Sets & Steps
Math Practice & Worked Solutions

Precalculus

Trig Foundations & Sinusoidal Models | 12-Topic Precalculus Bundle

12-topic bundle | 1,152 problems | Four activity formats

Included topics

The following pages list all 12 included resources in the suggested order, with a link to each product. Each resource retains its complete preview.

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

96 PDFs and 12 READMEs; 1,344 buyer PDF pages across the 12 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Sets & Steps
Math Practice & Worked Solutions

Precalculus

Included topics

12-topic bundle | 1,152 problems | Four activity formats

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01 **Periodic Functions, Radians, and the Unit Circle**

View individual product and its full preview

02 **Trig Ratios from Triangles and Coordinates**

View individual product and its full preview

03 **Exact Trig Values at Special Angles**

View individual product and its full preview

04 **Trig Signs and Reference Angles**

View individual product and its full preview

05 **Tangent Values and Domain**

View individual product and its full preview

06 **Sine and Cosine Parent Graphs**

View individual product and its full preview

07 **Sinusoidal Amplitude and Midline**

View individual product and its full preview

08 **Period and Frequency of Sinusoids**

View individual product and its full preview

09 **Phase Shifts of Sine and Cosine**

View individual product and its full preview

10 **Writing Sinusoidal Equations from Features**

View individual product and its full preview

11 **Interpreting Sinusoidal Transformations**

View individual product and its full preview

12 included resources | Complete resource previews follow this guide.

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Sets & Steps
Math Practice & Worked Solutions

Precalculus

Included topics

12-topic bundle | 1,152 problems | Four activity formats

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12 **Sinusoidal Modeling**

View individual product and its full preview

12 included resources | Complete resource previews follow this guide.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Periodic Functions, Radians, and the Unit Circle | APPC-U03-COMB-P01-P02

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
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05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for directed radian measure, supported periodicity claim, recurrence rates and range comparison, exact degrees, exact radians and exact ordering.

8 PDFs and a README; 131 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Express $-2\frac{\pi}{9}$ radians as an exact directed angle in degrees.

Bank label: _____

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

On a circle of radius 4 m, a point rotates through $\frac{3}{8}$ radians. Find the directed arc displacement s and state whether the motion is clockwise or counterclockwise.	For the directed angle $13\frac{\pi}{6}$, describe its terminal ray and list every coterminal representative in the interval $(\pi, 5\pi]$. Respect the endpoint symbols.
An exact periodic graph has period 9. A cycle marker occurs at $t = 5$; the graph reaches a specified decreasing phase 4 time units after every marker. Is $t = 38$ that same phase? Give an equivalent cycle window beginning at the marker.	For the directed angle $15\frac{\pi}{8}$, describe its terminal ray and list every coterminal representative in the interval $(0, 4\pi)$. Respect the endpoint symbols.

AP Precalculus

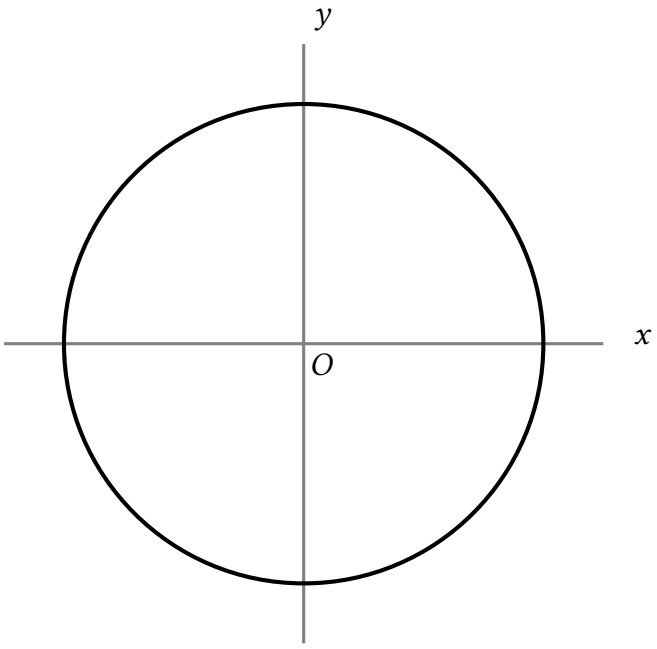
Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: 105°

Point $P = (\frac{8}{17}, y)$ lies on the unit circle and is below the x-axis. Find y and verify the point.



Your response: _____

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims that the smallest period is 7 time units for these exact repeating (sample index, value) pairs: $(0, 78)$; $(1, 71)$; $(2, 76)$; $(3, 73)$; $(4, 72)$; $(5, 73)$; $(6, 82)$; $(7, 78)$; $(8, 71)$; $(9, 76)$; $(10, 73)$; $(11, 72)$; $(12, 73)$; $(13, 82)$. Consecutive sample indices are 3 time units apart. Diagnose the claim, state the corrected smallest period, and cite one decisive comparison.

Identify the error or first invalid claim, give the corrected conclusion, and justify the correction.

sample index	value
0	78
1	71
2	76
3	73
4	72
5	73
6	82

sample index	value
7	78
8	71
9	76
10	73
11	72
12	73
13	82

A circle centered at O has radius 18 cm. A point moves counterclockwise along the circle through an arc of length $\frac{162}{5}$ cm. Find the directed angle in radians and explain how the sign records the sweep direction. Before solving, choose and justify the valid method. Method A: Use radius divided by arc length. Method B: Use signed arc length divided by radius.

Periodic Functions, Radians, and the Unit Circle

Full Worked Solutions - excerpt

Worked example

Problem. Convert the directed rotation 225° to an exact radian measure. Preserve its direction.

Answer. $5\frac{\pi}{4}$ radians

Explanation and check.

Multiply by $\frac{\pi}{180}^\circ$: $225(\frac{\pi}{180}) = 5\frac{\pi}{4}$. The sign is preserved.

Worked example

Problem. Convert the directed rotation -24° to an exact radian measure. Preserve its direction.

Answer. $-2\frac{\pi}{15}$ radians

Explanation and check.

Multiply by $\frac{\pi}{180}^\circ$: $-24(\frac{\pi}{180}) = -2\frac{\pi}{15}$. The sign is preserved.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Trig Ratios from Triangles and Coordinates

Trig Ratios from Triangles and Coordinates | APPC-U03-P03

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
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05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for exact coordinate ratios, radius and three ratios, selected exact ratios, scale, ray relation, and ratio behavior, domain and ratio comparison and ray directions and quotient behavior.

8 PDFs and a README; 112 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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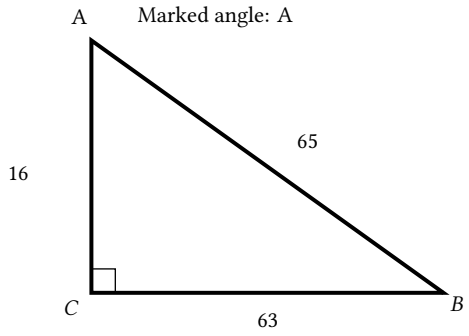
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

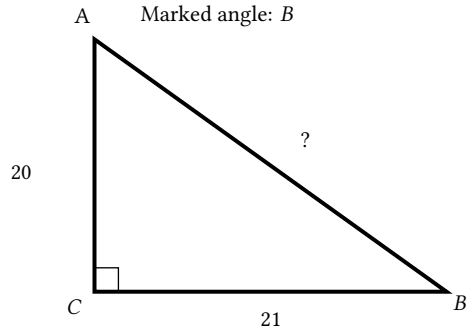
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

In right triangle ABC, $\angle C = 90^\circ$, AC = 16, BC = 63, and AB = 65. The marked angle is $\angle$A; the drawing may be rotated. Identify the opposite, adjacent, and hypotenuse sides relative to $\angle$A, then write its sine, cosine, and tangent.



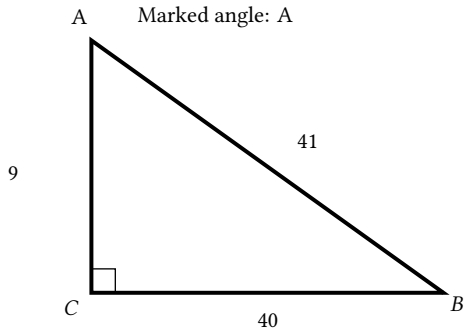
Bank label: _____

Right triangle ABC has $\angle C = 90^\circ$, AC = 20, BC = 21, and marked angle $\angle B$. First recover AB, then use the side roles relative to $\angle B$ to find sine, cosine, and tangent.



