Sets & Steps
Math Practice & Worked Solutions

Precalculus

Precalculus Activity Library | 94 Current Topics

94-topic bundle | 9,024 problems | Four activity formats

Included topics

The following pages list all 94 included resources in the suggested order, with a link to each product. Each resource retains its complete preview.

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

752 PDFs and 94 READMEs; 8,731 buyer PDF pages across the 94 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Math Practice & Worked Solutions

Precalculus

Included topics

94-topic bundle | 9,024 problems | Four activity formats

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01 **Function Representations and Average Rates of Change**

View individual product and its full preview

02 **Comparing Average Rates of Change**

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03 **Polynomial Degree, Zeros, and End Behavior**

View individual product and its full preview

04 **Function Symmetry, Finite Differences, and End Behavior**

View individual product and its full preview

05 **Rational Function Domains and Sign Charts**

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06 **Vertical Asymptotes and One-Sided Behavior**

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07 **Holes in Rational Functions**

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08 **Equivalent Polynomial Forms**

View individual product and its full preview

09 **Rational Expression Operations and Restrictions**

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10 **Polynomial Division and Slant Asymptotes**

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11 **Function Translations**

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12 **Function Reflections, Stretches, and Compressions**

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14 **Building and Selecting Function Models**

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19 **Interpreting and Comparing Function Models**

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20 **Residuals and Model Validation**

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21 **Arithmetic vs Geometric Sequences**

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22 **Sequences to Linear and Exponential Models**

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View individual product and its full preview

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53 **Period and Frequency of Sinusoids**

View individual product and its full preview

54 **Phase Shifts of Sine and Cosine**

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55 **Writing Sinusoidal Equations from Features**

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89 **Determinants and Area**

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Function Representations and Average Rates of Change

Function Representations and Average Rates of Change | APPC-COMB-U01-P01-P02

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for exact average rate, degrees Celsius per day, kilopascals per hour, function decision with justification, value comparison and behavior statement and labeled qualitative graph.

8 PDFs and a README; 107 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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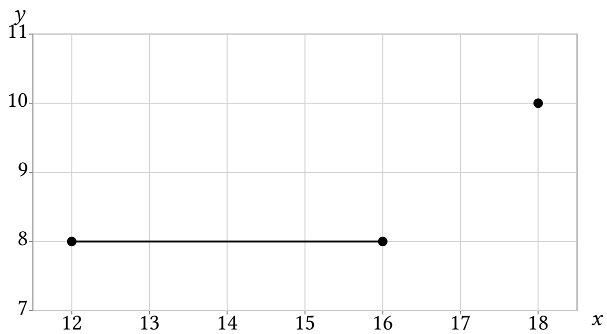
AP Precalculus

Function Representations and Average Rates of Change

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An exact graph consists of a line segment from $(12, 8)$ (solid) to $(16, 8)$ (solid); the isolated solid point $(18, 10)$. Find every input satisfying $y = 8$, respecting all endpoint markers.

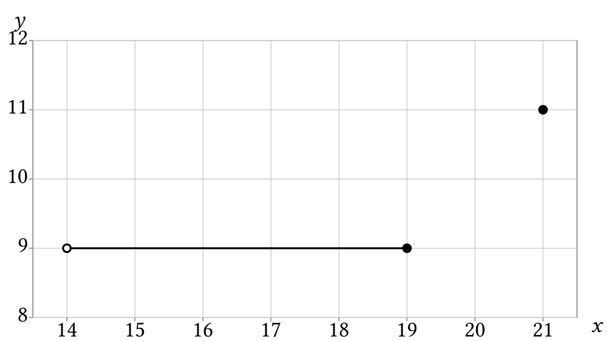


Bank label: ______

For a mixing tank, solution volume is $F(t) = 10 + 3t$ liters after t continuous minutes. Operation may last at most 8 minutes, but capacity is 22 liters. Give the feasible domain and output range.

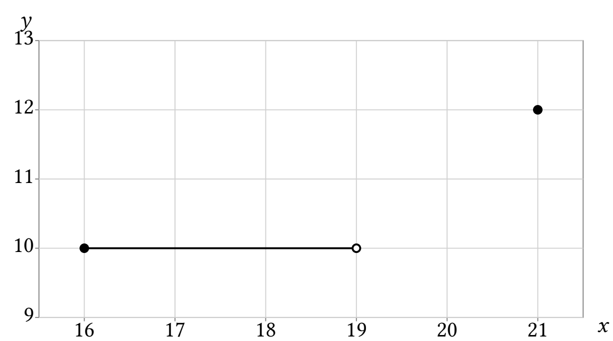
Bank label: ______

An exact graph consists of a line segment from $(14, 9)$ (open) to $(19, 9)$ (solid); the isolated solid point $(21, 11)$. Find every input satisfying $y = 9$, respecting all endpoint markers.



Bank label: ______

An exact graph consists of a line segment from $(16, 10)$ (solid) to $(19, 10)$ (open); the isolated solid point $(21, 12)$. Find every input satisfying $y = 10$, respecting all endpoint markers.



Bank label: ______

AP Precalculus

Function Representations and Average Rates of Change

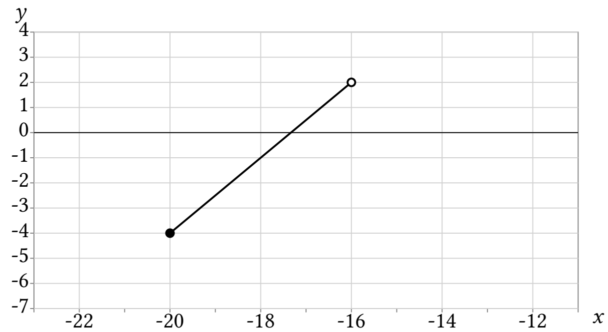
Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A study controls number of concert tickets purchased and records total charge. The model is $f(x) = 4x + 2$, and $f(3) = 14$. Identify the input and output quantities with units, then interpret $f(3) = 14$.

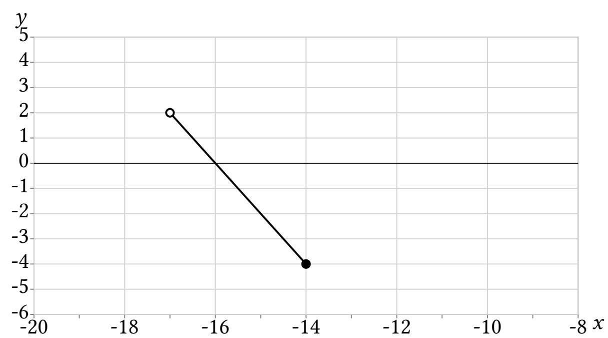
Measure the input in tickets and the total charge in dollars.

An exact graph uses one unit per tick and consists of a line segment from $(-20, -4)$ (solid) to $(-16, 2)$ (open). Determine the domain and range of the plotted relation; do not use the viewing-window edges.



For sealed sample tubes loaded into a rack, rack mass is $F(n) = 7 + 3n$. At most 4 items fit, and n must be a whole number. Give the feasible domain and range.

An exact graph uses one unit per tick and consists of a line segment from $(-17, 2)$ (open) to $(-14, -4)$ (solid). Determine the domain and range of the plotted relation; do not use the viewing-window edges.



AP Precalculus

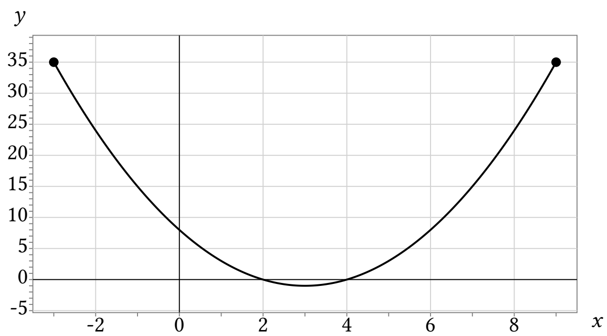
Function Representations and Average Rates of Change

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

An exact graph consists of the graph of $y = (x-3)^2 - 1$ for $-3 \le x \le 9$, with the left endpoint included and right endpoint included. Find every input satisfying $y = 15$, respecting all endpoint markers.



Your response: ______________

Incoming: Rates are F: $0, T$: $5, G$: 0; ranking: $T > F = G$.

A relation is shown by isolated plotted points: $(29, 18), (30, 23), (32, 18), (34, 25)$. Is the relation a function of the first coordinate? Justify using the input-output condition.

Context/domain: precalculus

29	18
30	23
32	18
34	25

Your response: ______________

Incoming: Rates are F: $-1, T$: $2, G$: 5; ranking: $G > T > F$.

