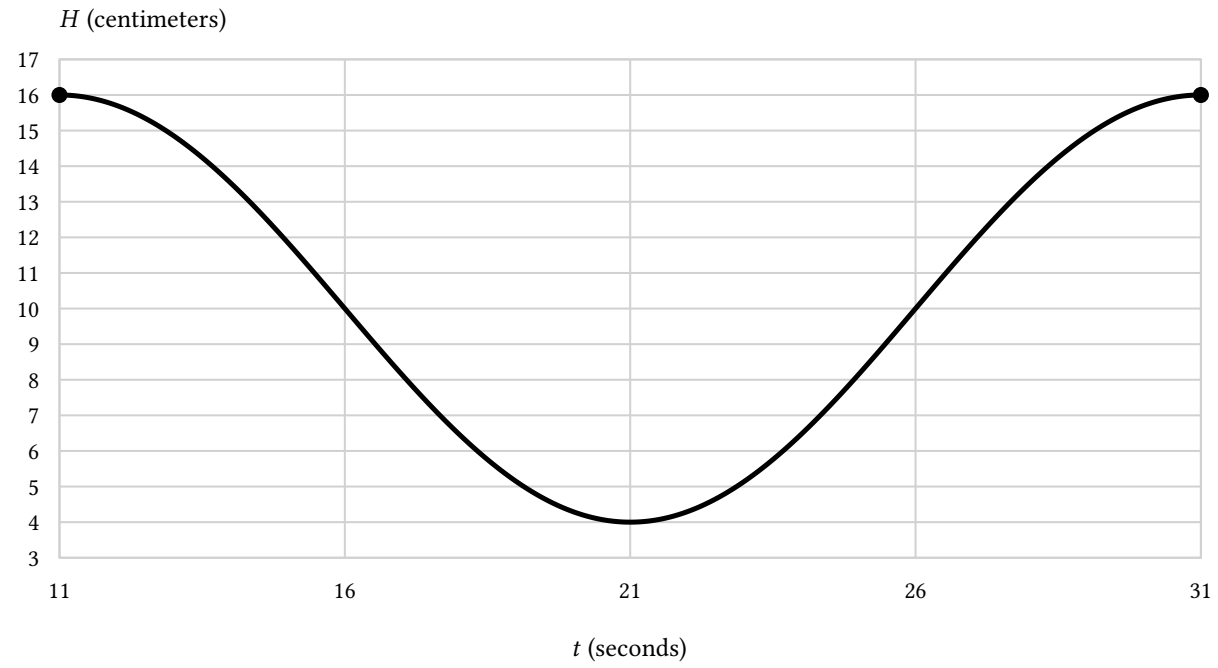
AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Interpret the supplied graph only on its physical operating window. State the attainable output interval, midline meaning, and whether displayed boundaries are attained; do not substitute the full mathematical range when the window clips an extremum.



AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{16})$; only changed category: period; original formula: $P(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{8})$; output unit: units; time unit: hours.

Your response: _____

Incoming: period doubles from 8 to 16 time units, so recurrence halves; range and the anchor at $t = \frac{7}{2}$ stay fixed.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: new datum is 52 cm above the old datum, so $R = Q - 52$; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 220 + 60\cos(2\pi\frac{t-60}{240})$; output unit before: centimeters.

Your response: _____

Incoming: $R(t) = 168 + 60\cos(2\pi\frac{t-60}{240})$; event times are unchanged while every height label decreases by 52 cm.

Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: meters; supplied formula: $Q(t) = 10 + 4\sin(2\pi\frac{t+\frac{3}{2}}{10})$; valid window: $0, 40$.

Your response: _____

Incoming: amplitude 4 meters; midline 10 meters; period 10 hours; frequency $\frac{1}{10}$ cycle per hour; $h = -\frac{3}{2}$ hours labels an upward midline crossing.

Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 6 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12})$, $B(t) = 8 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12})$; practical measure: same-period.

Your response: _____

Incoming: same period 12 hours and same spread 10, but B's baseline and range are 2 units higher.

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(\tau) = 8 + 5\sin(2\pi\frac{\tau}{10})$; only changed category: clock; original formula: $P(t) = 8 + 5\sin(2\pi\frac{t-4}{10})$; output unit: units; time unit: hours.

Your response: _____

Incoming: with t = $\tau + 4$, the same physical anchor is labeled $\tau = 0$ rather than $t = 4$; range and physical period are unchanged.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 200 + 40\cos(2\pi\frac{t}{360})$; output unit before: centimeters.

Your response: _____

AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Test the student's contextual parameter statement against the supplied model. Identify the first misleading claim and replace it with a quantitatively correct statement including units or window limits where relevant. context units: stated in formula or prompt; student statement: The period is 4π.; supplied model: $Q(t) = 4 + 3\sin(4\pi(t - 2))$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Interpreting Sinusoidal Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 2\sin(2\pi\frac{t+2}{8}), B(t) = 5 + 2\sin(2\pi\frac{t+2}{16})$; practical measure: same-range.

Answer. same range $[3, 7]$, but B recurs every 16 rather than 8 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[3, 7]$, but B recurs every 16 rather than 8 hours.

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 4\sin(2\pi\frac{t-1}{10}), B(t) = 5 + 4\sin(2\pi\frac{t-1}{20})$; practical measure: same-range.

Answer. same range $[1, 9]$, but B recurs every 20 rather than 10 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[1, 9]$, but B recurs every 20 rather than 10 hours.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Tangent Parent Function Graphs

Tangent Parent Function Graphs | APPC-U03-P14

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for signed unbounded behavior without an assigned value, method/conclusion choice, justification, and endpoint-aware output interval, tangent entries and branch conclusion, one disconnected increasing branch, domain, range, and monotonicity and endpoint-aware output interval.

8 PDFs and a README; 140 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Tangent Parent Function Graphs

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Select the candidate that can represent $y = \tan(x)$. Defend the choice with structural evidence and explain why local steepness or a cropped rising segment, by itself, cannot settle the match.

The axes have exact radian labels: yes.

Compare these candidates: A = decreasing branches between the same cosine-zero gaps; B = a continuous bounded wave with range $[-1, 1]$; $C = U$-shaped reciprocal branches with no x-intercepts; D = an odd graph through $(0, 0)$ whose central component approaches -∞ and +∞ at its excluded ends.

Apply this decision constraint: Distinguish increasing tangent branches from decreasing cotangent branches.

Some candidate cards are cropped: no.

Work on the domain structural identification of the tangent parent.

Bank label: _____

Complete the tangent column using $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Mark a row undefined rather than dividing by zero, then use the ordered rows to state one graph conclusion; do not connect points across an undefined input.