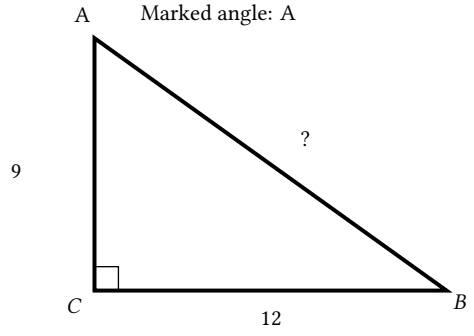
Bank label: _____

In right triangle ABC, $\angle C = 90^\circ$, AC = 9, BC = 40, and AB = 41. The marked angle is $\angle$A; the drawing may be rotated. Identify the opposite, adjacent, and hypotenuse sides relative to $\angle$A, then write its sine, cosine, and tangent.



Bank label: _____

Right triangle ABC has $\angle C = 90^\circ$, AC = 9, BC = 12, and marked angle $\angle$A. First recover AB, then use the side roles relative to $\angle$A to find sine, cosine, and tangent.



Bank label: _____

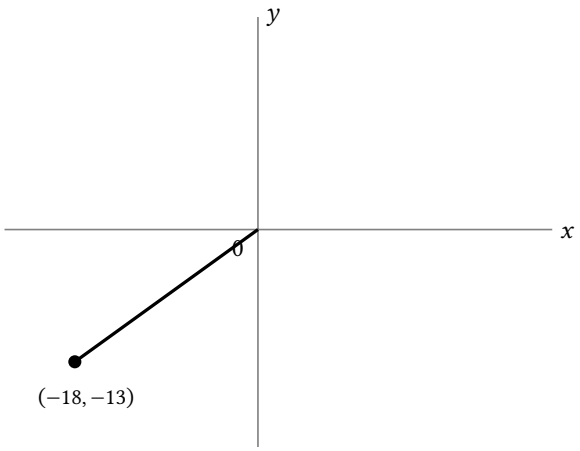
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A graphed ray starts at the origin and passes through $(-18, -13)$. Find its slope and interpret that quotient as $\tan\theta$ for the ray's directed angle.



AP Precalculus

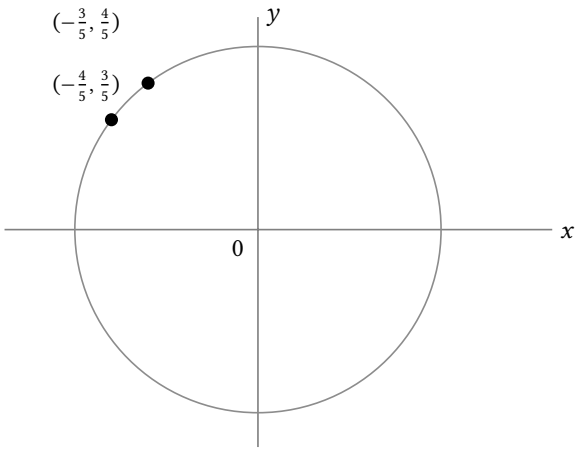
Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

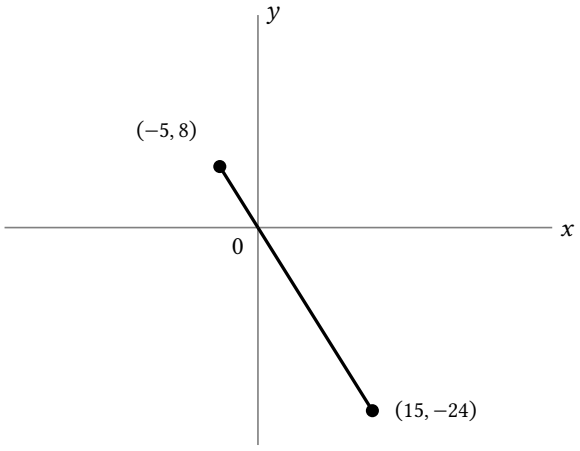
A unit-circle point moves along the shorter counterclockwise arc in the second quadrant from $\left(-\frac{3}{5}, \frac{4}{5}\right)$ to $\left(-\frac{4}{5}, \frac{3}{5}\right)$. Describe the endpoint changes in sine and cosine and compare the initial and final tangent ratios.



Your response: _____

Incoming: sin: $\frac{4}{5} \to \frac{3}{5}$; cos: $-\frac{3}{5} \to -\frac{4}{5}$; tan: $-\frac{4}{3} \to -\frac{3}{4}$

Rays OP and OQ pass through $P = (-5, 8)$ and $Q = (15, -24)$. Compare their directions, slopes, and tangent values. Explain what slope loses that sine and cosine would retain.



Your response: _____

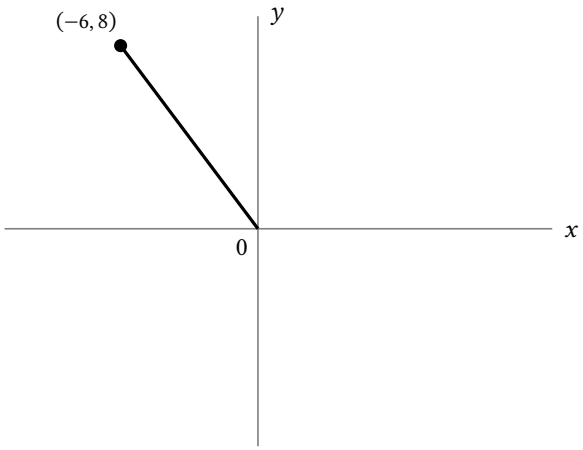
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For ray point $(-6, 8)$, a student assigns a negative radius because a coordinate is negative. Diagnose the error, state the correct r, and repair sine and cosine.



Trig Ratios from Triangles and Coordinates

Full Worked Solutions - excerpt

Worked example

Problem. The only information about a nonzero terminal ray is $\tan\theta = -\frac{7}{4}$. Describe the ray or rays that are determined, give representative point(s), and explain the scale or sign ambiguity that remains.

Answer. Two opposite rays, represented by $(4,-7)$ and $(-4,7)$; any positive scaling on either ray also works.

Explanation and check.

Tangent fixes $\frac{y}{x}$, so it fixes a line through the origin but not which of the two opposite rays or the radial scale.

Worked example

Problem. Can a real nonzero ray satisfy $\sin\theta = \frac{6}{5}$ under the coordinate definition? Decide and prove the conclusion using $|\text{coordinate}| \le r$.

Answer. No real ray satisfies the claim.

Explanation and check.

For every real point, the magnitude of the relevant coordinate is at most r, so $|\sin\theta| \le 1$. Here $\left|\frac{6}{5}\right| > 1$, a contradiction.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exact Trig Values at Special Angles

Exact Trig Values at Special Angles | APPC-U03-P04

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
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08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for correct equality and approximation, mode diagnosis and exact value, valid exact and approximate relations, exact equality groups and ordering, all permitted angle labels and completed exact row.

8 PDFs and a README; 128 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A special-angle table row has $(\sin\theta, \cos\theta) = (1, 0)$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate.

Required response

Angle	sin θ	cos θ
_____	1	0

Bank label: _____

On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos\theta = \frac{1}{2}$. Use the unit-circle line intersection and check both endpoint flags.

Bank label: _____

AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

The pair at $\frac{\pi}{3}$ is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. The target ray $\frac{\pi}{6}$ is related by reverse complementary transfer. Track coordinate exchange and any signs to infer its exact pair.

Audit this special-angle table row: $\theta = \frac{\pi}{3}, \sin\theta = \frac{1}{2}, \cos\theta = \frac{1}{2}$. Identify the earliest exact inconsistency and replace the row with the correct pair.

Required response

Angle	sin θ	cos θ
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{1}{2}$

AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A unit-circle table row for $\theta = 7\frac{\pi}{6}$ shows $\cos\theta$ missing and the other coordinate equal to $-\frac{1}{2}$. Complete the row exactly and use the angle's location to justify the square-root sign.

Required response

Angle	sin θ	cos θ
$7\frac{\pi}{6}$	$-\frac{1}{2}$	_____

Your response: _____

Incoming: $\sin\theta = -\frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}$

A special-angle table row has $(\sin\theta, \cos\theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate.

Required response

Angle	sin θ	cos θ
_____	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

Your response: _____

Incoming: $\theta = \frac{\pi}{6}$

On the finite one-turn window $(-\pi, \pi]$, locate every special-angle ray satisfying $\cos\theta = 1$. Use the unit-circle line intersection and check both endpoint flags.

Your response: _____

Incoming: $\theta = 0$

On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos\theta = -1$. Use the unit-circle line intersection and check both endpoint flags.

Your response: _____

AP Precalculus

Exact Trig Values at Special Angles

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A calculator in degree mode evaluates the radian expression $\cos(\frac{\pi}{3})$ and displays 0.9998. Diagnose the mode error, give the correct exact value, and explain why the display is not evidence of equality.

Exact Trig Values at Special Angles

Full Worked Solutions - excerpt

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{3}$ on the unit circle and evaluate the ordered pair $(\sin\theta, \cos\theta)$ exactly.

Answer. $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{3}$ has $x = \frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$. Therefore $(\sin\theta, \cos\theta) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$.