A finite table gives $x = 3$, $f(x) = 14$; $x = 6$, $f(x) = 19$; $x = 9$, $f(x) = 17$. The function is continuous on the displayed interval, but no interpolation rule is specified. Evaluate the claim: 'The maximum output on $[3, 9]$ is 19.' State whether it is supported and identify exactly what the evidence does or does not determine.

Context/domain: precalculus

3	14
6	19
9	17

Your response: ______________

Incoming: -2

Sketch one continuous qualitative graph of balloon altitude, measured in meters, versus elapsed time using breakpoints $[0, 4, 6, 9, 13]$. In order, the stages are: Stage A: increases while steepening; Stage B: remains constant; Stage C: decreases while flattening; Stage D: increases at a constant rate. Also satisfy this global constraint: the final level is above the initial level. Label both axes. Exact vertical values are not specified, so show only the required direction, pace, and sign information.

Your response: ______________

AP Precalculus

Function Representations and Average Rates of Change

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A table gives $x = -5, f(x) = 35$; $x = 1, f(x) = ?$; $x = 5, f(x) = 54$. The average rate of change from $x = -5$ to $x = 1$ is 1. The table also claims an average rate of 3 from $x = 1$ to $x = 5$. After recovering the missing endpoint, decide whether all the information is consistent. Find the missing output and show the signed net-change equation. Student work/claim: The missing output is 41, so every supplied condition is satisfied. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Context/domain: precalculus

-5	35
1	____
5	54

Diagnose the claim: 'A negative average rate means the function decreased throughout the interval.' Construct an exact one-unit-per-tick polyline counterexample on $[-3, 4]$ with endpoints $(-3, 13)$ and $(4, 8)$, and verify both its average rate and contradictory interior behavior. Student work/claim: A negative average rate means the function decreased throughout the interval. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Context/domain: precalculus

-3	13
4	8

Function Representations and Average Rates of Change

Full Worked Solutions - excerpt

Worked example

Problem. Let $f(x) = kx^2 + 2x$ for real k. Its average rate on $[1, 4]$ is 12. Determine k.

Answer. 2

Explanation and check.

The average rate is $k(1 + 4) + 2 = 5k + 2$. Thus $5k + 2 = 12$ and $k = 2$.

Worked example

Problem. For $f(x) = x^3 - x$ and $a > 0$, simplify the average rate on $[-a, a]$. Explain which symmetry is used.

Answer. $a^2 - 1$

Explanation and check.

The odd function has $f(-a) = -f(a)$. The quotient is $2(a^3 - a\frac{)}{2a} = a^2 - 1$.

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Math Practice & Worked Solutions

AP Precalculus
Comparing Average Rates of Change

Comparing Average Rates of Change | APPC-U01-P03

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
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08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for family with difference evidence, secant, interior comparison, curvature explanation, simplified rule in s, concavity and signed interval changes, secant rates, midpoint comparison, diagnosis and missing output or incompatibility proof.

8 PDFs and a README; 75 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Comparing Average Rates of Change

Name: ______ **Date:** ______ **Class:** ______

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

For $f(x) = -x + 29$, derive the average rate of change on $[s, s+3]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Bank label: ______

For the unknown quadratic $f(x) = ax^2 + bx + c$, two width-2 secants begin at $x = -2$ and $x = 0$ and have rates 2 and -6, respectively. Also $f(-1) = 6$. Recover the coefficients supported by these constraints.

Bank label: ______

A function is declared linear on the equally spaced inputs in this exact table: $x = -2, y = -19$; $x = -1, y = -16$; $x = 0, y = ?$; $x = 1, y = -10$; $x = 2, y = -5$. Find the missing output if the declaration is consistent; otherwise explain why no value can make all entries fit.

Required output: missing output or incompatibility proof

x	**y**
-2	-19
-1	-16
0	______
1	-10
2	-5

Bank label: ______

For $f(x) = x^2 + 4x + 17$, derive the average rate of change on $[s, s+4]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Bank label: ______

AP Precalculus

Comparing Average Rates of Change

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $f(x) = 3x^2 - 2x - 1$, derive the average rate of change on $[s, s+3]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

For $f(x) = -2x^2 - 3x - 4$, derive the average rate of change on $[s, s+2]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

For $f(x) = -2x^2 - 4x + 20$, derive the average rate of change on $[s, s+5]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

A model is known to be linear, quadratic, or outside those two families. Its exact samples are $x = -2, y = 12$; $x = 1, y = 12$; $x = 4, y = 12$; $x = 7, y = 12$; $x = 10, y = 12$. Identify the consistent family and cite the decisive difference pattern.

Required output: family with difference evidence

x	**y**
-2	12
1	12
4	12
7	12
10	12

AP Precalculus

Comparing Average Rates of Change

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

On $[-2, 3]$, $L(x) = -x + 6$ and $Q(x) = L(x) - 2(x + 2)(x - 3)$. Verify that their endpoint secant rates agree, compare $L(0)$ and $Q(0)$, and use curvature to explain the interior difference.

Your response: ______________________

Incoming: Both endpoint rates are -1; $L(0) = 6$ and $Q(0) = 18$. Q is concave down.

The inputs are not equally spaced in the exact table $(-1, 17)$; $(1, 17)$; $(4, 47)$; $(5, 65)$. Normalize every output change by its actual width, attach each rate to the interval midpoint, and test the quadratic requirement that those midpoint-rate pairs lie on a line.

Required output: secant rates, midpoint comparison, diagnosis

x	y
-1	17
1	17
4	47
5	65

Your response: ______________________

Incoming: Rates: $0, 10, 18$; midpoints: $0, \frac{5}{2}, \frac{9}{2}$. The evidence is consistent with a quadratic.

For the unknown quadratic $f(x) = ax^2 + bx + c$, two width-1 secants begin at $x = -3$ and $x = -1$ and have rates -11 and -3, respectively. No output value is supplied. Recover the coefficients supported by these constraints.

Your response: ______________________

Incoming: a = 2 and $b = -1$, while c is undetermined without an output anchor.

The inputs are not equally spaced in the exact table $(2, -2)$; $(3, -15)$; $(6, -78)$; $(10, -218)$. Normalize every output change by its actual width, attach each rate to the interval midpoint, and test the quadratic requirement that those midpoint-rate pairs lie on a line.

Required output: secant rates, midpoint comparison, diagnosis

x	y
2	-2
3	-15
6	-78
10	-218

Your response: ______________________

AP Precalculus

Comparing Average Rates of Change

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use the exact equally spaced table and the stated candidate families. A student reports slope 5 because successive outputs differ by 5. Identify the invalid inference and give the corrected conclusion supported by the exact data.

Required output: family with difference evidence

x	**y**
−4	0
−2	5
0	10
2	15
4	20

Use the exact equally spaced table and the stated candidate families. A student insists that a straight-line pattern is a nondegenerate quadratic because its second differences are constant. Identify the invalid inference and give the corrected conclusion supported by the exact data.

Required output: family with difference evidence

x	**y**
−3	−4
−2	−3
−1	−2
0	−1
1	0

Comparing Average Rates of Change

Full Worked Solutions - excerpt

Worked example

Problem. For $f(x) = x^2 + 4x + 17$, derive the average rate of change on $[s, s+4]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Answer. $R(s) = 2s + 8$; it increases with interval location.

Explanation and check.

The endpoint change factors as $4[2s + 8]$. Dividing by the nonzero width 4 gives $R(s) = 2s + 8$. Thus the secant rate increases with interval location.

Worked example

Problem. For $f(x) = 3x^2 + x + 35$, derive the average rate of change on $[s, s+5]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Answer. $R(s) = 6s + 16$; it increases with interval location.

Explanation and check.

The endpoint change factors as $5[6s + 16]$. Dividing by the nonzero width 5 gives $R(s) = 6s + 16$. Thus the secant rate increases with interval location.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polynomial Degree, Zeros, and End Behavior

Polynomial Degree, Zeros, and End Behavior | APPC-COMB-U01-P04-P06

96 problems · Four activity formats

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07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for left/right signs and oriented behavior, dominant term and both tails by case, decision with structural reason, qualified local relative-size conclusion, justified eventual comparison and qualified zero-set and equality conclusion.