Required response

$\cos(x)$	$\sin(x)$	x
-1.000	0.025	$-\pi - 0.025$
-1.000	0.000	$-\pi$
-1.000	-0.025	$-\pi + 0.025$

Bank label: _____

AP Precalculus

Tangent Parent Function Graphs

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

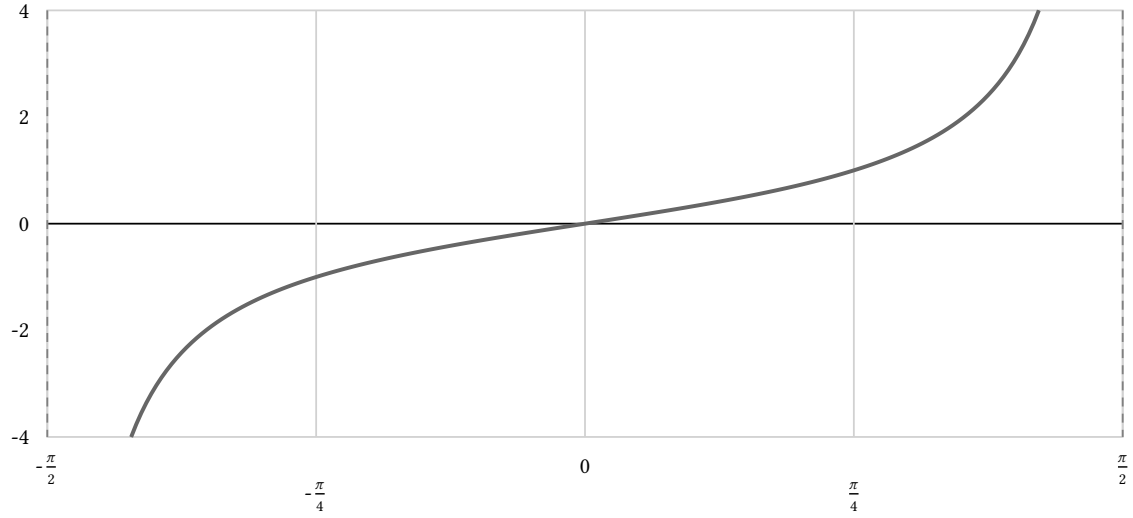
Reconstruct $y = \tan(x)$ on the displayed window using half-turn translations of the supplied central branch. List every visible zero and vertical asymptote, keep the branches disconnected, and state how many connected branch pieces meet the interior of the window.

The source branch is $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

The target window is display endpoints included is yes; left is $-9\frac{\pi}{4}$; right is $3\frac{\pi}{4}$.

The translation rule is (x, y) maps to $(x + k\pi, y)$.

Work on the domain half-turn repetition of the tangent parent.



AP Precalculus

Tangent Parent Function Graphs

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: the claim is false on $[5\frac{\pi}{4}, 7\frac{\pi}{4})$. The component ranges are $[1, \infty)$ and $(-\infty, -1)$; tangent increases on each component, but the gap at $3\frac{\pi}{2}$ prevents a single increasing interval claim.

Complete the tangent column using $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Mark a row undefined rather than dividing by zero, then use the ordered rows to state one graph conclusion; do not connect points across an undefined input.

Required response

$\cos(x)$	$\sin(x)$	x
0.999	-0.050	$-2\pi - 0.05$
1.000	0.000	-2π
0.999	0.050	$-2\pi + 0.05$

Your response: _____

AP Precalculus

Tangent Parent Function Graphs

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Identify the defining contradiction in the student's tangent graph or statement. Give the smallest mathematical repair and support it with the quotient definition or branch structure; do not redraw unrelated valid features.

Evaluate claim 1.

Limit your correction to change only the contradicted feature.

Evaluate this student work: The graph labels $x = \frac{\pi}{2}$ as an x-intercept at $(\frac{\pi}{2}, 0)$.

Work on the domain diagnosis of a proposed tangent graph.

Identify the defining contradiction in the student's tangent graph or statement. Give the smallest mathematical repair and support it with the quotient definition or branch structure; do not redraw unrelated valid features.

Evaluate claim 2.

Limit your correction to change only the contradicted feature.

Evaluate this student work: The graph's y-axis states that the range of tangent is $[-1, 1]$.

Work on the domain diagnosis of a proposed tangent graph.

Tangent Parent Function Graphs

Full Worked Solutions - excerpt

Worked example

Problem. Use the supplied sine and cosine sign evidence to determine the one-sided behavior of $\tan(x)$ at the excluded input. Justify each quotient sign and explicitly state that the function has no value at the boundary.

The excluded input is $-5\frac{\pi}{2}$.

The requested order is both sides.

The supplied sign information is cos from left is -; cos from right is +; sin near boundary is $-$.

Work on the domain one-sided quotient behavior.

Answer. as $x \to -5\frac{\pi}{2^-}, \tan(x) \to +\infty$; as $x \to -5\frac{\pi}{2^+}, \tan(x) \to -\infty$; $\tan(-5\frac{\pi}{2})$ is undefined.

Explanation and check.

On the left, numerator and small denominator have the same sign, so the quotient is large positive. On the right their signs differ, so it is large negative. A one-sided trend does not define a value where cosine equals zero.

Worked example

Problem. Use the supplied sine and cosine sign evidence to determine the one-sided behavior of $\tan(x)$ at the excluded input. Justify each quotient sign and explicitly state that the function has no value at the boundary.

The excluded input is $-15\frac{\pi}{2}$.

The requested order is left side then right side.

The supplied sign information is cos changes sign is + to -; sin at boundary is $+1$.

Work on the domain one-sided quotient behavior.

Answer. as $x \to -15\frac{\pi}{2^-}, \tan(x) \to +\infty$; as $x \to -15\frac{\pi}{2^+}, \tan(x) \to -\infty$; $\tan(-15\frac{\pi}{2})$ is undefined.

Explanation and check.

On the left, numerator and small denominator have the same sign, so the quotient is large positive. On the right their signs differ, so it is large negative. A one-sided trend does not define a value where cosine equals zero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Tangent Function Transformations

Tangent Function Transformations | APPC-U03-P15

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for compatible pattern or precise contradiction, invariants, center outputs, and orientation, period, zero family, and asymptote family, failed relation and corrected feature statement, invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result and included zeros and asymptote locations.

8 PDFs and a README; 151 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Tangent Function Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

List every zero and every vertical-asymptote location inside the specified window. Solve integer bounds for the two families separately and honor each open or closed display endpoint exactly.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition	Supplied value
Given vertical-asymptote family	$x = \frac{\pi}{2} + k(\frac{\pi}{2})$
Required window	$(-3\frac{\pi}{4}, 5\frac{\pi}{4}]$
Given zero family	$x = \frac{\pi}{4} + k(\frac{\pi}{2})$

Bank label: _____

AP Precalculus

Tangent Function Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Compare the two supplied tangent models without solving a general tangent equation. State what happens to period, asymptotes, branch-center outputs, former zeros, and branch orientation after the vertical translation, with special attention to the required evidence focus. The supplied models are $f(x) = 3\tan(\frac{1}{2}(x))$ and $g(x) = 3\tan(\frac{1}{2}(x)) + 1$. The branch-center family is $x = 0 + k(2\pi)$. The required evidence focus is to state exactly which invariants come from the inner map.

Solve the problem and show enough exact work for the requested result.