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{6}$ on the unit circle and evaluate the ordered pair $(\sin\theta, \cos\theta)$ exactly.

Answer. $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{6}$ has $x = \frac{\sqrt{3}}{2}$ and $y = \frac{1}{2}$. Therefore $(\sin\theta, \cos\theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Trig Signs and Reference Angles

Trig Signs and Reference Angles | APPC-U03-P06

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
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Topic focus

Across 96 tasks this topic asks for three signs with coordinate evidence, all allowed angles, terminal location and reference angle, correct boundary classification, possible location or inconsistency and all consistent pairings or contradiction.

8 PDFs and a README; 107 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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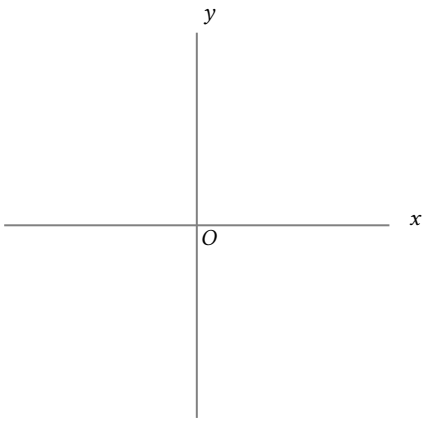
AP Precalculus

Trig Signs and Reference Angles

Name: _____ **Date:** _____ **Class:** _____

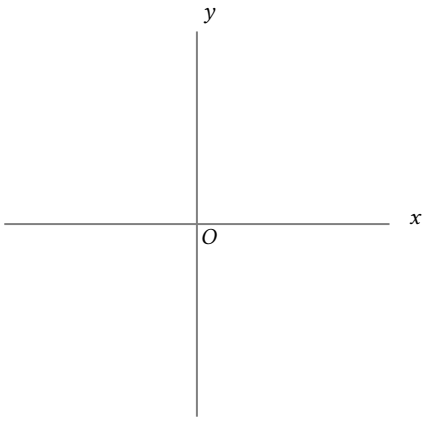
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Intersect the constraints $\sin\theta > 0, \cos\theta \neq 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.



Bank label: _____

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(2\pi - \alpha)$ using $\cos\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.



Bank label: _____

Intersect the constraints $\cos\theta < 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Bank label: _____

Construct every angle in $[0, 2\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant IV. Respect both endpoint flags and include all admitted turns.

Bank label: _____

AP Precalculus

Trig Signs and Reference Angles

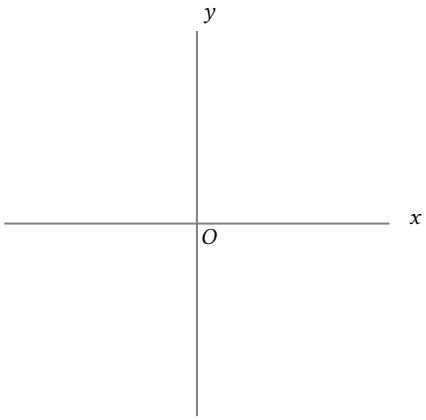
Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For quadrant IV coordinate box with $x > 0, y < 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.

Given evidence	Sine	Cosine	Tangent
$x > 0, y < 0$	_____	_____	_____

For positive y-axis ray with $x = 0, y > 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.



Reduce the standard-position rotation $\theta = 7\frac{\pi}{6}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

Reduce the standard-position rotation $\theta = 11\frac{\pi}{4}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

AP Precalculus

Trig Signs and Reference Angles

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\theta = 2\frac{\pi}{3}$

For negative x-axis ray with $x < 0, y = 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.

Given evidence	Sine	Cosine	Tangent
$x < 0, y = 0$	_____	_____	_____

Your response: _____

Incoming: $\theta = 7\frac{\pi}{6}$

Intersect the constraints $\sin\theta = 0, \cos\theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Your response: _____

Incoming: $5\frac{\pi}{6}$

Intersect the constraints $\sin\theta > 0, \cos\theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Your response: _____

Incoming: quadrant I

Construct every angle in $[0, 4\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant III. Respect both endpoint flags and include all admitted turns.

Your response: _____

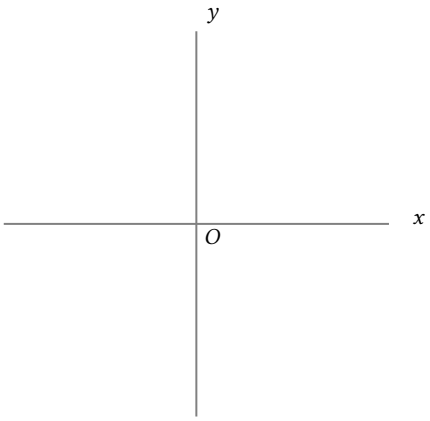
AP Precalculus

Trig Signs and Reference Angles

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(3\pi - \alpha)$ using $\cos\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α. First select and justify the valid method. Method A: Reduce $3\pi - \alpha$ to $\pi - \alpha$ and use the reflected x-coordinate. Method B: Cancel 3π as though it were an integer multiple of a full turn.



Trig Signs and Reference Angles

Full Worked Solutions - excerpt

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\sin(3\pi - \alpha)$ using $\sin\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.

Answer. $\sin(3\pi - \alpha) = \sin\alpha$

Explanation and check.

The reference magnitude is $\sin\alpha$; reduce to $\pi - \alpha$ in quadrant II. Therefore $\sin(3\pi - \alpha) = \sin\alpha$.

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\tan(2\pi - \alpha)$ using $\tan\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.

Answer. $\tan(2\pi - \alpha) = -\tan\alpha$

Explanation and check.

The reference magnitude is $\tan\alpha$; quadrant IV has opposite coordinate signs. Therefore $\tan(2\pi - \alpha) = -\tan\alpha$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Tangent Values and Domain

Tangent Values and Domain | APPC-U03-P05

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for classification and decisive evidence, exact slope and sign evidence, classification and earliest error, classification and evidence, exact ordering and earliest error and repair.

8 PDFs and a README; 146 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Give exact answers and all requested reasoning; record a bank letter. If bank entries give equivalent final responses for the same quantity and context, either label is valid and may be reused. A shared numerical value alone does not make entries interchangeable.

Order these exact scalar values from least to greatest, grouping ties and using no decimals: $\tan(2\frac{\pi}{3}), \sin(3\frac{\pi}{2}), \cos(5\frac{\pi}{6})$.

Solve the problem and show enough exact work for the requested result.

$\tan(2\frac{\pi}{3})$
$\sin(3\frac{\pi}{2})$
$\cos(5\frac{\pi}{6})$

Bank label: _____

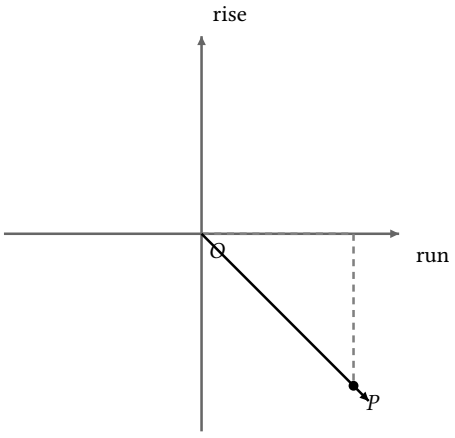
Given $\tan(\frac{\pi}{3}) = \sqrt{3}$, use "negate angle" to infer $\tan(-\frac{\pi}{3})$. Explain whether one or both coordinate signs change.

Bank label: _____

A terminal ray passes through $(-4, -4\sqrt{3})$. Evaluate its exact tangent from rise/run and explain why radius normalization is unnecessary.

Bank label: _____

A marked ray has signed run 12 and rise -12. Calculate its exact tangent and explain which geometric datum controls the sign.



Bank label: _____

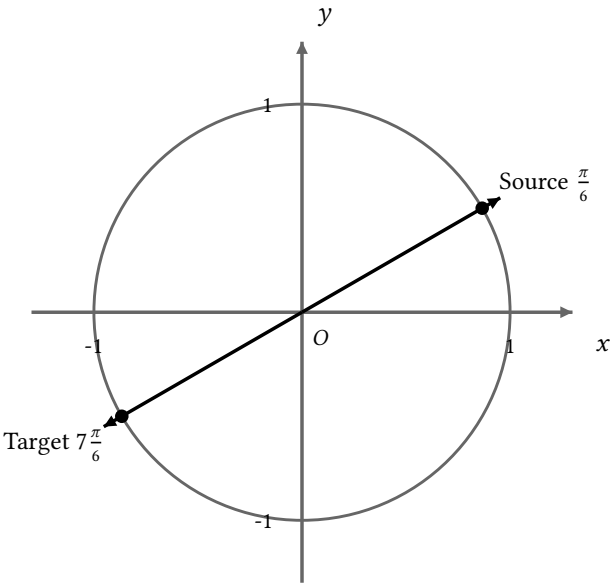
AP Precalculus

Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Given $\tan(\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$, use "add π" to infer $\tan(7\frac{\pi}{6})$. Explain whether one or both coordinate signs change.