8 PDFs and a README; 88 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polynomial Degree, Zeros, and End Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

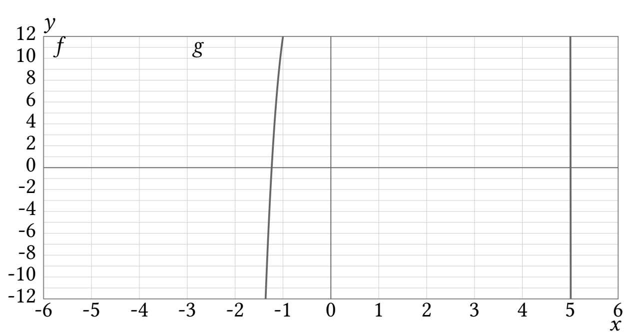
For $P(x) = 4x^3 - 4x^3 + 7 - 7$, determine the actual degree, leading term, and leading coefficient. Simplify only as much as needed and handle any constant or zero exception.

Bank label: ______

On the stated domain, the rule is $f(x) = x^4 + \frac{3}{x}$. Does it define a polynomial function? Simplify if useful and justify from coefficient, exponent, and domain structure.

Bank label: ______

For $f(x) = -x^6 + 5x^5 + 18$ and $g(x) = -x^6 - 17$, compare the requested difference after leading cancellation. Distinguish signed tail direction from eventual absolute magnitude and inspect cancellation when relevant.



Bank label: ______

For $P\_p(x) = (p - 1)x^4 - 2x^3 + 17$, describe both tails for $p > 1$, $p < 1$, and $p = 1$. Recompute degree at cancellation rather than assigning a zero leading coefficient.

Bank label: ______

AP Precalculus

Polynomial Degree, Zeros, and End Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

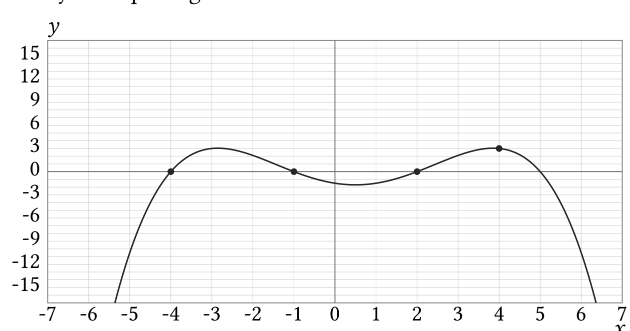
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For the polynomial $f(x) = x^6 + 2x^5 + 18$, describe both end behaviors. State the x-direction and output direction for each tail, and justify from the leading structure.

On the stated domain, the rule is $f(x) = \sqrt{2}x^1 - 2x + 11$. Does it define a polynomial function? Simplify if useful and justify from coefficient, exponent, and domain structure.

For $P(x) = (4x^3 + 1)(3x + 1)(-x + 1)$, determine the actual degree, leading term, and leading coefficient. Simplify only as much as needed and handle any constant or zero exception.

A continuous polynomial graph uses one unit per tick and includes exact visible points $(-4, 0), (-1, 0), (2, 0), (4, 3)$. Its certified arrows state $y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$. Infer the necessary degree parity and leading-coefficient sign. Does this identify a unique degree?

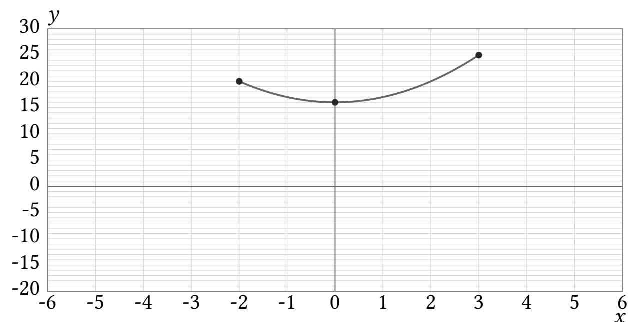


AP Precalculus

Polynomial Degree, Zeros, and End Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START	**Incoming:** Degree 3; leading term $-2x^3$; leading coefficient -2.
For $P(x) = 5x^5 - 5x^5 - 2x^3 + 11$, determine the actual degree, leading term, and leading coefficient. Simplify only as much as needed and handle any constant or zero exception.	An exact polynomial graph uses rule $x^2 + 16$, one unit per tick, domain $[-2, 3]$, supplied feature points $(-2, 20), (0, 16), (3, 25)$, and certified behavior: no full-domain tails are asserted. Determine which absolute extrema are justified on the stated domain; distinguish local from global evidence. 
Your response: ______________	Your response: ______________

AP Precalculus

Polynomial Degree, Zeros, and End Behavior

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Construct one polynomial of minimum allowed degree at least 2, with even degree, positive leading coefficient, and constant term 1. Give a formula and verify both tails. A student writes only the final conclusion and omits the substitution, algebra, domain check, or justification required by the prompt. Diagnose the incomplete reasoning and supply the missing work.

Polynomial Degree, Zeros, and End Behavior

Full Worked Solutions - excerpt

Worked example

Problem. For $P(x) = 5x^5 - 5x^5 - 2x^3 + 11$, determine the actual degree, leading term, and leading coefficient. Simplify only as much as needed and handle any constant or zero exception.

Answer. Degree 3; leading term $-2x^3$; leading coefficient -2.

Explanation and check.

After the needed combining or leading-factor multiplication, the highest surviving power is 3 with coefficient -2.

Worked example

Problem. For $P(x) = 4x^3 - 4x^3 + 7 - 7$, determine the actual degree, leading term, and leading coefficient. Simplify only as much as needed and handle any constant or zero exception.

Answer. The result is the zero polynomial, whose ordinary degree and leading coefficient are undefined.

Explanation and check.

All terms cancel. The zero polynomial has no highest nonzero power, so it has no ordinary degree or leading coefficient.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Function Symmetry, Finite Differences, and End Behavior

Function Symmetry, Finite Differences, and End Behavior | APPC-COMB-U01-P07-P09

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for classification with identity check, classification with mapped-point evidence, unit-bearing conditional model comparison, scaled difference and unchanged degree, two tail sides and crossing qualifier and forced values or exact conflict.

8 PDFs and a README; 94 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Function Symmetry, Finite Differences, and End Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Find all real k for which $f_k(x)=kx^3 + 17x$ is even. Collect like powers and set every coefficient of the incompatible parity to zero; report whether the result is unique, free, or inconsistent.

Bank label: ______

Classify $f(x) = -x^2 + 3$ as even, odd, both, or neither. Compute and simplify $f(-x)$; do not use the parity of the highest degree alone.

Bank label: ______

A polynomial is declared even. Its exact partial table gives $f(0) = 6$. Recover every forced entry and diagnose any conflict, enforcing the paired-input law and the odd-function origin condition.

Complete response required: forced values or exact conflict.

x	f(x)
0	6

Bank label: ______

Construct an odd polynomial with $f(0) = 1$. If impossible, prove the contradiction from the symmetry identity.

Bank label: ______

AP Precalculus

Function Symmetry, Finite Differences, and End Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Classify $f(x) = 0$ as even, odd, both, or neither. Compute and simplify $f(-x)$; do not use the parity of the highest degree alone.	Classify $f(x) = 3x^9 - 5x$ as even, odd, both, or neither. Compute and simplify $f(-x)$; do not use the parity of the highest degree alone.
A polynomial is declared odd and satisfies $f(1) = 5$. Infer both $f(-1)$ and $f(0)$, including the forced graph feature at the origin.	Construct an odd polynomial of degree at most 5 with $f(1) = 1$ and $f(2) = 8$. Give one formula and verify both values.
Find all real k for which $f_k(x)=kx^4 + 4x^2$ is even. Collect like powers and set every coefficient of the incompatible parity to zero; report whether the result is unique, free, or inconsistent.	Construct an even polynomial of degree at most 4 with $f(0) = 1$ and $f(1) = 3$. Give one formula and verify symmetry.

AP Precalculus

Function Symmetry, Finite Differences, and End Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The data are inconsistent with a declared odd function: $f(-1)$ must equal -5, not 6.

Find all real k for which $f_k(x)=(k-5)x^3+6x^2$ is even. Collect like powers and set every coefficient of the incompatible parity to zero; report whether the result is unique, free, or inconsistent.

Your response: ______________

Incoming: Yes. The inputs are equally spaced (the step need not be 1), so ordinary successive differences may be compared.

A polynomial is declared even. Its exact partial table gives $f(3)=10$, $f(-3)=11$, $f(0)=1$. Recover every forced entry and diagnose any conflict, enforcing the paired-input law and the odd-function origin condition.