Condition	Supplied value
Given branch-center input family	$x = 0 + k(2\pi)$
Required evidence focus	state exactly which invariants come from the inner map
Given output translation	1

AP Precalculus

Tangent Function Transformations

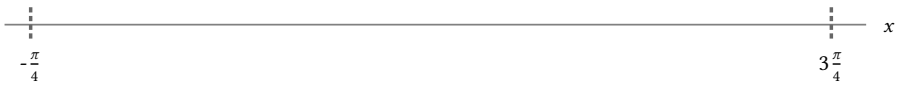
Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

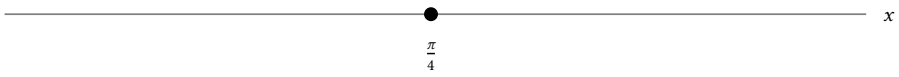
Incoming: zeros are $-15\frac{\pi}{4}, -7\frac{\pi}{4}, \frac{\pi}{4}, 9\frac{\pi}{4}, 17\frac{\pi}{4}$; asymptote locations are $-19\frac{\pi}{4}, -11\frac{\pi}{4}, -3\frac{\pi}{4}, 5\frac{\pi}{4}, 13\frac{\pi}{4}, 21\frac{\pi}{4}$.

Recover the horizontal period and a phase-anchor class from the graph evidence. Decide whether the asymptote spacing is one period or several, and state exactly what sign ambiguity remains; use orientation only when it is supplied.

Observed asymptote locations



Marked branch-center input



Feature	Exact x-location(s)
Observed asymptote locations	$-\frac{\pi}{4}, 3\frac{\pi}{4}$
Marked branch-center input	$\frac{\pi}{4}$

Condition	Supplied value
Given adjacency information	certified consecutive
Given branch direction	not supplied
Given outer coefficient	not orientation-resolving

Your response: _____

AP Precalculus

Tangent Function Transformations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “Omit $x = -7\frac{\pi}{3}$ and $x = 7\frac{\pi}{6}$ because boundary locations can never appear in requested zero or asymptote lists.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Use this route: audit both endpoint flags. Enumerate zeros from $x = -\frac{\pi}{3} + k(\pi)$ and poles from $x = \frac{\pi}{6} + k(\pi)$ inside $[-7\frac{\pi}{3}, 7\frac{\pi}{6}]$, honoring every endpoint.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition	Supplied value
Given vertical-asymptote family	$x = \frac{\pi}{6} + k(\pi)$
Required window	$[-7\frac{\pi}{3}, 7\frac{\pi}{6}]$
Given zero family	$x = -\frac{\pi}{3} + k(\pi)$

Tangent Function Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Find the first invalid relation in the worked tangent-feature calculation, repair only that relation, and verify the correction with one representative zero, asymptote, or spacing check.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition	Supplied value
Step to examine	calls every branch center an x-intercept after adding $d = 4$

Answer. centers have output 4 and need not be x-intercepts.

Explanation and check.

Check the tangent-specific period π, the separate inner-angle conditions for zeros and poles, and the role of any vertical offset. Preserve steps that remain valid after the single correction. At an old center, the inner angle is 0 and $\tan(0) = 0$. After the stated translation by 4, the output there is 4, not 0. This direct substitution verifies that the old center is not an x-intercept.

Worked example

Problem. Find the first invalid relation in the worked tangent-feature calculation, repair only that relation, and verify the correction with one representative zero, asymptote, or spacing check.

Compare the stated objects or approaches and justify the conclusion with exact evidence.

Condition	Supplied value
Step to examine	reports $T = -\frac{\pi}{3}$ for $b = -3$

Answer. period is positive $\frac{\pi}{3}$.

Explanation and check.

Check the tangent-specific period π, the separate inner-angle conditions for zeros and poles, and the role of any vertical offset. Preserve steps that remain valid after the single correction. With $b = -3$ and $T = \frac{\pi}{3}$, the inner-angle change is $-3(\frac{\pi}{3}) = -\pi$. Tangent has the same value after that change, while T is positive. The claimed negative value is not a period length.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Trigonometric Functions

Inverse Trigonometric Functions | APPC-U03-P16

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for valid inverse pair and reason for discarding a row, reason proposal fails and a valid restriction, validity and approximate corrected sign, defined or undefined, real-defined or undefined with failed restriction and validity and location of output.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Inverse Trigonometric Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Evaluate $\arcsin(-\frac{\sqrt{3}}{2})$ exactly. Identify the compatible unit-circle angle that lies in the principal range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Bank label: _____	Let $\theta = \arcsin(\frac{5}{13})$. A right triangle for θ has opposite side 5 and hypotenuse 13. Find $\cos\theta$ and explain why its sign is forced by the arcsine branch. Bank label: _____
Evaluate $\arcsin(\sin(5\frac{\pi}{6}))$. Do not cancel the two function names; use a symmetry relation to place the result in the arcsine range. Bank label: _____	Find the exact principal value of $\arccos(-\frac{\sqrt{2}}{2})$. Use the sign of cosine and the required range $[0, \pi]$ to select the angle. Bank label: _____
Set $\theta = \arccos(-\frac{8}{17})$ and represent θ by a point (x, y) on a circle of radius 17 with $x = -8$. Determine $\tan\theta$, including its sign, from the principal arccos range. Bank label: _____	In radian mode, estimate $\arcsin(0.98)$ to the nearest thousandth. Verify that the rounded result lies in the correct principal interval. Bank label: _____
Reduce $\arccos(\cos(7\frac{\pi}{6}))$ to its principal value. Explain the reflection that produces an angle in $[0, \pi]$. Bank label: _____	Evaluate $\sin(\arctan(-\frac{7}{24}))$ exactly. Build a signed coordinate triangle compatible with the inverse-tangent range rather than treating the inner ratio as a sine value. Bank label: _____

AP Precalculus

Inverse Trigonometric Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Is $\arccos(\frac{6}{5})$ defined as a real number? Name the restriction that decides the question.

AP Precalculus

Inverse Trigonometric Functions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $-\frac{\pi}{6}$

Place $13\frac{\pi}{6}$ on the tangent graph and remove enough periods to reach the inverse-tangent branch. Evaluate the composition.

Your response: _____

Incoming: y = arccos *x*.

Both $-\frac{\pi}{6}$ and $5\frac{\pi}{6}$ have tangent $-\frac{\sqrt{3}}{3}$. Which one equals $\arctan(-\frac{\sqrt{3}}{3})$, and why is the other rejected?

Your response: _____

Incoming: $\sqrt{(1-v^2)}$, for $-1 \le v \le 1$

Estimate $\arccos(-0.6)$ to the nearest tenth of a degree. Use the negative input to check the calculator's quadrant.

Your response: _____

Incoming: 126.9 degrees

For $\theta = \arccos(-\frac{3}{5})$, place a point of radius 5 with $x = -3$ in the correct half of the unit-circle model. Find $\sin\theta$.

Your response: _____

Incoming: 0.045 radians

Without finding θ itself, evaluate $\cot(\arctan(-\frac{9}{40}))$. State why this reciprocal is defined.

Your response: _____

Incoming: $\frac{\pi}{3}$

A calculation table records $\arcsin(0.73) = 0.818321$ in radian mode. Report the value to the nearest hundredth and verify its range.

Your response: _____

AP Precalculus

Inverse Trigonometric Functions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student writes, "$\arccos(\cos(4\frac{\pi}{3})) = 4\frac{\pi}{3}$ because inverse functions cancel." Identify the failed condition and replace the conclusion with the correct principal angle.