AP Precalculus

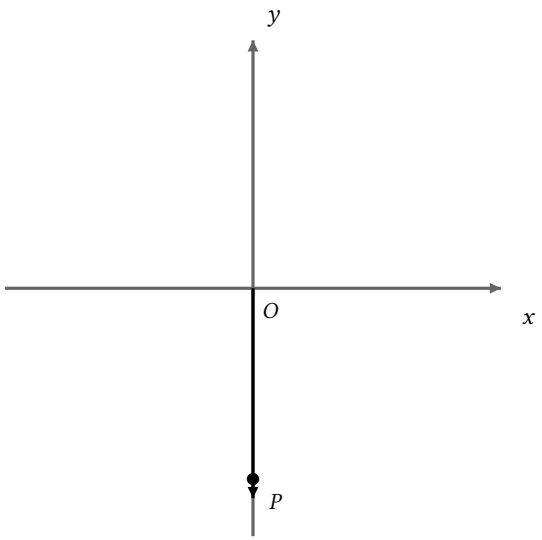
Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\tan(-5\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$

A ray is represented by the exact point $(0, -3)$. Classify its tangent as zero, nonzero, or undefined and cite numerator and denominator roles.



zero
nonzero
undefined

Your response: _____

AP Precalculus

Tangent Values and Domain

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the claim "$\tan(\pi)$ is undefined because sine is zero." Classify tangent correctly and identify the first quotient or domain error.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

denominator
permission
sign

A student claims: "Because the x-coordinate contains a radical, only a decimal estimate is possible." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Evidence case: an exact point $(2\sqrt{3}, 2)$. Decide whether it determines an exact tangent, only an estimate, or no domain conclusion. State the decisive evidence and any exact datum still needed.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Value status	x	y
exact	$2\sqrt{3}$	2

Tangent Values and Domain

Full Worked Solutions - excerpt

Worked example

Problem. At $\theta = \pi$, $(\sin\theta, \cos\theta) = (0, -1)$. Evaluate $\tan\theta$ as a signed coordinate quotient and simplify exactly.

Answer. $\tan(\pi) = 0$

Explanation and check.

Cosine -1 is the denominator. The quotient $\frac{0}{-1}$ simplifies to 0.

Worked example

Problem. Given $\tan(3\frac{\pi}{4}) = -1$, use "add π" to infer $\tan(7\frac{\pi}{4})$. Explain whether one or both coordinate signs change.

Answer. $\tan(7\frac{\pi}{4}) = -1$

Explanation and check.

Adding π takes the opposite ray, so both x and y change sign. The quotient $\frac{y}{x}$ is unchanged, giving $\tan(7\frac{\pi}{4}) = -1$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sine and Cosine Parent Graphs

Sine and Cosine Parent Graphs | APPC-U03-P07

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for direction, sign, turning, and bending description, exact average rate and interpretation, missing projection and graph identity, parent graph and projection explanation, projection error and correction and missing anchors and verified curve.

8 PDFs and a README; 191 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

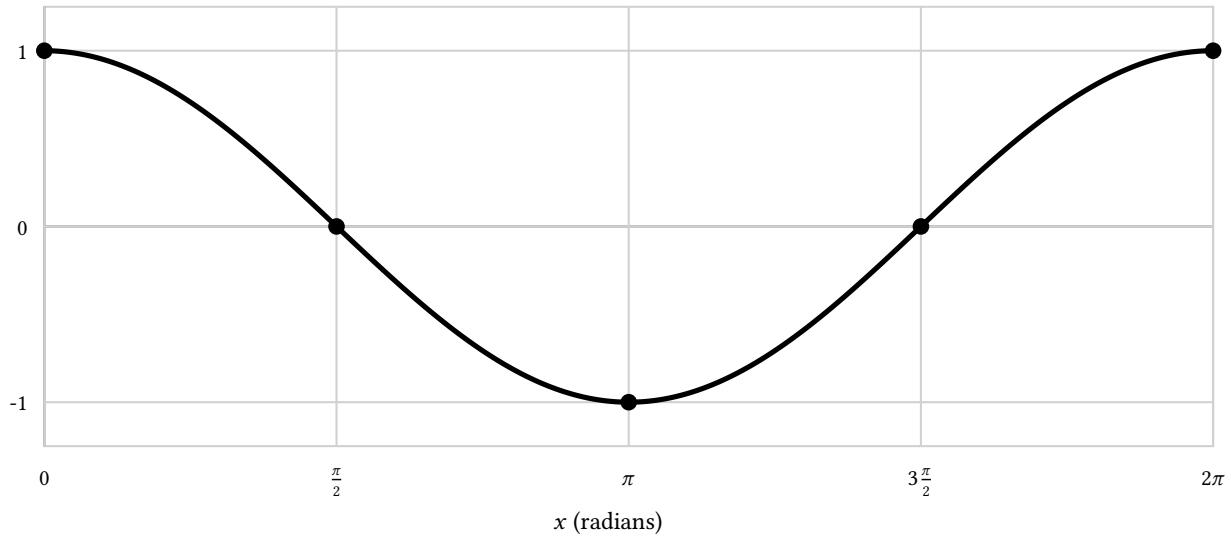
AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

From the labeled parent graph $y = \cos x$ on the closed window $[0, 2\pi]$, identify its zeros. List event coordinates rather than confusing x-locations with output values, and include boundary events.



x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$3\frac{\pi}{2}$	0
2π	1

Bank label: _____

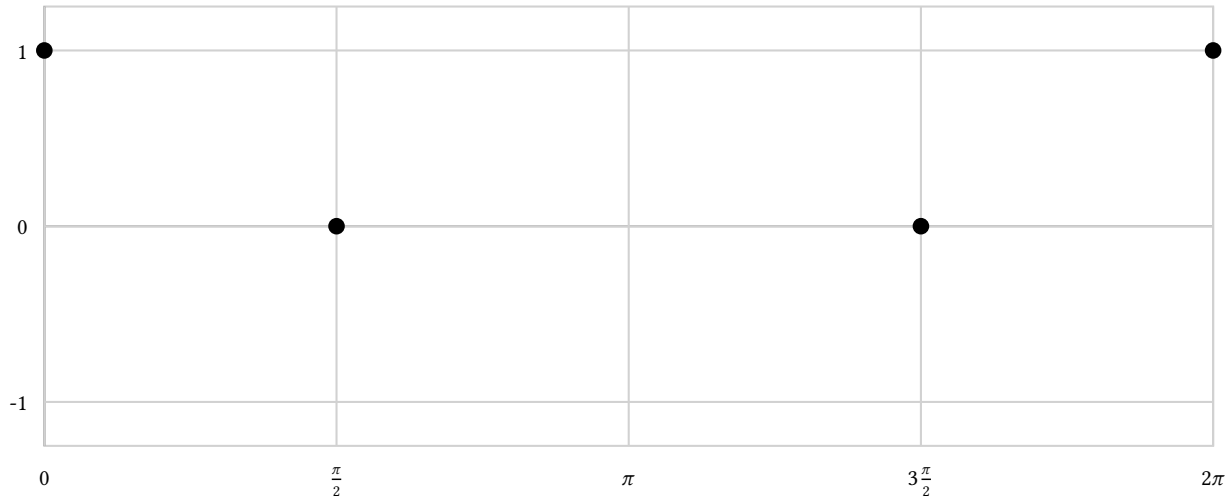
AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For the stated parent $y = \cos x$ on $[0, 2\pi]$, restore every question-mark quarter-turn anchor from the partial data, then sketch one smooth continuous parent curve with exact endpoints.