Complete response required: forced values or exact conflict.

x	**f(x)**
3	10
−3	11
0	1

Your response: ______________

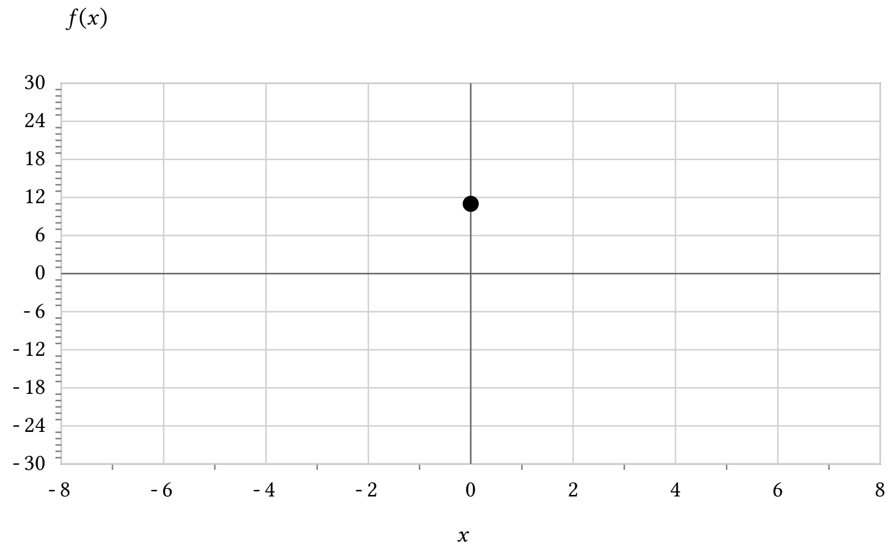
AP Precalculus

Function Symmetry, Finite Differences, and End Behavior

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A polynomial is declared odd. Its exact partial graph gives $f(0) = 11$. Recover every forced entry and diagnose any conflict, enforcing the paired-input law and the odd-function origin condition. A student reads only one visible feature of the graph and states the conclusion without checking scale, domain, endpoints, or marked exclusions. Diagnose the incomplete reasoning and give the corrected justification.



0	11

Viewing window	**Minimum**	**Maximum**
x	-8	8
y	-30	30

x	**f(x)**
0	11

Function Symmetry, Finite Differences, and End Behavior

Full Worked Solutions - excerpt

Worked example

Problem. Construct an even polynomial with $f(1) = 3$ and $f(-1) = 4$. If impossible, identify the exact paired-value conflict.

Answer. Impossible: even symmetry requires $f(-1) = f(1) = 3$, not 4.

Explanation and check.

Apply the symmetry identity directly to the prescribed values; the contradiction prevents any polynomial from satisfying all conditions.

Worked example

Problem. Construct an odd polynomial with $f(0) = 1$. If impossible, prove the contradiction from the symmetry identity.

Answer. Impossible: odd symmetry forces $f(0) = 0$, contradicting $f(0) = 1$.

Explanation and check.

Apply the symmetry identity directly to the prescribed values; the contradiction prevents any polynomial from satisfying all conditions.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Rational Function Domains and Sign Charts

Rational Function Domains and Sign Charts | APPC-U01-P10

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for compatible graph with decisive feature, verified formula or impossibility proof, exact real domain, repaired solution with domain/sign reason, qualified exact zero conclusion and complete real parameter set.

8 PDFs and a README; 59 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Rational Function Domains and Sign Charts

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Determine the real domain of the original rule $f(x) = \frac{x+40}{(x-6)^2(x-10)(x^2+23)}$. Factor the denominator as needed and preserve exclusions even if a factor cancels.

Bank label: ______

Find all real zeros and x-intercepts of $f(x) = \frac{(x+3)(x+1)(x-2)}{(x+1)(x-5)(x^2+15)}$. List numerator roots first, then enforce the original denominator restrictions.

Bank label: ______

The table rows (x, reported $f(x)$) are $(-9, 0.03), (-8, 0.00), (-7, -0.02)$. Additional exact information: $f(-8) = \frac{1}{500}$ exactly. Decide what zero claim is warranted, using the stated precision and definition status.

Required output: qualified exact zero conclusion

x	**reported** $f(x)$
-9	0.03
-8	0.00
-7	-0.02

Bank label: ______

Find all real zeros and x-intercepts of $f(x) = \frac{(x+9)(x+7)(x+4)}{(x+7)(x+1)(x^2+9)}$. List numerator roots first, then enforce the original denominator restrictions.

Bank label: ______

AP Precalculus

Rational Function Domains and Sign Charts

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Determine the real domain of the original rule $f(x) = \frac{(x+2)x}{(x+2)(x-4)}$. Factor the denominator as needed and preserve exclusions even if a factor cancels.	Construct one real rational function with zeros $\{-6, -3\}$, exclusions $\{0, 2\}$, and common factor $(x-2)$, or prove the conditions incompatible. No hole height is prescribed.
A student argues: "For $\frac{(x+3)x}{x} = 0$, cancel to $(x+3) = 0$ and include both $x = -3$ and $x = 0$ as zeros." Diagnose the exact error and repair the solution while preserving original-domain and sign conditions.	Determine the real domain of the original rule $f(x) = \frac{(x-14)}{(x-11)(x^2+27)}$. Factor the denominator as needed and preserve exclusions even if a factor cancels.
Solve $f(x) = \frac{0}{(x-11)(x-13)} \le 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly.	Construct one real rational function with zero set containing 5 and excluded set also containing 5, or prove the conditions incompatible. No hole height is prescribed.

AP Precalculus

Rational Function Domains and Sign Charts

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Solve $f(x) = \frac{(x+8)}{(x+6)} > 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly.

Your response: ______________________

Incoming: Solution: $(-\infty, -8) \cup (-6, \infty)$.

Find all real zeros and x-intercepts of $f(x) = \frac{(x+13)(x+11)}{(x+13)(x+11)(x^2+5)}$. List numerator roots first, then enforce the original denominator restrictions.

Your response: ______________________

Incoming: Real zeros: none; there are no x-intercepts.

Solve $f(x) = \frac{0}{(x+9)(x+7)} \leq 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly.

Your response: ______________________

Incoming: Solution: $(-\infty, -9) \cup (-9, -7) \cup (-7, \infty)$.

Determine all real k such that $x = -5$ is a real zero of $f_k(x) = \frac{x-k}{x}$. Enforce numerator zero and original denominator nonzero simultaneously.

Your response: ______________________

Incoming: k = −5.

Solve $f(x) = \frac{(x+6)(x+4)}{(x+4)(x+1)} < 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly.

Your response: ______________________

Incoming: Solution: $(-6, -4) \cup (-4, -1)$.

Solve $f(x) = \frac{(x+10)(x+8)}{(x+8)(x+5)} < 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly.

Your response: ______________________

AP Precalculus

Rational Function Domains and Sign Charts

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student argues: "For $\frac{x(x-3)}{x-3} = 0$, cancel to $x = 0$ and include both $x = 0$ and $x = 3$ as zeros." Diagnose the exact error and repair the solution while preserving original-domain and sign conditions. Identify the first invalid inference, give the corrected solution set, and justify it using the original equation.

A student argues: "For $\frac{(x-3)(x-6)}{x-6} = 0$, cancel to $(x-3) = 0$ and include both $x = 3$ and $x = 6$ as zeros." Diagnose the exact error and repair the solution while preserving original-domain and sign conditions. Identify the first invalid inference, give the corrected solution set, and justify it using the original equation.

A student argues: "For $\frac{x-1}{x-4} = 1$, cross-multiply and include $x = 4$ because both sides become 0." Diagnose the exact error and repair the solution while preserving original-domain and sign conditions. Identify the first invalid inference, give the corrected solution set, and justify it using the original equation.

Rational Function Domains and Sign Charts

Full Worked Solutions - excerpt

Worked example

Problem. Construct one real rational function with zero set containing 5 and excluded set also containing 5, or prove the conditions incompatible. No hole height is prescribed.

Answer. No such rational function exists.

Explanation and check.

Impossible: a rational zero must be in the domain, while $x = 5$ is required to be outside the domain.

Worked example

Problem. Determine all real k such that $x = 2$ is a real zero of $f_k(x) = \frac{x-k}{x-k}$. Enforce numerator zero and original denominator nonzero simultaneously.

Answer. No real value of k works.

Explanation and check.

The numerator condition at $x = 2$ forces $k = 2$, but then the denominator is also zero at $x = 2$. The only candidate is forbidden.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Vertical Asymptotes and One-Sided Behavior | APPC-U01-P11

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for two signed one-sided limits, complete generic and exceptional classification, verified formula or impossibility proof, evidence-based pole decision, qualified local limit conclusion and exact pole set with residual orders.