A solution interprets $\sin^{-1}(\frac{1}{2})$ as the reciprocal $\frac{1}{\sin(\frac{1}{2})}$. Diagnose the notation error, rewrite the inverse with unambiguous notation, and evaluate it exactly.

A symbolic solution states $\tan(\arcsin u) = \frac{u}{\sqrt{1-u^2}}$ for $-1 \le u \le 1$. Repair the domain and explain what happens at $u = \pm 1$ in the original composition, not just in the formula.

A calculator returns 45 for $\arctan(1)$, and a student reports 45 radians. Diagnose the display and give the correct radian answer.

Inverse Trigonometric Functions

Full Worked Solutions - excerpt

Worked example

Problem. Use radian mode to approximate $\arccos(0.2)$ to three decimal places.

Answer. 1.369 radians

Explanation and check.

The calculator value $1.369438\ldots$ rounds to 1.369 radians.

Worked example

Problem. Find the domain of $G(x) = \arcsin(3 - x)$. Preserve the inequality reversal when isolating x.

Answer. $[2, 4]$

Explanation and check.

Require $-1 \le 3 - x \le 1$. The two bounds give $x \le 4$ and $x \ge 2$, hence $2 \le x \le 4$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Solving Trigonometric Equations

Solving Trigonometric Equations | APPC-U03-P17

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for count by h-region, count by level case, count for each L, number of distinct solutions for each case, solution count by case and all roots.

8 PDFs and a README; 47 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Solving Trigonometric Equations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Solve $\sin x = \frac{1}{2}$ for $0 \le x \le 2\pi$. Use both unit-circle branches and check the endpoints. Bank label: _____	Solve $\sin(2x) = 0$ on $0 \le x \le \pi$. First map the x-interval to the complete $2x$-interval, then divide the enumerated angle values by 2. Bank label: _____
Solve $2\sin^2 x - 3\sin x + 1 = 0$ for $0 \le x \le 2\pi$. Treat $u = \sin x$, solve both feasible levels, and merge any overlapping angle sets. Bank label: _____	Find every x in $[0, 2\pi)$ satisfying $\sin x \cos x = 0$. Solve the two zero factors separately and combine without duplicating any root. Bank label: _____
Solve $\frac{\sin x}{1-\cos x}=0$ on $[0, 2\pi]$. Record denominator exclusions before using numerator-zero logic. Bank label: _____	Using $\sin^2 x = 1\text{-}\cos^2 x$, solve $\sin^2 x = 1\text{-}\cos x$ on $[0, 2\pi)$. Keep every root produced by the resulting factored equation. Bank label: _____
Solve $\cos x = -\frac{\sqrt{2}}{2}$ for $-\pi \le x < \pi$. Use the signed unit-circle interval rather than converting the final answers to $[0, 2\pi)$. Bank label: _____	Solve $\cos(x - \frac{\pi}{3}) = \frac{1}{2}$ on $[0, 2\pi]$. Enumerate every allowed angle $x - \frac{\pi}{3}$ in the mapped window, including boundary hits. Bank label: _____

AP Precalculus

Solving Trigonometric Equations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Solve $2\sin x + 1 = 2$ for $0 \le x \le 2\pi$. Normalize the output before selecting sine branches.	For $\sin x = c$ on $[0, 2\pi)$, determine the solution count when $c = 1$, when $0 < \lvert c\rvert < 1$, when $c = -1$, and when $\lvert c\rvert > 1$. Base the count on intersections with the extrema.
Solve $-3\cos x + 2 = 5$ on $[0, 2\pi]$. After isolating cosine, use the extreme level to determine multiplicity.	Solve $\sqrt{1 - \cos x} = \sin x$ on $[0, 2\pi]$. After squaring, substitute every candidate into the original equation to enforce the nonnegative left side.
Solve $\sin x = 2$ on $[0, 2\pi]$. Give the range-based reason for your conclusion.	Solve $\sin^2 x = \sin x$ on $[0, 2\pi)$. Explain which solutions would disappear if both sides were divided by $\sin x$.

AP Precalculus

Solving Trigonometric Equations

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\{0\}$

Solve $\tan(2x - \frac{\pi}{4}) = 1$ for $0 \le x \le \pi$. Filter the angle family $\frac{\pi}{4} + k\pi$ through the mapped window.

Your response: _____

Incoming: $[0, \frac{\pi}{2}]$ union $[3\frac{\pi}{2}, 2\pi]$

Solve $\sin^2 x - 2\sin x + 1 = 0$ on $[0, 2\pi]$. Explain why a repeated auxiliary root still produces a one-element angle set.

Your response: _____

Incoming: $\{0, \frac{\pi}{2}, \pi, 2\pi\}$

A nonnegative graph of $(\sin x - 0.6)^2$ on $[0, \pi]$ touches zero at two certified minima. Report both roots to the nearest thousandth and explain why sign-change bracketing detects neither.

Required response

0.643501109	0
2.498091545	0

Your response: _____

Incoming: $\{5\frac{\pi}{3}, 7\frac{\pi}{3}\}$

Solve $(2\sin x + 1)(2\sin x - 1) = 0$ on $[0, 2\pi)$. Enumerate the positive and negative half-levels separately.

Your response: _____

Incoming: $\{\frac{\pi}{4}, 5\frac{\pi}{4}\}$

Solve $\cos(-x + \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ on $[-\pi, \pi]$. Reverse the mapped interval correctly before filtering the two cosine branches.

Your response: _____

Incoming: $\{3.000\}$

Solve $\frac{\sin x}{\cos x} = 1$ on $[0, 2\pi)$. Preserve the cosine exclusions while using the equivalent tangent level where defined.

Your response: _____

AP Precalculus

Solving Trigonometric Equations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student reports only $x = \frac{\pi}{3}$ for $\sin x = \frac{\sqrt{3}}{2}$ on $[0, 2\pi]$. Identify the missing branch and give the corrected complete set.

A student uses $a2\pi$ repeat and lists $\{-2\pi, 0, 2\pi\}$ for $\tan x = 0$ on $[-2\pi, 2\pi]$. Repair the period and include every missing root.

A solution lists both 0 and 2π for $\cos x = 1$ on $[0, 2\pi)$. Repair the set by applying the interval notation.

A solution factors $\sin^2 x + \sin x - 2 = 0$ but tries to include an angle for $\sin x = -2$. Remove the impossible auxiliary level and finish the solution on $[0, 2\pi]$.

Solving Trigonometric Equations

Full Worked Solutions - excerpt

Worked example

Problem. Solve $\cos x = -1$ on $-2\pi \le x \le 2\pi$. Filter the extreme-value family $\pi + 2k\pi$ through the symmetric interval.

Answer. $\{-\pi,\ \pi\}$

Explanation and check.

The family $x = \pi + 2k\pi$ yields $-\pi$ for $k = -1$ and π for $k = 0$. Adjacent values lie outside the interval.

Worked example

Problem. Solve $5\tan x + 2 = 2$ on $[-\pi, \pi]$. Apply tangent's $\pi-$spaced zero family to the closed endpoints.

Answer. $\{-\pi,\ 0,\ \pi\}$

Explanation and check.