x	**y**
0	1
$\frac{\pi}{2}$	0
π	_____
$3\frac{\pi}{2}$	0
2π	1

AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $y = \cos x$ on $[0, \frac{\pi}{2}]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Required response

x	y
0	1
$\frac{\pi}{2}$	0

Your response: _____

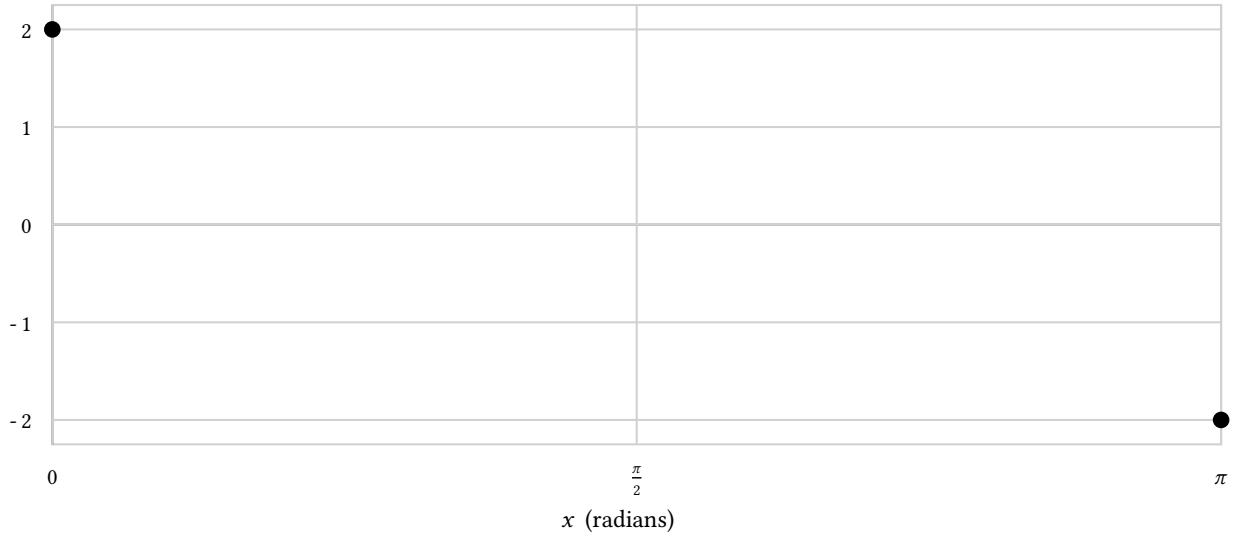
AP Precalculus

Sine and Cosine Parent Graphs

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the proposed $y = \cos x$ sketch using value at zero, quarter-turn anchors, range, cycle joins, and smoothness. Identify the first structural error and give the smallest sufficient repair.



x	y
0	2
π	-2

Sine and Cosine Parent Graphs

Full Worked Solutions - excerpt

Worked example

Problem. For $y = \cos x$ on $[\frac{\pi}{2}, 2\pi]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Answer. average rate = $\frac{2}{3\pi}$; output rises overall

Explanation and check.

Compute $\frac{(1)-(0)}{(2\pi)-(\frac{\pi}{2})} = \frac{2}{3\pi}$. Thus output rises overall.

Worked example

Problem. For $y = \cos x$ on $[3\frac{\pi}{2}, 2\pi]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Answer. average rate = $\frac{2}{\pi}$; output rises overall

Explanation and check.

Compute $\frac{(1)-(0)}{(2\pi)-(3\frac{\pi}{2})} = \frac{2}{\pi}$. Thus output rises overall.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sinusoidal Amplitude and Midline

Sinusoidal Amplitude and Midline | APPC-U03-P08

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for exact transformed local range, feature comparison and invariant, global pointwise relation and proof, vertical transformation equation, first error and minimal repair and invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

For $y = -4\cos x + 1$, complete the table by applying the vertical output map to each listed parent output. Use the sign of the scale rather than reading a decorative graph height.

Required response

parent output	transformed output
0	_____
1	_____

Bank label: _____

Under $y = 3\sin x + 2$, a transformed output $Y = \frac{23}{4}$ is requested. Reverse the output map to find the required parent output, then decide whether that output is available from the sin parent.

Required response

required parent output	transformed output
_____	$\frac{23}{4}$

Bank label: _____

For $y = 2\cos x + 5$, complete the table by applying the vertical output map to each listed parent output. Use the sign of the scale rather than reading a decorative graph height.

Required response

parent output	transformed output
-1	_____
0	_____
1	_____

Bank label: _____

A complete cycle has minimum 2 and maximum 6. For the supplied parent $p(x) = \sin x$, the labeled anchor has parent output -1 and transformed output 6. Recover amplitude, midline, and every coefficient sign supported by this evidence.

Required response

feature	output	model output	parent output
global minimum	2	_____	_____
global maximum	6	_____	_____
anchor $x = 3\frac{\pi}{2}$	_____	6	-1

Bank label: _____

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $y = (-\frac{1}{2})\cos x + 2$, the horizontal rule is fixed as the cos parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	On the stated x-window $[0, \frac{\pi}{2}]$, the sin parent has certified local output range $[0, 1]$. Under $y = 2\sin x + 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.
For $y = -3\sin x - 2$, the horizontal rule is fixed as the sin parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	On the stated x-window $[-\frac{\pi}{2}, 0]$, the sin parent has certified local output range $[-1, 0]$. Under $y = 4\sin x + 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.
For $y = 4\sin x - 3$, the horizontal rule is fixed as the sin parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	Compare $y = 3\cos x - 1$ and $y = -3\cos x - 1$. State amplitudes, midlines, full-cycle ranges, whether the ranges overlap, relative vertical orientation, and the horizontal invariant.

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

On the stated x-window $[\frac{\pi}{3}, \pi]$, the cos parent has certified local output range $[-1, \frac{1}{2}]$. Under $y = -2\cos x + 2$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.

Your response: _____

Incoming: $[1, 4]$; order reversed under the negative scale

Starting from $p(x) = \cos x$, perform this vertical-only task: Translate parent outputs by $\frac{3}{2}$, scale the result by 2, and finally translate by -5. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Your response: _____

Incoming: $y = 2(\cos x + \frac{3}{2}) - 5$

On the stated x-window $[0, \frac{\pi}{2})$, the cos parent has certified local output range $(0, 1]$. Under $y = 3\cos x - 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.

Your response: _____

Incoming: $(-1, 2]$; order preserved under the positive scale

A complete cycle has minimum -9 and maximum -1. For the supplied parent $p(x) = \cos x$, the labeled anchor has parent output 1 and transformed output -1. Recover amplitude, midline, and every coefficient sign supported by this evidence.

Required response

feature	output	model output	parent output
global minimum	-9	_____	_____
global maximum	-1	_____	_____
anchor $x = 0$	_____	-1	1

Your response: _____

AP Precalculus

Sinusoidal Amplitude and Midline

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the student's vertical-feature inference using full-cycle extrema, the affine output map, operation order, and visible-window limits as relevant. Identify the first mathematical error and give the smallest sufficient repair. output map: $Y = -2p + 3$; restricted parent range: $[0, 1]$; student range: $[1, 5]$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

A student claims: "Using the first two rows gives $a = 1$ and $d = 3$, so $Y = \cos x + 3$ fits all three rows." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Fit $Y = ap + d$ to the first two distinct parent-output rows in the supplied cos table, then audit the third row. State whether one vertical-only sinusoid fits all observations.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

parent output	target output
-1	-2
0	1
1	4

Sinusoidal Amplitude and Midline

Full Worked Solutions - excerpt

Worked example

Problem. Starting from $p(x) = \cos x$, perform this vertical-only task: First multiply every cos output by -3, then translate outputs by -1. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Answer. y = $-3\cos x - 1$

Explanation and check.

Apply the actions to the output p in order. $Y = -3p - 1$; hence $y = -3\cos x - 1$.

Worked example

Problem. Starting from $p(x) = \cos x$, perform this vertical-only task: Reflect the graph $y = \cos x$ across the horizontal line $y = 4$. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Answer. y = - $\cos x + 8$

Explanation and check.

Apply the actions to the output p in order. $Y = 2(4) - p = -p + 8$; hence $y = -\cos x + 8$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Period and Frequency of Sinusoids

Period and Frequency of Sinusoids | APPC-U03-P09

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for period and ordinary frequency, normalized frequencies and comparison, possible b values and verification, complete traversals, residual cycle, ending phase, first error and repair and supported candidates and needed evidence.

8 PDFs and a README; 75 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

For $y = \cos((-\frac{\pi}{4})x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: _____	Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 4π; input is measured in radian units. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units before using $\lvert b\rvert = \frac{2\pi}{T}$. Bank label: _____
A sinusoid has period $\frac{4}{3}$ seconds. An inclusive observation window lasts 5 seconds and begins $\frac{1}{2}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal. Bank label: _____	For $y = \cos((4)x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: _____
Construct $y = \sin(bx)$ or $y = \cos(bx)$ with frequency $\frac{1}{8}$ cycles per hour; input is measured in hours. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units if needed before using $\lvert b\rvert = \frac{2\pi}{T}$. Bank label: _____	A sinusoid has period 8 seconds. An inclusive observation window lasts 17 seconds and begins $\frac{7}{8}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal. Bank label: _____
For $y = \cos((3\frac{\pi}{2})x)$, the argument is in radians and x is measured in hours. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: _____	Construct $y = \sin(bx)$ or $y = \cos(bx)$ with frequency 2 cycles per second; input is measured in seconds. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units if needed before using $\lvert b\rvert = \frac{2\pi}{T}$. Bank label: _____

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

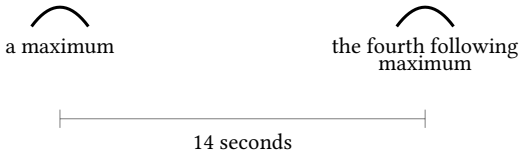
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Map the listed parent-angle anchors u to the graph of $y = \cos(-3x)$. Complete the transformed x-inputs using $u = bx$, retain each parent output, and state whether left-to-right anchor order is preserved or reversed.