8 PDFs and a README; 120 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Determine every vertical asymptote of $f(x) = \frac{(x-2)^2(x-6)}{(x-2)^2(x-6)^2(x^2+19)}$. Collect common factors and compare multiplicities; do not call every original denominator zero a pole.

Bank label: ______

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Determine every vertical asymptote of $f(x) = \frac{(x-11)^2(x-15)}{(x-11)^2(x-15)^2(x^2+28)}$. Collect common factors and compare multiplicities; do not call every original denominator zero a pole.

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $f(x) = \frac{3}{(x-1)(x^2+18)}$, analyze the two one-sided limits at $x = 1$. Use the residual denominator parity and the sign of the nonzero local cofactor.

Your response: ____

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

An exact display uses $f(x) = \frac{1}{x-10}$ on window $[0, 4]$. A student claims: "No spike appears in the window $[0, 4]$, so the function has no vertical asymptote." Diagnose the claim using residual denominator factors rather than visual height alone. Diagnose the no-spike claim by locating the actual pole relative to the displayed window.

Vertical Asymptotes and One-Sided Behavior

Full Worked Solutions - excerpt

Worked example

Problem. Construct a simple rational example satisfying one pole at $x = -6$ with left limit $+\infty$ and right limit $-\infty$, or prove it impossible. Verify pole order and every prescribed one-sided sign.

Answer. One example is $f(x) = -\frac{1}{x+6}$.

Explanation and check.

The denominator power 1 has the required parity and numerator sign -1; direct local sign analysis verifies both branches.

Worked example

Problem. Construct a simple rational example satisfying one pole at $x = 0$ with left limit $-\infty$ and right limit $+\infty$, or prove it impossible. Verify pole order and every prescribed one-sided sign.

Answer. One example is $f(x) = \frac{1}{x}$.

Explanation and check.

The denominator power 1 has the required parity and numerator sign 1; direct local sign analysis verifies both branches.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Holes in Rational Functions

Holes in Rational Functions | APPC-U01-P12

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for verified original ratio or impossibility proof, height membership conclusion with alternate-preimage check, corrected value-limit-hole statement, separate limit and point-value claims, exact removable point coordinates and required point value or impossibility.

8 PDFs and a README; 116 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Holes in Rational Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Locate every removable hole of $f(x) = \frac{(x+6)(x+2)(-x-1)}{(x+6)(x+2)}$. The reduced rule is not the original value at an excluded input; report full coordinates and ignore any residual pole except to keep it distinct.

Bank label: ______

The exact approach table follows the continuation $g(x) = -x + 1$ near $x = -8$. The point definition says: $f(c)$ is undefined. State the two-sided limit when justified and state $f(-8)$ separately.

Given original rule: $f(x) = \frac{(x+8)(-x+1)}{(x+8)}, x \neq -8$

x	reported f(x)
-9	10
$-\frac{17}{2}$	$\frac{19}{2}$
$-\frac{81}{10}$	$\frac{91}{10}$
$-\frac{79}{10}$	$\frac{89}{10}$
$-\frac{15}{2}$	$\frac{17}{2}$
-7	8

Bank label: ______

AP Precalculus

Holes in Rational Functions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Locate every removable hole of $f(x) = \frac{x(-x-2)}{x(x-5)}$. The reduced rule is not the original value at an excluded input; report full coordinates and ignore any residual pole except to keep it distinct.

The exact approach table follows the continuation $g(x) = 2x - 1$ near $x = 5$. The point definition says: $f(5) = 11$ is separately defined. State the two-sided limit when justified and state $f(5)$ separately.

Given original rule: $f(x) = \frac{(x-5)(2x-1)}{(x-5)}, x \neq 5$

x	**reported f(x)**
4	7
$\frac{9}{2}$	8
$\frac{49}{10}$	$\frac{44}{5}$
$\frac{51}{10}$	$\frac{46}{5}$
$\frac{11}{2}$	10
6	11

AP Precalculus

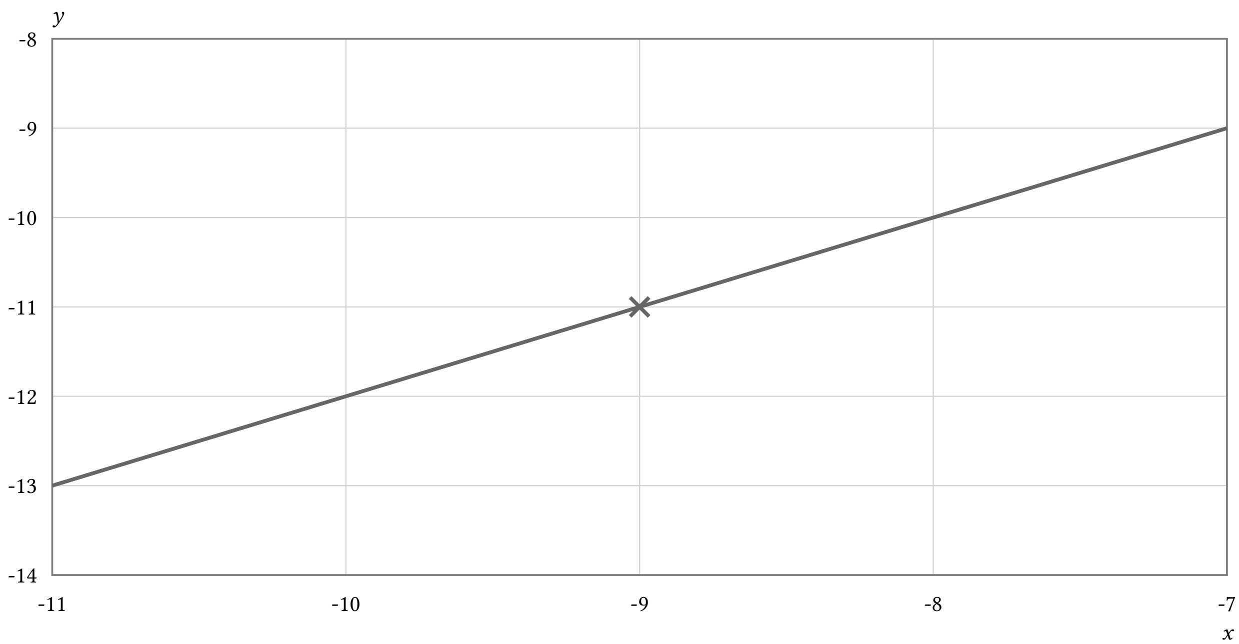
Holes in Rational Functions

Name: __________ **Date:** __________ **Class:** __________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Construct an original rational function satisfying reduced graph $y = x - 2$ with one hole at $(-9, -11)$, or prove it impossible. Verify every target height against the reduced graph.



Your response: __________

AP Precalculus

Holes in Rational Functions

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: "For $f(x) = \frac{(x-2)(x-1)}{x-2}$, $\frac{0}{0}$ means $f(2) = 0$." Diagnose the implication and distinguish nearby equality, the limit, and the original point value. Identify the invalid inference. Give the nearby simplified rule, the two-sided limit when justified, and the original point value separately.

A student claims: "For $f(x) = \frac{(x+1)(x-1)}{x+1}$, $\frac{0}{0}$ means $f(-1) = 0$." Diagnose the implication and distinguish nearby equality, the limit, and the original point value. Identify the invalid inference. Give the nearby simplified rule, the two-sided limit when justified, and the original point value separately.

Holes in Rational Functions

Full Worked Solutions - excerpt

Worked example

Problem. Determine the point value, if any, that makes this function continuous at $x = 9$: $f(x) = \frac{1}{x-9}$ for $x \neq 9$, with a proposed finite $f(9)$. Verify a finite two-sided limit before assigning the point.

Answer. No finite point value can make the function continuous.

Explanation and check.

The one-sided outputs diverge with opposite signs at $x = 9$, so no finite two-sided limit exists to use as a fill value.

Worked example

Problem. Determine the point value, if any, that makes this function continuous at $x = 0$: $f(x) = \frac{1}{x}$ for $x \neq 0$, with a proposed finite $f(0)$. Verify a finite two-sided limit before assigning the point.

Answer. No finite point value can make the function continuous.

Explanation and check.

The one-sided outputs diverge with opposite signs at $x = 0$, so no finite two-sided limit exists to use as a fill value.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Equivalent Polynomial Forms

Equivalent Polynomial Forms | APPC-U01-P13

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for form choice and exact revealed feature, consistent candidate with decisive evidence, first error, corrected identity, and lost graph feature, complete exact polynomial and requested coefficient, factored form plus newly visible feature and feature meaning with units and domain check.

8 PDFs and a README; 78 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Equivalent Polynomial Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An answer may be used more than once.