The equation reduces to $\tan x = 0$, whose zeros $x = k\pi$ in the interval are $-\pi, 0, \pi$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Trigonometric Inequalities

Trigonometric Inequalities | APPC-U03-P18

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for closed interval to tolerance, interval to tolerance, interval union to tolerance, correct intersection and counterexample, correct interval and boundary counterexample and correct set and boundary counterexample.

8 PDFs and a README; 111 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Trigonometric Inequalities

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Certified numerical evidence locates the crossings of $\sin x$ and $0.2x + 0.1$ at 0.125411 and 2.498473 on $[0, 4]$. Solve the nonstrict inequality to the nearest thousandth.

Certified numerical evidence locates the crossings of $\sin x$ and $0.2x + 0.1$ at 0.125411 and 2.498473 on $[0, 4]$. Solve the nonstrict inequality to the nearest thousandth.

Supplied evidence	Value
equality points	0.125410601 2.498473192

Condition or evidence	Supplied value
Original inequality	$\sin x \geq 0.2x + 0.1$
Requested precision	nearest thousandth
Given shape evidence	the difference is nonnegative exactly between the crossings
Given window	$[0, 4]$

Bank label: _____

Solve $-2\cos x + 1 \leq 0$ on $[0, 2\pi)$. Show the order reversal caused by division by -2.

Solve $-2\cos x + 1 \leq 0$ on $[0, 2\pi)$. Show the order reversal caused by division by -2.

Condition or evidence	Supplied value
Original inequality	$-2\cos x + 1 \leq 0$
Input interval	$[0, 2\pi)$

Bank label: _____

Solve -sin($2x - \frac{\pi}{2}$) ≥ 0 on $[0, \pi]$. Track the output reversal before mapping the inner-angle intervals.

Solve -sin($2x - \frac{\pi}{2}$) ≥ 0 on $[0, \pi]$. Track the output reversal before mapping the inner-angle intervals.

Condition or evidence	Supplied value
Original inequality	-sin(2x-pi/2)>=0
Input interval	$[0, \pi]$

Bank label: _____

Solve $\sin x(1 + \cos x) \geq 0$ on $[0, 2\pi]$. Use that $1 + \cos x$ never becomes negative but does vanish at π.

Solve $\sin x(1 + \cos x) \geq 0$ on $[0, 2\pi]$. Use that $1 + \cos x$ never becomes negative but does vanish at π.

Condition or evidence	Supplied value
Original inequality	$\sin x(1 + \cos x) \geq 0$
Input interval	$[0, 2\pi]$

Bank label: _____

AP Precalculus

Trigonometric Inequalities

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Solve $\sin x < 0$ OR $\cos x \le 0$ on $[0, 2\pi)$. Decide which boundaries are contributed by only one condition.

Solve $\sin x < 0$ OR $\cos x \le 0$ on $[0, 2\pi)$. Decide which boundaries are contributed by only one condition.

Condition or evidence	Supplied value
Given conditions	$\sin x < 0$ $\cos x \le 0$
Logical connective	OR
Input interval	$[0, 2\pi)$

During $0 \le t \le 12$ hours, a temperature follows $T(t) = 50 + 10\sin(\pi\frac{t}{6})$. When is $T > 55$, and for how many total hours?

During $0 \le t \le 12$ hours, a temperature follows $T(t) = 50 + 10\sin(\pi\frac{t}{6})$. When is $T > 55$, and for how many total hours?

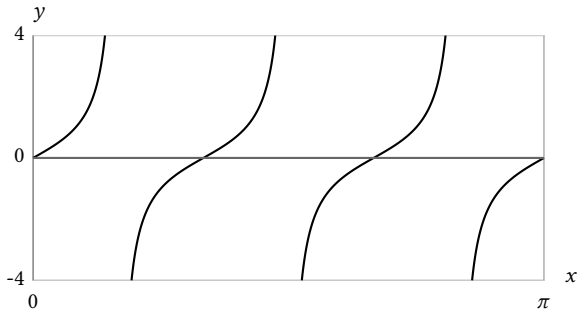
Condition or evidence	Supplied value
Required condition	$T > 55$
Given model	$T(t) = 50 + 10\sin(\pi\frac{t}{6})$
Time interval	$[0, 12]$ hours

For $0 \le t \le 10, V(t) = 12 + 4\cos(\pi\frac{t}{5})$. When is $V \le 8$, and what is the total duration?

For $0 \le t \le 10, V(t) = 12 + 4\cos(\pi\frac{t}{5})$. When is $V \le 8$, and what is the total duration?

Condition or evidence	Supplied value
Required condition	$V \le 8$
Given model	$V(t) = 12 + 4\cos(\pi\frac{t}{5})$
Time interval	$[0, 10]$ hours

Solve $\tan(3x) < 0$ on $[0, \pi]$, marking the three scaled poles before reading signs.



Condition or evidence	Supplied value
Original inequality	$\tan(3x) < 0$
Input interval	$[0, \pi]$

AP Precalculus

Trigonometric Inequalities

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[0, 0.125] \cup [2.498, 4]$

Solve $\tan x \geq -\sqrt{3}$ on $[0, \pi)$, treating the two sides of $\frac{\pi}{2}$ as separate branches.

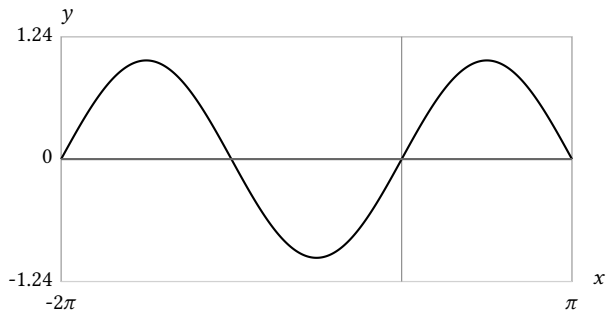
Solve $\tan x \geq -\sqrt{3}$ on $[0, \pi)$, treating the two sides of $\frac{\pi}{2}$ as separate branches.

Condition or evidence	Supplied value
Original inequality	$\tan x \geq -\sqrt{3}$
Input interval	$[0, \pi)$

Your response: _____

Incoming: $(\frac{\pi}{2}, \pi)$

Solve $\sin x \geq 0$ on $[-2\pi, \pi]$, using signed cycles and all equality zeros.



Condition or evidence	Supplied value
Original inequality	$\sin x \geq 0$
Input interval	$[-2\pi, \pi]$

Your response: _____

Incoming: $[-2\pi, \pi]$

Let $\alpha = \arccos(0.2)$. Solve $\cos x \leq 0.2$ on $[0, 2\pi)$ in terms of α.

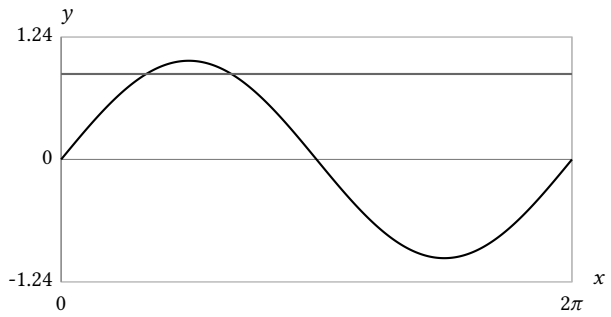
Let $\alpha = \arccos(0.2)$. Solve $\cos x \leq 0.2$ on $[0, 2\pi)$ in terms of α.