Required response

parent angle	parent output	transformed input
0	1	_____
$\frac{\pi}{2}$	0	_____
π	-1	_____
$3\frac{\pi}{2}$	0	_____
2π	1	_____

A sinusoidal graph labels a maximum and the fourth following maximum 14 seconds apart. The graph explicitly certifies the stated events as consecutive or gives the exact number skipped, and crossing directions are shown. Determine the smallest period and justify the cycle fraction used.



Map the listed parent-angle anchors u to the graph of $y = \cos(\frac{1}{3}x)$. Complete the transformed x-inputs using $u = bx$, retain each parent output, and state whether left-to-right anchor order is preserved or reversed.

Required response

parent angle	parent output	transformed input
0	1	_____
$\frac{\pi}{2}$	0	_____
π	-1	_____
$3\frac{\pi}{2}$	0	_____

Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 360; input is measured in degrees of physical input. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units before using $|b| = \frac{2\pi}{T}$.

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A sinusoid has period 3 seconds. An inclusive observation window lasts 8 seconds and begins 0 of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal.

Your response: _____

Incoming: 2 complete cycles; residual $\frac{2}{3}$ cycle; ending phase $\frac{2}{3}$ cycle

Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 5; input is measured in minutes. The parent phase at the origin is fixed. Respect the stated sign information (positive) and list every supported b, converting units before using $|b| = \frac{2\pi}{T}$.

Your response: _____

Incoming: b = $2\frac{\pi}{5}$

For $y = \cos((\frac{1}{2})x)$, the argument is in radians and x is measured in radian input. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Your response: _____

Incoming: period 4π; frequency $\frac{1}{4\pi}$ cycles per input unit

A sinusoid has period 2 seconds. An inclusive observation window lasts 6 seconds and begins $\frac{1}{4}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal.

Your response: _____

AP Precalculus

Period and Frequency of Sinusoids

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The supplied cos sample table is finite. Audit the listed candidate rules against every row and decide whether the period is identified. Explain any aliasing and name one minimal additional sample or bound that would resolve or narrow it. Measure the trigonometric arguments in radians. Compare the candidate rules $\cos(2\pi t), \cos(4\pi t)$. Work with sinusoidal recurrence with fixed origin and phase. No additional constraint is supplied. Only the displayed finite samples are given. The supplied samples are $(\text{output } 1;\ t0), (\text{output } -\frac{1}{2};\ t\frac{1}{3}), (\text{output } -\frac{1}{2};\ t\frac{2}{3}), (\text{output } 1;\ t1)$.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

output	t
1	0
$-\frac{1}{2}$	$\frac{1}{3}$
$-\frac{1}{2}$	$\frac{2}{3}$
1	1

Audit the proposed horizontal-scaling explanation. Check nonconstancy, full-cycle phase change, landmark direction, sampling limits, and input units as relevant. Identify the first error and give the smallest sufficient repair. Work with sinusoidal recurrence with fixed origin and phase. Use $y = \sin(\pi\frac{t}{30})$. A student claims: frequency is $\frac{\pi}{30}$ cycles/s Measure t in seconds.

Period and Frequency of Sinusoids

Full Worked Solutions - excerpt

Worked example

Problem. For $y = \sin((\pi)x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Answer. period 2; frequency $\frac{1}{2}$ cycles per input unit

Explanation and check.

For a nonconstant parent, $T = \frac{2\pi}{|b|} = 2$. Then ordinary frequency is $\frac{1}{T} = \frac{1}{2}$; b instead measures radians of phase per input unit.

Worked example

Problem. For $y = \sin((6)x)$, the argument is in radians and x is measured in days. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Answer. period $\frac{\pi}{3}$; frequency $\frac{3}{\pi}$ cycles per input unit

Explanation and check.

For a nonconstant parent, $T = \frac{2\pi}{|b|} = \frac{\pi}{3}$. Then ordinary frequency is $\frac{1}{T} = \frac{3}{\pi}$; b instead measures radians of phase per input unit.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Phase Shifts of Sine and Cosine

Phase Shifts of Sine and Cosine | APPC-U03-P10

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
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07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for equivalence and canonical representatives, first error and minimal repair, new phase shift and event verification, shifted anchors and window qualification, phase classes or consistency conclusion and phase class and canonical representative.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \cos x$ horizontally by $h = 3\frac{\pi}{2}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** -1	_____
x $-\frac{\pi}{2}$ **y** 0	_____
x 0 **y** 1	_____
x $\frac{\pi}{2}$ **y** 0	_____

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: upward midline; x: 1); period: 4; signed amplitude: 1

Required response

event	x
upward midline	1

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (event: minimum; x: 7); period: 10; signed amplitude: 1

Required response

event	x
minimum	7

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: downward midline; x: 5); period: 8; signed amplitude: 1

Required response

event	x
downward midline	5

Bank label: _____

AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \sin x$ horizontally by $h = 13\frac{\pi}{4}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** 0	_____
x 0 **y** 0	_____
x π **y** 0	_____
x 2π **y** 0	_____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (direction: not specified; level: midline; x: 3); period: 8; signed amplitude: 1

Required response

direction	level	x
not specified	midline	3

Translate every listed landmark of $y = \cos x$ horizontally by $h = -4\frac{\pi}{3}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** -1	_____
x $-\frac{\pi}{2}$ **y** 0	_____
x 0 **y** 1	_____
x $\frac{\pi}{2}$ **y** 0	_____

For y=-cos(2π(x-h)/10), amplitude sign and period are fixed. A labeled maximum occurs at x=7. Find h modulo the period and give the representative in [0,10).

AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: First issue: a positive-sine peak occurs a quarter period after h, so $h = 0$; repair/defense: subtract $\frac{T}{4}$ from the peak location.

Rewrite the radian argument of y=cos((7/3)x-14π/9) in the form b(x-h). State h in x-units and distinguish it from the angular constant -14π/9.

Your response: _____

Incoming: not equivalent; representatives $\frac{9}{2}$ and 2 differ modulo 5

Two sine rules have the same signed amplitude and period 7, with shifts h_1=−2 and h_2=12. Decide whether they define the same graph. Reduce each shift modulo the period; a half-period difference does not count as equivalent when the amplitude sign is fixed.

Your response: _____

Incoming: $h \equiv \frac{3}{2} \pmod 5$; canonical $h = \frac{3}{2}$

Rewrite the radian argument of y=sin((-4/3)x+10π/9) in the form b(x-h). State h in x-units and distinguish it from the angular constant 10π/9.

Your response: _____

Incoming: equivalent; difference 12 = 1 period(s); common representative 7

For y=sin(2π(x-h)/(5/2)), amplitude sign and period are fixed. A labeled minimum occurs at x=19/8. Find h modulo the period and give the representative in [0,5/2).

Your response: _____

Incoming: argument $\frac{7}{3}(x - 2\frac{\pi}{3})$; $h = 2\frac{\pi}{3}$

Rewrite the radian argument of y=sin((-9/4)x-27π/8) in the form b(x-h). State h in x-units and distinguish it from the angular constant -27π/8.

Your response: _____

Incoming: argument $10(x - 3\frac{\pi}{10})$; $h = 3\frac{\pi}{10}$

Rewrite $y = 2\cos(2\pi\frac{x-\frac{3}{2}}{9})$ using the sin reference while holding period and midline fixed. Track the signed coefficient and explain the quarter-cycle phase convention; for advanced cases give both same-sign and opposite-sign target representatives.

Your response: _____

AP Precalculus

Phase Shifts of Sine and Cosine

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the proposed phase-shift claim. Normalize the argument, use the named event and signed family, and reduce representatives modulo the fixed period. Identify the first error or defend a valid noncanonical shift.

Given: domain: phase analysis with amplitude and period fixed; rule: $y = \sin(2x - \pi)$; student claim: shift right π

A student applies the horizontal shift to output values instead of translating the input landmarks with the stated sign. Diagnose the first mathematical error and state the correction. Then complete the original task: Translate every listed landmark of $y = \sin x$ horizontally by $h = \frac{\pi}{2}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x 0 **y** 0	_____
x $\frac{\pi}{2}$ **y** 1	_____
x π **y** 0	_____

Phase Shifts of Sine and Cosine

Full Worked Solutions - excerpt

Worked example

Problem. Rewrite the radian argument of y=cos((-7/2)x+21π/4) in the form b(x-h). State h in x-units and distinguish it from the angular constant 21π/4.

Answer. argument $-\frac{7}{2}(x-3\frac{\pi}{2})$; $h=3\frac{\pi}{2}$

Explanation and check.