Factor $P(x) = 2x^2 - 16x + 32$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Rewrite $f(x) = 2x^2 + 1$ in vertex form. On the stated domain $-2 \le x \le 3$, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible. Bank label: _______
Factor $P(x) = 2x^2 + 18x$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Rewrite $f(x) = x^2 + 8x + 13$ in vertex form. On the stated domain all real x, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible. Bank label: _______
Factor $P(x) = 3x^2 - 42x + 147$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Rewrite $f(x) = 3x^2 + 12x + 11$ in vertex form. On the stated domain $-1 \le x \le 2$, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible. Bank label: _______
Factor $P(x) = 3x^2 - 18x$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Two equivalent descriptions of the same polynomial are vertex: $(x + 1)^2 + 2$; standard: $x^2 + 2x + 3$. Which named form most directly answers the request for vertex? State the feature it reveals and explain why equivalence alone does not make both forms equally efficient. Bank label: _______

AP Precalculus

Equivalent Polynomial Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Rewrite $f(x) = 2x^2 - 12x + 17$ in vertex form. On the stated domain $1 \le x \le 6$, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible.

Factor $P(x) = 2x^2 + 32x + 128$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots.

Use the supplied complete Pascal table to expand $(4x - 4)^5$ and explain how signs and nonunit factors enter each coefficient.

Context/domain: precalculus

j	C(n,j)	power of x	power of constant term	resulting coefficient
0	1	5	0	1024
1	5	4	1	−5120
2	10	3	2	10240
3	10	2	3	−10240
4	5	1	4	5120
5	1	0	5	−1024

Determine all real parameter values that make the two forms $a(x - 3)^2 + k$ and $x^2 - 6x + 8$ identical for every x. Match only decisive coefficients first, then verify every coefficient.

AP Precalculus

Equivalent Polynomial Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Two equivalent descriptions of the same polynomial are factored: $2(x-2)(x-5)$; standard: $2x^2 - 14x + 20$. Which named form most directly answers the request for initial output $f(0)$? State the feature it reveals and explain why equivalence alone does not make both forms equally efficient.

Your response: ______________

Incoming: The roots t = 0 and $t = 9$ mean the object is at height 0 feet at launch and landing; both are feasible endpoints.

Use the supplied complete Pascal table to expand $(x+1)^2$ and explain how signs and nonunit factors enter each coefficient.

Context/domain: precalculus

j	C(n,j)	power of x	power of constant term	resulting coefficient
0	1	2	0	1
1	2	1	1	2
2	1	0	2	1

Your response: ______________

Incoming: Use the factored form; it directly reveals $x = -1$ and $x = 2$.

Factor $P(x) = x^2$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots.

Your response: ______________

Incoming: $(x-1)^3 = x^3 - 3x^2 + 3x - 1$.

Two equivalent descriptions of the same polynomial are vertex: $3(x+2)^2 - 1$; standard: $3x^2 + 12x + 11$. Which named form most directly answers the request for quadratic extremum and where it occurs? State the feature it reveals and explain why equivalence alone does not make both forms equally efficient.

Your response: ______________

AP Precalculus

Equivalent Polynomial Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims $3(x-2)^2-1$ is identically equal to x^2-4x+3. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the missing factor of 3 in expansion; compare the values at $x=0$. Give the corrected identity and explain why the claimed expressions are not equivalent.

A student claims $3x^2+2$ is identically equal to x^2+2. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the omitted coefficient 3; compare the values at $x=1$. Give the corrected identity and explain why the claimed expressions are not equivalent.

A student claims $(x+2)^2$ is identically equal to x^2+4. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the omitted cross term; compare both vertices and the values at $x=1$. Give the corrected identity and explain why the claimed expressions are not equivalent.

A student claims $(x+5)^2$ is identically equal to x^2+25. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the omitted cross term; compare the values at $x=1$. Give the corrected identity and explain why the claimed expressions are not equivalent.

Equivalent Polynomial Forms

Full Worked Solutions - excerpt

Worked example

Problem. Factor $P(x) = x^2$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots.

Answer. $P(x) = xx = x^2$; the repeated factor shows a double zero at $x = 0$.

Explanation and check.

The factorization $xx = x^2$ is an identity. The repeated factor shows a double zero at $x = 0$.

Worked example

Problem. Determine all real parameter values that make the two forms $a(x+3)^2 + k$ and $x^2 + 6x + 7$ identical for every x. Match only decisive coefficients first, then verify every coefficient.

Answer. a = $1, k = -2$.

Explanation and check.

Coefficient matching yields the unique pair $a = 1, k = -2$, and substitution reproduces $x^2 + 6x + 7$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Rational Expression Operations and Restrictions

Rational Expression Operations and Restrictions | APPC-U01-P14

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for recommended representation with preserved domain, function equality and exact agreement domain, verified original ratio and exact exclusion set, correct rewrite, exclusion, and test-input evidence, simplified rule plus every original exclusion and single ratio, reduced rule, and sourced exclusions.

8 PDFs and a README; 55 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Rational Expression Operations and Restrictions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Rewrite $R(x) = \frac{(x+2)(x-6)(2x+3)}{(x+2)(x-6)(x-2)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: ______	Choose the more useful of the two supplied forms to determine the horizontal asymptote and the nonzero remainder in a quotient-remainder form. A (single ratio): $\frac{-3(x+2)+1}{x+2}$; B (supplied quotient-remainder): $-3 + \frac{1}{x+2}$. State the feature, explain why that form exposes it, and retain every original excluded input. Bank label: ______
Rewrite $R(x) = 6(x+1)\frac{x-5}{(x+1)(x-5)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: ______	Determine all real k for which $\frac{x+k}{x-7}$ and $\frac{x+2}{x-7}$ define identical functions. First compare the algebraic identity, then check zero denominators and whether the domains coincide. Bank label: ______
Rewrite $R(x) = 5(x+7)\frac{x+1}{(x+7)(x+1)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: ______	Determine all real k for which $\frac{(x+2)(x+k)}{(x+2)(x-3)}$ and $\frac{x+k}{x-3}$ define identical functions. First compare the algebraic identity, then check zero denominators and whether the domains coincide. Bank label: ______
Rewrite $R(x) = 4(x-2)\frac{x-8}{(x-2)(x-8)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: ______	Choose the more useful of the two supplied forms to determine the horizontal asymptote and the nonzero remainder in a quotient-remainder form. A (single ratio): $\frac{3(x+5)+3}{x+5}$; B (supplied quotient-remainder): $3 + \frac{3}{x+5}$. State the feature, explain why that form exposes it, and retain every original excluded input. Bank label: ______

AP Precalculus

Rational Expression Operations and Restrictions

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Construct an original rational ratio that reduces to $x-1$ but has requested original exclusions $\{x=-1\}$. The reduced rule already excludes none; do not add redundant common factors or unintended restrictions.	Rewrite $(\frac{x+7}{x+4}) \div (\frac{x-3}{x})$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.
Rewrite $R(x) = \frac{(x+8)x(2x-3)}{(x+8)x(x+2)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator.	Given $f(x) = \frac{(x+6)(2x-3)}{(x+6)(x+1)}$ with excluded inputs $-6, -1$, and $g(x) = \frac{2x-3}{x+1}$ with $x \neq -6, -1$ with excluded inputs $-6, -1$. Decide whether they are identical functions and state exactly where their values agree. A matching simplified formula is not sufficient without matching domains.
Construct an original rational ratio that reduces to $x-2$ but has requested original exclusions $\{x=-7\}$. The reduced rule already excludes none; do not add redundant common factors or unintended restrictions.	Rewrite $(\frac{x+5}{x+2})(\frac{x+2}{x-2})$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

AP Precalculus

Rational Expression Operations and Restrictions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Rewrite $\left(\frac{x-2}{x-5}\right) \div \left(\frac{x-12}{x-9}\right)$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Your response: ______________

Incoming: The single ratio is $\frac{(x-2)(x-9)}{(x-5)(x-12)}$, reducing to $\frac{(x-2)(x-9)}{(x-5)(x-12)}$, with all real x except $x = 5, x = 9, x = 12$.

Determine all real k for which $\frac{x+k}{x+2}$ and $\frac{x}{x+2}$ define identical functions. First compare the algebraic identity, then check zero denominators and whether the domains coincide.

Your response: ______________

Incoming: k = 0.

Rewrite $\left(\frac{x+10}{x+7}\right) \div \left(\frac{x}{x+3}\right)$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Your response: ______________

Incoming: The single ratio is $\frac{(x+10)(x+3)}{(x+7)x}$, reducing to $\frac{(x+10)(x+3)}{(x+7)x}$, with all real x except $x = -7, x = -3, x = 0$.