Condition or evidence	Supplied value
Original inequality	$\cos x \leq 0.2$
Input interval	$[0, 2\pi)$

Your response: _____

Incoming: $[-2, 2]$

Solve $\sin x > \frac{\sqrt{3}}{2}$ on $[0, 2\pi]$.



Condition or evidence	Supplied value
Original inequality	$\sin x > \frac{\sqrt{3}}{2}$
Input interval	$[0, 2\pi]$

Your response: _____

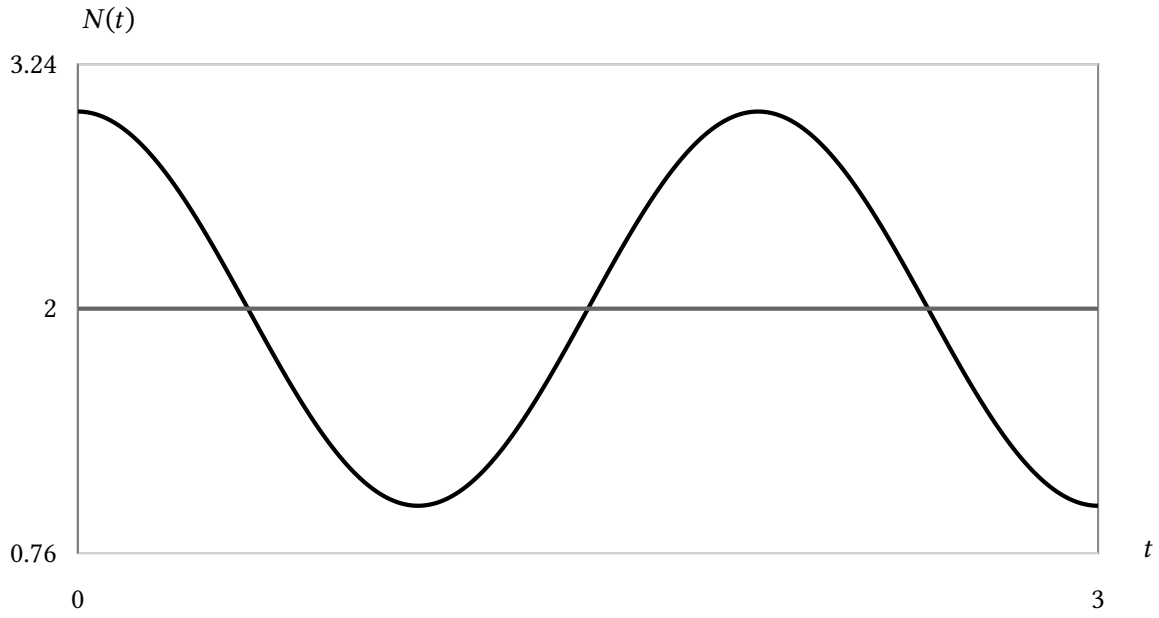
AP Precalculus

Trigonometric Inequalities

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $0 \le t \le 3, N(t) = 2 + \cos(\pi t)$. Find when $N > 2$ and add the unequal visible window lengths. First choose and justify a valid method. Method A: Add the actual lengths of the two admitted time windows. Method B: Multiply the number of windows by a common assumed length.



Condition or evidence	Supplied value
Required condition	$N > 2$
Given model	$N(t) = 2 + \cos(\pi t)$
Time interval	$[0, 3]$ hours

Trigonometric Inequalities

Full Worked Solutions - excerpt

Worked example

Problem. Determine the solution set of $\cos x > 1$ on $[0, 2\pi]$ without solving an equality first.

Determine the solution set of $\cos x > 1$ on $[0, 2\pi]$ without solving an equality first.

Condition or evidence	Supplied value
Original inequality	$\cos x > 1$
Input interval	$[0, 2\pi]$

Answer. $\varnothing$

Explanation and check.

Cosine's maximum value is 1, so it can never be strictly greater than 1. The solution set is empty.

Worked example

Problem. Solve $-3\sin x + 2 < -1$ on $[0, 2\pi]$, reversing the inequality when the sine term is isolated.

Solve $-3\sin x + 2 < -1$ on $[0, 2\pi]$, reversing the inequality when the sine term is isolated.

Condition or evidence	Supplied value
Original inequality	$-3\sin x + 2 < -1$
Input interval	$[0, 2\pi]$

Answer. $\varnothing$

Explanation and check.

Isolation gives $\sin x > 1$. Since the upper endpoint of sine's range is 1, the strict inequality is impossible.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Secant and Cosecant Functions

Secant and Cosecant Functions | APPC-U03-P19

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for cosecant order and magnitude, cosecant value and order, cosecant value order and magnitude, exact order, exact signed and magnitude relation and exact values and order.

8 PDFs and a README; 85 buyer pages.

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AP Precalculus

Secant and Cosecant Functions

Name: _____ **Date:** _____ **Class:** _____

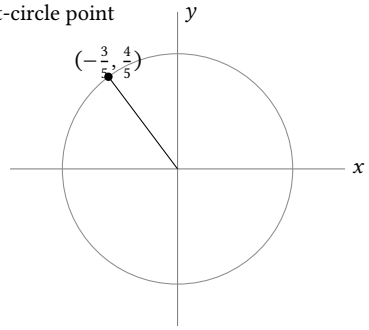
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A table gives $\cos\theta = \frac{1}{4}$ and $\sin\theta = \frac{\sqrt{15}}{4}$. Find $\sec\theta$ and $\csc\theta$ exactly.

Bank label: _____

The terminal point for θ is $(-\frac{3}{5}, \frac{4}{5})$ on the unit circle. Find $\sec\theta$ and $\csc\theta$.

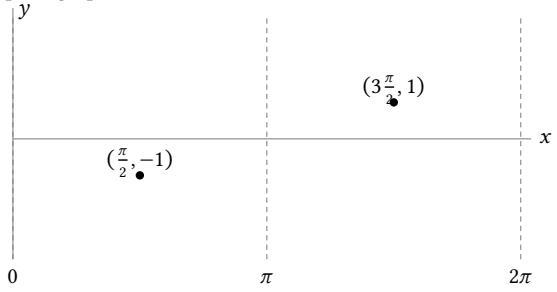
Supplied unit-circle point



Bank label: _____

A graph has poles at $0, \pi, 2\pi$ and vertices $(\frac{\pi}{2}, -1), (3\frac{\pi}{2}, 1)$. Which listed reciprocal formula matches it?

Supplied graph landmarks



$\frac{\pi}{2}$	-1
$3\frac{\pi}{2}$	1

Bank label: _____

If $\sin\theta = 0.8$, compute $\csc\theta$ and compare the two positive values.

Bank label: _____

AP Precalculus

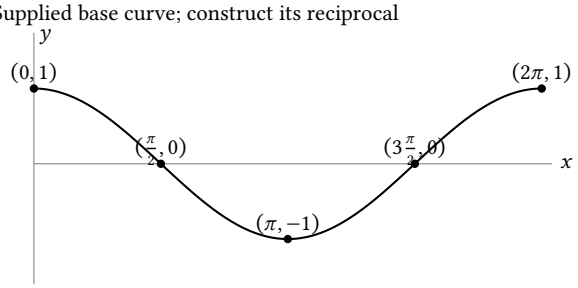
Secant and Cosecant Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Construct $y = \sec x$ on $[0, 2\pi]$ from the supplied cosine landmarks. State all poles, reciprocal vertices, and branch signs.