Solve $-\frac{7}{2}(x-h)=(-\frac{7}{2})x+21\frac{\pi}{4}$. Since $h=-\frac{c}{b}, h=3\frac{\pi}{2}$; the inside constant itself is not the displacement unless $b=-1$.

Worked example

Problem. Rewrite the radian argument of y=cos(10x-3π) in the form b(x-h). State h in x-units and distinguish it from the angular constant -3π.

Answer. argument $10(x-3\frac{\pi}{10})$; $h=3\frac{\pi}{10}$

Explanation and check.

Solve $10(x-h)=10x-3\pi$. Since $h=-\frac{c}{b}, h=3\frac{\pi}{10}$; the inside constant itself is not the displacement unless $b=-1$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Writing Sinusoidal Equations from Features

Writing Sinusoidal Equations from Features | APPC-U03-P11

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for existence decision and first contradiction, equivalence decision and decisive normalized evidence, one compatible rule and sufficiency explanation, one complete rule and parameter verification, one example rule or impossibility proof and one sine rule with directional check.

8 PDFs and a README; 152 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$-\frac{5}{2}$
next global maximum	1	$\frac{5}{2}$

Bank label: _____

Determine which sinusoidal parameters are forced by the partial exact features and which remain free. Demonstrate any nonuniqueness by checking the two explicit candidate constructions, or replace either if it fails.

The first proposed rule is $y = 3\cos(2\pi\frac{x+\frac{5}{2}}{6})$.

The second proposed rule is $y = 3\cos(2\pi\frac{x+\frac{5}{2}}{3})$.

The global range is $[-3, 3]$.

The two stated maxima are not specified to be consecutive.

Maxima occur at $x = -\frac{5}{2}$ and $x = \frac{7}{2}$.

Bank label: _____

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$\frac{3}{2}$
next global maximum	3	$\frac{21}{2}$

Bank label: _____

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 10.

The claimed range is $[-4, 4]$.

The reference input is $x = -\frac{3}{2}$.

The first proposed rule is $y = 4\sin(2\pi\frac{x+\frac{3}{2}}{10})$.

The second proposed rule is $y = 4\cos(2\pi\frac{x-1}{10})$.

Bank label: _____

AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

An exact partial graph is supplied with its global range, labeled events and directions, and the stipulation that the curve continues sinusoidally. Construct one compatible cosine rule and explain why clipping does or does not hide needed information.

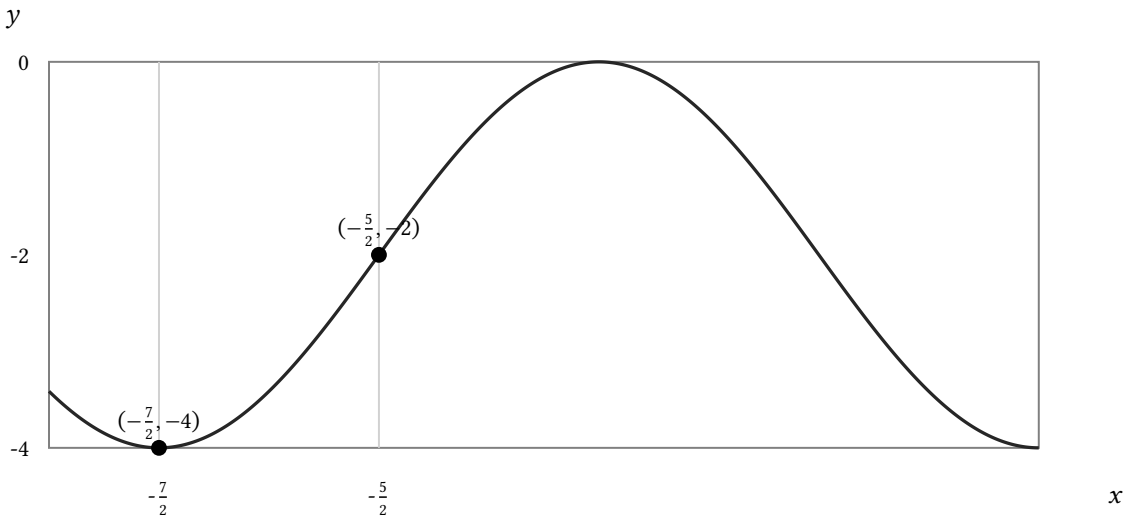
The curve continues sinusoidally beyond the displayed interval.

The global range is $[-4, 0]$.

The labeled minimum is $(-\frac{7}{2}, -4)$.

The labeled upward midline is $(-\frac{5}{2}, -2)$.

The displayed x-interval is $[-4, \frac{1}{2}]$.



event	x	y
minimum	$-\frac{7}{2}$	-4
upward midline	$-\frac{5}{2}$	-2

AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

event	value	x
global maximum	0	$\frac{21}{2}$
next global maximum	0	$\frac{57}{2}$
global minimum	-6	not specified

Your response: _____

AP Precalculus

Writing Sinusoidal Equations from Features

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the student's multi-parameter reconstruction against every exact feature. Identify the earliest failed parameter relation, retain valid deductions, give a corrected complete rule, and verify the repaired model.

Preserve the valid deductions when repairing the model.

The supplied features are: range $[0, 6]$, period 4, upward midline at $x = 1$.

The student proposes $y = 3 + 3\sin(2\pi\frac{x-1}{8})$.

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 14.

The claimed range is $[-3, 1]$.

The reference input is $x = \frac{11}{2}$.

The first proposed rule is $y = -1 + 2\sin(2\pi\frac{x-\frac{11}{2}}{14})$.

The second proposed rule is $y = -1 + 2\sin(2\pi\frac{x-9}{14})$.

Writing Sinusoidal Equations from Features

Full Worked Solutions - excerpt

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = 1$.

The global range is $[-2, 2]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-2+2}{2} = 0$, not 1. Therefore the feature set is inconsistent.

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = -1$.

The global range is $[-4, 0]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-4+0}{2} = -2$, not -1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-4+0}{2} = -2$, not -1. Therefore the feature set is inconsistent.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Interpreting Sinusoidal Transformations

Interpreting Sinusoidal Transformations | APPC-U03-P13

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for changed and invariant physical features, relevant common-scale comparison, first misleading claim and corrected quantitative statement, phase event and initial-value qualification, contextual attained range and boundary status and parameter meanings and derived features.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes and $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 230 + 40\cos(2\pi\frac{t-90}{300})$; output unit before: centimeters. Bank label: _____	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 11 + 3\cos(2\pi\frac{t-3}{16})$; valid window: $0, 64$. Bank label: _____
Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 210 + 40\cos(2\pi\frac{t-30}{360})$; output unit before: centimeters. Bank label: _____	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 9 + 5\cos(2\pi\frac{t-4}{8})$; valid window: $0, 32$. Bank label: _____

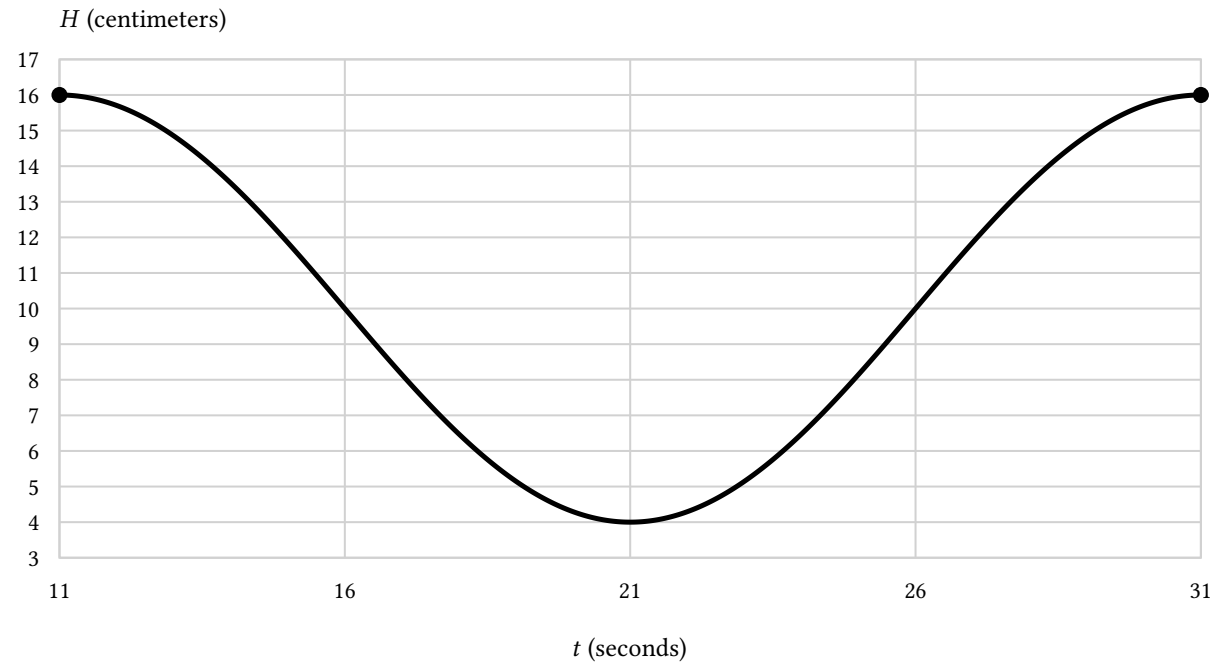
AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Interpret the supplied graph only on its physical operating window. State the attainable output interval, midline meaning, and whether displayed boundaries are attained; do not substitute the full mathematical range when the window clips an extremum.



AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{16})$; only changed category: period; original formula: $P(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{8})$; output unit: units; time unit: hours.

Your response: _____

Incoming: period doubles from 8 to 16 time units, so recurrence halves; range and the anchor at $t = \frac{7}{2}$ stay fixed.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: new datum is 52 cm above the old datum, so $R = Q - 52$; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 220 + 60\cos(2\pi\frac{t-60}{240})$; output unit before: centimeters.

Your response: _____

Incoming: $R(t) = 168 + 60\cos(2\pi\frac{t-60}{240})$; event times are unchanged while every height label decreases by 52 cm.

Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: meters; supplied formula: $Q(t) = 10 + 4\sin(2\pi\frac{t+\frac{3}{2}}{10})$; valid window: $0, 40$.

Your response: _____

Incoming: amplitude 4 meters; midline 10 meters; period 10 hours; frequency $\frac{1}{10}$ cycle per hour; $h = -\frac{3}{2}$ hours labels an upward midline crossing.

Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 6 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12})$, $B(t) = 8 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12})$; practical measure: same-period.

Your response: _____

Incoming: same period 12 hours and same spread 10, but B's baseline and range are 2 units higher.

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(\tau) = 8 + 5\sin(2\pi\frac{\tau}{10})$; only changed category: clock; original formula: $P(t) = 8 + 5\sin(2\pi\frac{t-4}{10})$; output unit: units; time unit: hours.

Your response: _____

Incoming: with t = $\tau + 4$, the same physical anchor is labeled $\tau = 0$ rather than $t = 4$; range and physical period are unchanged.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 200 + 40\cos(2\pi\frac{t}{360})$; output unit before: centimeters.

Your response: _____

AP Precalculus

Interpreting Sinusoidal Transformations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Test the student's contextual parameter statement against the supplied model. Identify the first misleading claim and replace it with a quantitatively correct statement including units or window limits where relevant. context units: stated in formula or prompt; student statement: The period is 4π.; supplied model: $Q(t) = 4 + 3\sin(4\pi(t - 2))$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Interpreting Sinusoidal Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 2\sin(2\pi\frac{t+2}{8}), B(t) = 5 + 2\sin(2\pi\frac{t+2}{16})$; practical measure: same-range.

Answer. same range $[3, 7]$, but B recurs every 16 rather than 8 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[3, 7]$, but B recurs every 16 rather than 8 hours.

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 4\sin(2\pi\frac{t-1}{10}), B(t) = 5 + 4\sin(2\pi\frac{t-1}{20})$; practical measure: same-range.

Answer. same range $[1, 9]$, but B recurs every 20 rather than 10 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[1, 9]$, but B recurs every 20 rather than 10 hours.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sinusoidal Modeling

Sinusoidal Modeling | APPC-U03-P12

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for family decision with evidentiary limit, model recommendation or insufficiency conclusion, invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result, invalid assumption and actionable repair, plausible fitted rule and uncertainty statement and signed-residual diagnosis and revision recommendation.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Sinusoidal Modeling

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 3\sin(2\pi\frac{t+\frac{5}{2}}{8})$. The event level is $y = 10$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[-\frac{31}{6}, \frac{27}{2}]$.

Bank label: _____

The contextual extrema are exact and the cycle is stipulated stationary only on the feasible time domain. Construct one sinusoidal model in the stated time unit, explain which events determine a full or half cycle, and retain the domain restriction. The feasible time domain is $[8, 80]$. Time is measured in days.

Required response

event	time	value
minimum	8	11
next minimum	26	11

Bank label: _____

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 20 + 3\cos(2\pi\frac{t+3}{8})$. The response is measured in units. The sample time window is $[-11, 13]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = -3$. Time is measured in hours.

Bank label: _____

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 3\sin(2\pi\frac{t-\frac{11}{2}}{14})$. The event level is $y = 12$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[\frac{5}{6}, \frac{67}{2}]$.

Bank label: _____

AP Precalculus

Sinusoidal Modeling

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compare the fitted sinusoids for the stated use case. Weigh observed-window error, physical range, prediction horizon, and divergence. If the evidence cannot distinguish them for the intended decision, recommend a specific validation measurement. The intended decision is at $t = 34$. The observed time window is $[0, 13]$. The response tolerance is 1 unit.

Required response

future prediction	model	observed window RMSE	period
21	A	0.43	11
26	B	0.45	14

The dataset and fully specified candidate regression outputs are supplied. Select every usable model in the stated angle mode. Verify at least one observed or held-out landmark and do not infer software behavior that is not shown. Use radians mode on the calculator.

Required response

formula	label
$y = 3 + 2\cos(2\pi\frac{t-10}{8})$	A
$y = 3 + 2\sin(2\pi\frac{t-8}{8})$	B

t	y
10	5
14	1
18	5

Use the signed residual definition shown. Assess both the pattern across phases and the held-out observation; a small total error does not excuse a systematic pattern. Recommend retention, revision, or investigation. The absolute tolerance is 0.75. At held-out time $t = 5$, the signed residual is 0.11.

Required response

observed minus predicted	t
0.11	0
-0.11	1
0.0	2
0.11	3
-0.11	4

Use several measurements rather than a single high or low value to estimate a stationary sinusoid. Give a plausible rule, identify the approximate baseline, amplitude, period, and phase, and state what the finite noisy sampling cannot establish exactly. The measured response is in units. Time is measured in hours.

Required response

measurement	time
24.2	$\frac{13}{2}$
19.8	$\frac{23}{2}$
20.2	$\frac{43}{2}$
23.8	$\frac{53}{2}$

AP Precalculus

Sinusoidal Modeling

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 23 + 4\cos(2\pi\frac{t-4}{12})$. The response is measured in units. The sample time window is $[-8, 28]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = 13$. Time is measured in hours.

Your response: _____

Incoming: Prediction 8: 23 units; interpolation inside the sampled window.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 4\sin(2\pi\frac{t-\frac{5}{2}}{8})$. The event level is $y = 12$. Find the second occurrence of the midline level in the window, counting either direction. Time is measured in hours. Use the time window $[\frac{5}{2}, \frac{21}{2}]$.

Your response: _____

Incoming: Event case 6: $t = \frac{13}{2}$ (downward).

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 9. Initially the point is initial vertical velocity zero, then downward. Use seconds as the time unit for t. The operating window is $\frac{1}{2} \le t \le \frac{85}{2}$. The radius is 6. The starting position is top. The starting time is $t = \frac{1}{2}$. One uniform rotation takes 14 seconds.

Your response: _____

Incoming: $y = 9 + 6\cos(2\pi\frac{t-\frac{1}{2}}{14})$

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Treat negative depth as physically valid. A fitted water-depth sinusoid has range $[-2, 8]$ meters.

Your response: _____

Incoming: Water depth in $[-2, 8]$ meters includes negative depth. Restrict use to nonnegative depths or refit with a nonnegative constrained model.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 6\sin(2\pi\frac{t-\frac{1}{2}}{14})$. The event level is $y = 10$. Find the all upward midline crossings in the window. Time is measured in hours. Use the time window $[\frac{1}{2}, \frac{57}{2}]$.

Your response: _____

Incoming: Event case 4: $t = \frac{1}{2}, \frac{29}{2}, \frac{57}{2}$.

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 7. Initially the point is initial vertical velocity zero, then upward. Use seconds as the time unit for t. The operating window is $-\frac{5}{2} \le t \le \frac{43}{2}$. The radius is 3. The starting position is bottom. The starting time is $t = -\frac{5}{2}$. One uniform rotation takes 8 seconds.

Your response: _____

AP Precalculus

Sinusoidal Modeling

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Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Combine times without a common clock origin. Tidal observations are in hours after midnight on different dates.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Repair only the phase of a stationary sinusoid. A sensor baseline rises steadily while residuals alternate.

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Full Worked Solutions - excerpt

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Use $T = 3$ with t measured in seconds. The fitted period is 3 minutes but t is entered in seconds.

Answer. Assumption repair 3: convert the period to 180 seconds, or convert every input to minutes.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to convert the period to 180 seconds, or convert every input to minutes.

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Extrapolate the fitted cycle indefinitely. Seasonal data cover one year during an unusual shutdown.

Answer. Assumption repair 4: limit claims to the observed window and gather ordinary-year validation data.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to limit claims to the observed window and gather ordinary-year validation data.