Construct an original rational ratio that reduces to $\frac{3x-2}{x+6}$ but has requested original exclusions $\{x = -6, x = -2\}$. The reduced rule already excludes $\{x = -6\}$; do not add redundant common factors or unintended restrictions.

Your response: ______________

Incoming: One valid predecessor is $\frac{(x+2)(3x-2)}{(x+2)(x+6)}$, with all real x except $x = -6, x = -2$.

Rewrite $\left(\frac{x+3}{x}\right) + \left(\frac{4}{x}\right)$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Your response: ______________

Incoming: The single ratio is $\frac{x+7}{x}$, reducing to $\frac{x+7}{x}$, with all real x except $x = 0$.

Choose the more useful of the two supplied forms to determine the horizontal asymptote and the nonzero remainder in a quotient-remainder form. A (single ratio): $\frac{-2(x-6)+4}{x-6}$; B (supplied quotient-remainder): $-2 + \frac{4}{x-6}$. State the feature, explain why that form exposes it, and retain every original excluded input.

Your response: ______________

AP Precalculus

Rational Expression Operations and Restrictions

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student rewrites $(x-3)\,\frac{x+2}{x-3}$ as $x+2$ for every real x. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = 3$.

A student rewrites $\frac{x-1}{x-4}$ as $-\frac{1}{-4}$. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = 1$.

A student rewrites $(x+3)\,\frac{x+2}{x+3}$ as $x+2$ for every real x. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = -3$.

A student rewrites $\frac{4-x}{x-4}$ as 1. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = 5$.

Rational Expression Operations and Restrictions

Full Worked Solutions - excerpt

Worked example

Problem. Rewrite $\left(\frac{x+3}{x}\right)+\left(\frac{4}{x}\right)$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Answer. The single ratio is $\frac{x+7}{x}$, reducing to $\frac{x+7}{x}$, with all real x except $x = 0$.

Explanation and check.

Combining the operation gives $\frac{x+7}{x}$. The common denominator supplies the sole restriction. Therefore the complete exclusion set is 0.

Worked example

Problem. Rewrite $\left(\frac{x-9}{x-12}\right)+\left(\frac{6}{x-12}\right)$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Answer. The single ratio is $\frac{x-3}{x-12}$, reducing to $\frac{x-3}{x-12}$, with all real x except $x = 12$.

Explanation and check.

Combining the operation gives $\frac{x-3}{x-12}$. The common denominator supplies the sole restriction. Therefore the complete exclusion set is 12.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polynomial Division and Slant Asymptotes

Polynomial Division and Slant Asymptotes | APPC-U01-P15

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for quotient line, remainder correction, and exclusion, quotient, remainder, and reconstructed identity, quotient and degree-valid remainder, validity verdict with both requirements, first invalid step and corrected continuation and missing polynomial or inconsistency proof.

8 PDFs and a README; 66 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Divide $12x^4 + 48x^3 + 3x^2 + 9x - 14$ by $3x + 12$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Bank label: ______

Determine the parameter value(s) so division of $k + x^2 + x$ by $x + 8$ has required remainder 0. Use the supplied exact constraint table and verify $p = dq + r$.

Complete response required: complete parameters with verified remainder.

input	**p_k(input)**	**required remainder**
-8	$56 + k$	0

Bank label: ______

Divide $2x^3 - 4x^2 + 2$ by $2x - 2$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Bank label: ______

In the identity $p(x) = d(x)q(x) + r(x), d(x) = x^2 + 3x + 8, q(x) = x - 1, r(x) = 1 - x$ are given. Determine the dividend $p(x)$. Then verify the identity and the strict condition $\deg r < \deg d$.

Bank label: ______

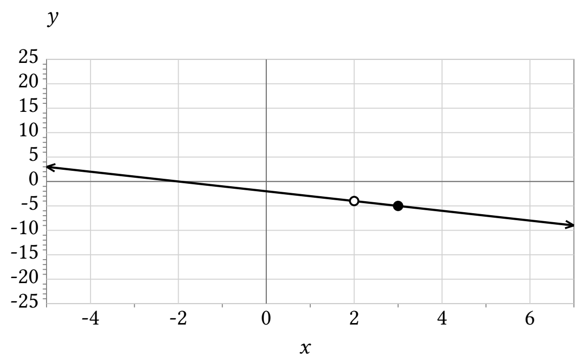
AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ______________ **Date:** ______________ **Class:** ______________

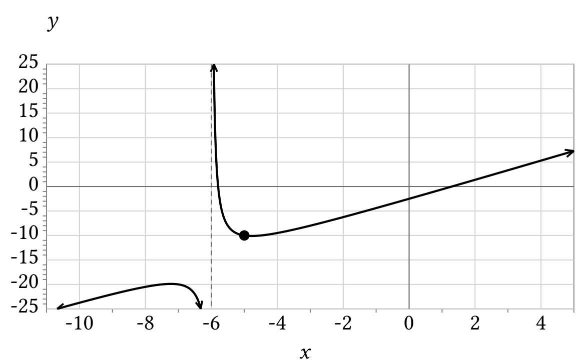
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Divide the numerator in $\frac{4-x^2}{x-2}$ by its denominator. Write the quotient-remainder form, derive the slant line, and explain why the correction tends to 0 while preserving $x = 2$ as excluded.



Divide $9x^4 + 9x^3 - 9x^2 - 3x + 7$ by $3x + 3$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Divide the numerator in $\frac{2x^2+9x-15}{x+6}$ by its denominator. Write the quotient-remainder form, derive the slant line, and explain why the correction tends to 0 while preserving $x = -6$ as excluded.



A student division of $3x^3 + 3x^2 + 6x$ by $3x$ first fails because the divisor leading coefficient was ignored when choosing the next quotient term. Explain the repair, finish the division, and verify the result.

AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $q(x) = 2x - 1, r(x) = -3.$ Divide $x^3 - x^2 + 16x - 32$ by $x^2 + x + 17$. Continue only while the current remainder degree is at least 2, then justify the stopping point. Your response: ______________	**Incoming:** $q(x) = x - 1, r(x) = x.$ In the identity $p(x) = d(x)q(x) + r(x)$, $p(x) = 2x^3 - 5x^2 + 7x - 3, q(x) = 2x - 1, r(x) = -x$ are given. Determine the divisor $d(x)$. Then verify the identity and the strict condition $\deg r < \deg d$. Your response: ______________
Incoming: $3x - 2 + \frac{4}{x+5}$; slant asymptote $y = 3x - 2$. Divide $x^3 - 3x^2 + 2x + 22$ by $x^2 - 5x + 11$. Continue only while the current remainder degree is at least 2, then justify the stopping point. Your response: ______________	**Incoming:** $q(x) = 2x + 1, r(x) = -3.$ Divide $4x^3 - 16x^2 - 8x + 12$ by $2x^2 - 10x + 6$. Continue only while the current remainder degree is at least 2, then justify the stopping point. Your response: ______________

AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student is dividing $2x^3 - x^2 + 2x - 2$ by $2x - 1$. Diagnose this specific student statement: The quotient coefficient list $[1, 1]$ represents $x^2 + 1$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

A student is dividing $x^3 - x - 1$ by $x - 1$. Diagnose this specific student statement: After subtracting $x^3 \text{-} x^2$ from $x^3 + 0x^2 \text{-} x - 1$, the remainder is $\text{-} x^2 \text{-} x - 1$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

A student is dividing $x^3 - 3x^2 + 2x - 2$ by $x - 2$. Diagnose this specific student statement: Subtracting $x^3 - 2x^2$ from the dividend gives $-5x^2 + 2x - 2$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

A student is dividing $2x^3 + 4$ by $2x + 2$. Diagnose this specific student statement: The descending coefficient list of $2x^3 + 4$ is $[2, 4]$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

Polynomial Division and Slant Asymptotes

Full Worked Solutions - excerpt

Worked example

Problem. Divide $x^2 - 7x - 18$ by $x - 9$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Answer. $q(x) = x + 2, r(x) = 0$.

Explanation and check.

Successive leading-term cancellation gives $q = x + 2$ and $r = 0$. Re-expansion verifies $x^2 - 7x - 18 = (x - 9)(x + 2)$; the zero remainder makes the division exact.

Worked example

Problem. Divide $x^3 - x^2 + 16x - 32$ by $x^2 + x + 17$. Continue only while the current remainder degree is at least 2, then justify the stopping point.

Answer. $q(x) = x - 2, r(x) = x + 2$.