Supplied base curve; construct its reciprocal



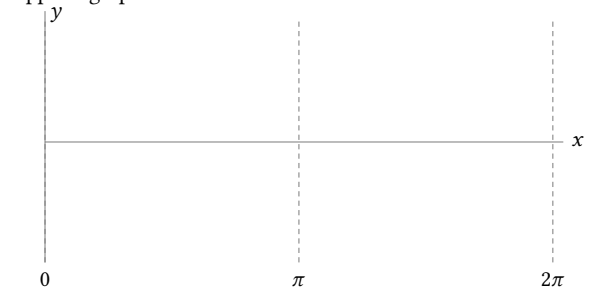
0	1
π	-1
2π	1

Derive the domain and range of $y = 2\sec x + 3$.

For $y = \csc(2x)$, give the period, all pole locations, and the reciprocal-vertex pattern.

A cropped graph has integer-π poles, a positive first branch, and a negative second branch. Match its reciprocal formula. Choose from $\csc x$, $-\csc x$, and $\sec x$.

Supplied graph landmarks



AP Precalculus

Secant and Cosecant Functions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

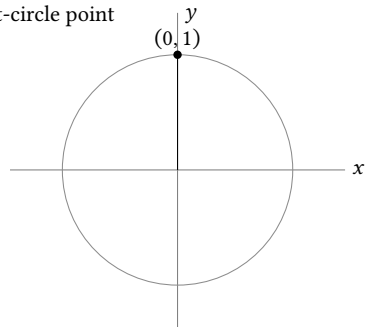
Give the repeating pole and vertex pattern of $y = \csc(3x)$.

Your response: _____

Incoming: period $2\frac{\pi}{3}$; poles $x = k\frac{\pi}{3}$; vertices $x = \frac{\pi}{6} + k\frac{\pi}{3}$ with alternating values $1, -1$

At the unit-circle point $(0, 1)$, evaluate $\sec\theta$ and $\csc\theta$.

Supplied unit-circle point



Your response: _____

Incoming: $\sec\theta$ is undefined; $\csc\theta = 1$

Restrict $y = \csc x$ to $[\frac{\pi}{6}, 5\frac{\pi}{6}]$. Determine its actual domain and range.

Your response: _____

Incoming: domain $[\frac{\pi}{6}, 5\frac{\pi}{6}]$; range $[1, 2]$

Recover the horizontal landmarks of $y = \sec(\frac{x}{3})$.

Your response: _____

AP Precalculus

Secant and Cosecant Functions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims $\sec(0) = 0$ because the angle is zero. Diagnose the reciprocal error.

A proposed $y = \csc x$ graph crosses the x-axis at $x = \frac{\pi}{2}$. Diagnose the error using the denominator value there and the reciprocal range.

A student marks $x = 0$ and $x = \pi$ as zeros of $\sec x$. Repair the graph landmarks on $[0, 2\pi]$.

A student copies $\cos\theta = 0.2$ into the secant column. Repair the table entry and state the operation required.

Secant and Cosecant Functions

Full Worked Solutions - excerpt

Worked example

Problem. A table gives $\cos\theta = \frac{1}{8}$. Find $\sec\theta$.

Answer. $\sec\theta = 8$

Explanation and check.

Invert the nonzero cosine value.

Worked example

Problem. A table lists $\sin\theta = 0$. Classify $\csc\theta$.

Answer. $\csc\theta$ is undefined

Explanation and check.

Cosecant would require division by zero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Cotangent Functions

Cotangent Functions | APPC-U03-P20

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for cotangent status and value, family of compatible rules and missing evidence, sign and domain on each interval, period consecutive pole and zero, range and existence of local extrema and values and domain explanation.

8 PDFs and a README; 142 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Cotangent Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$ to find $\cot\theta$.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given $\cos\theta$	$\frac{\sqrt{3}}{2}$
Given $\sin\theta$	$\frac{1}{2}$

Bank label: _____

AP Precalculus

Cotangent Functions

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$, find the period and give the first pole and first zero at or to the right of $x = \frac{\pi}{8}$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original rule	$y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$

AP Precalculus

Cotangent Functions

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(a, b) = (2, 2)$ and $(-2, -2)$

For $y = 3\cot(-x + \frac{\pi}{5})$, solve for its pole and zero families and state the period and branch direction.

Solve the card and move only to the card whose incoming header matches the entire requested response.

Condition or evidence	Supplied value
Original rule	$y = 3\cot(-x + \frac{\pi}{5})$

Your response: _____

AP Precalculus

Cotangent Functions

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Assess the statement "$\cot x = \frac{1}{\tan x}$ for every real x." Give a corrected domain-qualified statement and one counterexample to the original.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Statement to examine	$\cot x = \frac{1}{\tan x}$ for every real x

Cotangent Functions

Full Worked Solutions - excerpt

Worked example

Problem. For $y = 2\cot(3x)$, state the period, all pole locations, all zeros, and the direction of each branch.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original rule	$y = 2\cot(3x)$

Answer. period $\frac{\pi}{3}$; poles $x = k\frac{\pi}{3}$; zeros $x = \frac{\pi}{6} + k\frac{\pi}{3}$; decreasing branches

Explanation and check.

Set $3x = k\pi$ for poles and $3x = \frac{\pi}{2} + k\pi$ for zeros. The positive output factor preserves cotangent's decreasing direction.

Worked example

Problem. For $y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$, find the period and give the first pole and first zero at or to the right of $x = \frac{\pi}{8}$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original rule	$y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$

Answer. period $\frac{\pi}{4}$; pole $x = \frac{\pi}{8}$; zero $x = \frac{\pi}{4}$

Explanation and check.

Input scale 4 gives period $\frac{\pi}{4}$. Starting at the phase pole, a zero lies half a period, $\frac{\pi}{8}$, later.

Sets & Steps
Math Practice & Worked Solutions

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

Trigonometric Functions | APPC-U03-P21

96 problems · Four activity formats

Topic focus

Use reciprocal and quotient identities, derive Pythagorean forms, simplify expressions, and justify domain restrictions and sign choices.

8 PDFs and a README; 62 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values, method-choice labels, and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

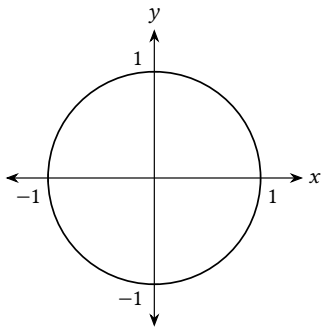
Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values and the requested justification. Angles are in radians; k is an integer.

Rewrite $\sec x$ using cosine and state the inputs allowed by the original expression.

Bank label: _____

A terminal ray lies in quadrant II and $\sin^2\theta = \frac{9}{25}$. Recover $\sin\theta$.



Bank label: _____

Use the Pythagorean identity to rewrite $1 - \sin^2\theta$ as one squared trigonometric function.

Bank label: _____

Simplify $\frac{1}{\csc x}$, but retain the domain of the written original expression.

Bank label: _____

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning. Use radians, exact values, and all stated domain restrictions. In excluded-input families, k is an integer.

Express $\tan x$ using sine and cosine, including its domain condition.	The expression $\frac{\sin x}{\tan x}$ simplifies to $\cos x$. On which inputs is the simplification equal to the original, and which inputs does the simplified formula newly admit?
Divide $\sin^2 x + \cos^2 x = 1$ by $\cos^2 x$ and name every quotient. State the required condition.	Refute the proposed identity $\sec x = \cos x$ using an input where both sides are defined.
Rewrite $\frac{\tan x}{\cot x}$ entirely in sine and cosine and simplify. Preserve the original restrictions.	After replacing $1 - \cos^2 x$, the expression $\frac{1-\cos^2 x}{\sin x}$ becomes $\sin x$. Compare the domains of the two formulas.

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\cot x$ for $\sin x \neq 0$ Rewrite $\sin^2 x + 2\cos^2 x$ as 1 plus one squared trig term. Your response: _____	**Incoming:** $\sin\theta = \frac{7}{8}$ Use the reciprocal definition to express $\sec x \sin x$ as a quotient function, retaining its domain. Your response: _____
Incoming: $\cos\theta = -2\frac{\sqrt{6}}{5}$ Rewrite $\tan x \cos^2 x$ using only sine and cosine and simplify. Your response: _____	**Incoming:** $\cot^2 t = \csc^2 t - 1$ for $\sin t \neq 0$ Rewrite $2(\cos^2 t - 1)$ in terms of $\sin^2 t$. Your response: _____

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statements or methods. Explain your decision and complete the requested task. Not every prompt contains an error.

A derivation divides $\sin^2 x + \cos^2 x = 1$ by $\sin^2 x$ "for every real x." Repair the step and state the resulting identity.

Test the claim $1 - \cos^2 x = \sin x$. Give a counterexample and replace it with the correct identity.

Repair the step $\sqrt{\cos^2 x} = \cos x$. Give a correct all-real formula and explain when the student's version happens to agree.

Refute $\sqrt{\sin^2 x} = \sin x$ as an all-real identity, then write the correct version.

Reciprocal, Quotient, and Pythagorean Trig Identities

Full Worked Solutions - excerpt

Worked example

Problem. Rewrite $\sec x$ using cosine and state the inputs allowed by the original expression.

Answer. $\sec x = \frac{1}{\cos x}$ for $\cos x \neq 0$

Explanation and check.

$\sec x = \frac{1}{\cos x}$

Cosine is defined for every real x, but the denominator must be nonzero. Therefore $\cos x \neq 0$, or equivalently $x \neq \frac{\pi}{2} + k\pi$.

Worked example

Problem. A terminal ray lies in quadrant II and $\sin^2\theta = \frac{9}{25}$. Recover $\sin\theta$.

Answer. $\sin\theta = \frac{3}{5}$

Explanation and check.

$\sin^2\theta = \frac{9}{25}$

$\sin\theta = \pm(\frac{3}{5})$

Sin is positive on quadrant II. Select the matching sign.

$\sin\theta = \frac{3}{5}$

Check the given square: $\frac{9}{25}$; the selected sign also meets the location condition.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

Angle Sum, Difference, and Double-Angle Identities | APPC-U03-P22

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for cos(α-β), cos 2x, equivalent expression, rewrite and range, exact value and rewrite and maximum.

8 PDFs and a README; 75 buyer pages.

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AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

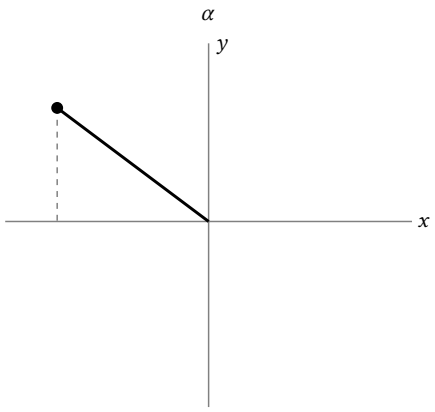
Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

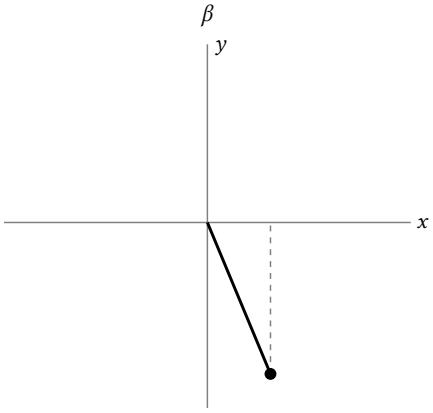
Use a difference identity to obtain the exact value of $\cos 15^\circ$.

Bank label: _____

Angle α lies in quadrant II with $\tan\alpha = -\frac{3}{4}$, and β lies in quadrant IV with $\sec\beta = \frac{13}{5}$. Find $\sin(\alpha - \beta)$.



$(-4, 3)$



$(5, -12)$

Bank label: _____

AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Prove $\frac{1-\cos 2x}{\sin 2x} = \tan x$ by transforming the left side, and state the common domain.	Use $120^\circ + 45^\circ$ to evaluate $\cos 165^\circ$. Write the result so its negative sign is visually clear.
Verify $\frac{1+\cos 2x}{\sin 2x} = \cot x$ from the left side and identify every excluded family in the original equality.	Find $\sin 105^\circ$ exactly and use its quadrant to check the sign of your result.
Starting from $\cos 2x = \frac{5}{13}$, select two forms of the cosine double-angle identity to determine $\cos^2 x$ and $\sin^2 x$.	A rewrite begins with $\sin(\alpha + \beta) = \sin\alpha + \sin\beta$. Identify the invalid assumption and replace it with a correct first line.

AP Precalculus

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Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Evaluate $\sin 75^\circ$ exactly by decomposing the angle into familiar angles.

Your response: _____

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Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A short route cancels $\cos x$ and announces an identity for all real x. A second route checks domains first. Decide which conclusion is valid and repair the quantifier.

Evaluate the claim $\frac{\sin 2x}{2\cos x} = \sin x$ for all real x.

Work on the domain real x.

The route A is cancel $\cos x$.

The route B is compare original domains first.

The output is corrected claim.

A student writes $\cos 2x = 2\cos x - 1$. Repair the rule and give one input that reveals the missing square.

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Full Worked Solutions - excerpt

Worked example

Problem. Transform $\sin 2x - 2\cos x$ into a product that displays its two zero-producing factors. Do not find the roots.

Answer. $2\cos x(\sin x - 1)$

Explanation and check.

Substitute $2\sin x\cos x$ for $\sin 2x$ and factor the common $2\cos x$.

Worked example

Problem. Rewrite $2\cos^2 x = 1 + \sin x$ as a polynomial equation in $\sin x$, without solving for x.

Answer. $2\sin^2 x + \sin x - 1 = 0$

Explanation and check.

Substitute $\cos^2 x = 1 - \sin^2 x$ and collect: $2 - 2\sin^2 x = 1 + \sin x$, so $2\sin^2 x + \sin x - 1 = 0$.