Explanation and check.

Multiplication gives $(x^2 + x + 17)(x - 2)$ and subtraction leaves $x + 2$. Its degree 1 is below 2, so the division is complete.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Function Translations

Function Translations | APPC-U01-P16

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for translation candidate and failure evidence, identify and correct mismatched row, corrected conclusions, translated rows plus rule, new formula and zero-time value and translated contextual model with units.

8 PDFs and a README; 79 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Function Translations

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

From the table, evaluate $g(-7)$ and $g(4)$ for $g(x) = f(x+5) + 2$.

Complete response required: ordered pair of exact values.

u	$f(u)$
-8	-4
-2	6
3	0
9	11

Bank label: ______

If $g(x) = f(x-2) - 4$, use the table to find $g(4)$.

Complete response required: exact value.

x	$f(x)$
-3	12
2	-5
7	1

Bank label: ______

Write g in terms of f when the graph of f is translated 4 units left and 2 units down. Explain the sign used inside f.

Bank label: ______

To determine $g(-3)$ for $g(x) = f(x+8) - 2$, which source-table row is needed? If that row is $(5, 14)$, compute $g(-3)$.

Bank label: ______

AP Precalculus

Function Translations

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A graph consists of closed line segments joining $(-4, 1)$ to $(-1, 5)$ to $(2, -2)$. Translate the graph 2 units left and 3 units up. Give the image of every listed vertex and retain the connections.

-4	1
-1	5
2	-2

AP Precalculus

Function Translations

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Rows $(-13, 3), (-7, -3), (0, -11)$, and $(6, 2)$.

First set $p(x) = f(x) - 9$, then set $q(x) = p(x + 4)$. Collapse the two steps into one formula involving f and describe the two-coordinate displacement.

Your response: ____________

Incoming: Rows $(-11, 2), (-6, 8), (2, -1)$; $g(x) = f(x + 7) + 2$.

Using only the displayed table for f, evaluate every determined value among $g(-4)$, $g(1)$, $g(5)$, and $g(0)$ when $g(x) = f(x + 1) - 3$. Identify any value that cannot be found.

Complete response required: determined values and unsupported query.

u	$f(u)$
-7	2
-3	9
2	-1
6	4

Your response: ____________

Incoming: New y-intercept $(0, -4)$; old y-intercept maps to $(-7, 7)$.

The same translation, left 3 and up 4, is applied three times. Replace the repeated action by one vector and one rule g in terms of f.

Your response: ____________

Incoming: h = 3, $k = -6$, so $g(x) = f(x - 3) - 6$.

Move the graph of f right 2 and up 10. Write g in terms of f and map the source point $(8, -3)$.

Your response: ____________

AP Precalculus

Function Translations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The table gives exact values of f. If $g(x) = f(x+2) - 1$, use the displayed data to find $g(0)$. A student claims $g(0) = f(0) - 1 = -3$. Diagnose the horizontal-input error using the exact table, and give the complete corrected result.

Complete response required: exact value.

x	$f(x)$
-3	7
0	-2
2	5
5	1

Only the displayed values of f are known. For $g(x) = f(x-3) - 1$, find every x supported by the table for which $g(x) = 5$. Do not assume f is one-to-one or extend the table. A student claims the supported answers are the source inputs -4 and 2 without any horizontal change. Diagnose the horizontal-input error using the exact table, and give the complete corrected result.

Complete response required: all displayed-data solutions.

u	$f(u)$
-4	6
-1	2
2	6
3	-3
7	2

Function Translations

Full Worked Solutions - excerpt

Worked example

Problem. Move the graph of f right 2 and up 10. Write g in terms of f and map the source point $(8, -3)$.

Answer. $g(x) = f(x - 2) + 10$; $(8, -3)$ maps to $(10, 7)$.

Explanation and check.

A right shift uses $x - 2$ inside, while an upward shift adds 10 outside. Adding $(2, 10)$ to the point gives $(10, 7)$.

Worked example

Problem. The complete solution of $f(u) > 4$ is $(-7, -1)$ union $(3, 8]$. If $g(x) = f(x + 2) - 9$, solve $g(x) > -5$.

Answer. $(-9, -3)$ union $(1, 6]$.

Explanation and check.

The target inequality becomes $f(x + 2) > 4$. Thus $x + 2$ must lie in the given source set; subtract 2 from every endpoint and preserve all inequality memberships.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Function Reflections, Stretches, and Compressions

Function Reflections, Stretches, and Compressions | APPC-U01-P17

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for supported conclusion and limitation, a and possible b, a and all b, forced and unidentifiable factors, description and a,b and explanation.

8 PDFs and a README; 71 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

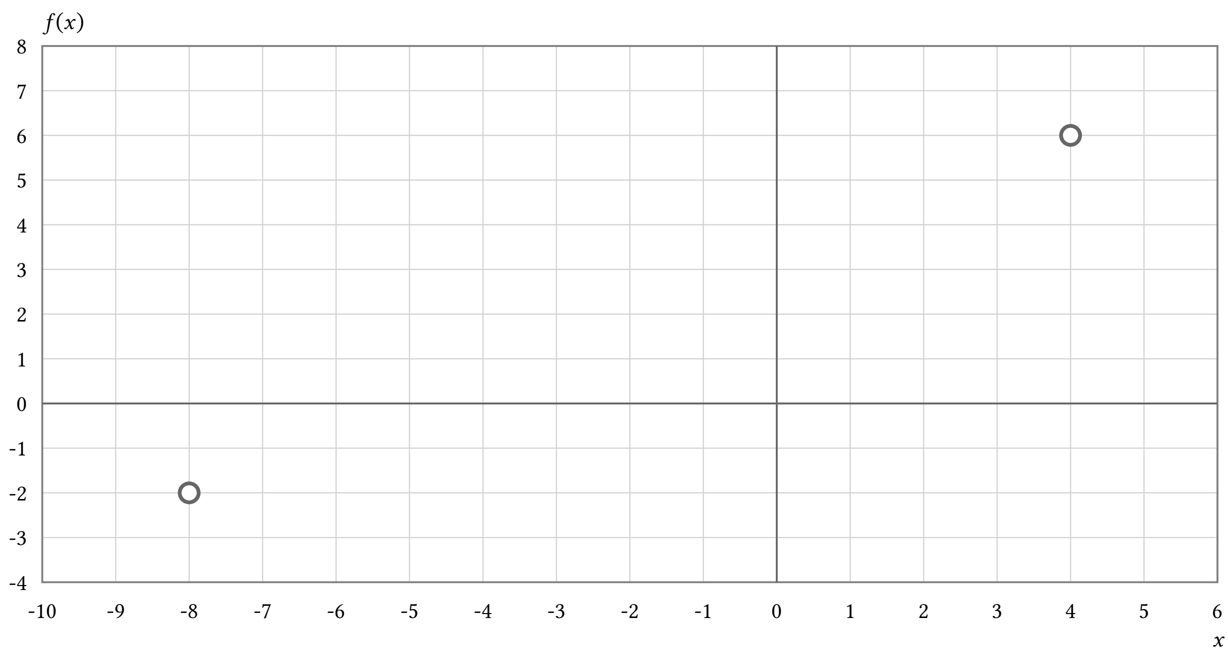
AP Precalculus

Function Reflections, Stretches, and Compressions

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A curve runs between open endpoints $(-8, -2)$ and $(4, 6)$. Where are its open endpoints under $g(x) = -f(\frac{x}{2})$?



Bank label: ______

AP Precalculus

Function Reflections, Stretches, and Compressions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

The table lists exact values of f. If $g(x) = 2f(-\frac{x}{2})$, find $g(6)$.

Required output: exact value

x	f(x)
-6	4
-3	-5
1	7
4	0

The domain of f is $[-6, 3)$ and its range is $(-2, 8]$. Determine the domain and range of $g(x) = -4f(-2x)$.

AP Precalculus

Function Reflections, Stretches, and Compressions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A table row for f is $(6, -4)$. Find its corresponding row for $g(x) = -5f(-2x)$.

Given f-table row

x	f(x)
6	-4

Your response: ______________

Incoming: $[-3, 20]$

If f has domain $[2, 10]$ and range $[-4, 6]$, find the domain and range of $g(x) = 3f(2x)$.

Your response: ______________

AP Precalculus

Function Reflections, Stretches, and Compressions

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $g(x) = f(-3x)$, choose: horizontal stretch by 3, or horizontal compression by $\frac{1}{3}$ with y-axis reflection. Justify by point mapping. Diagnose this student claim: Multiplying the input by -3 stretches the graph horizontally by 3. Use the actual rule and supplied data to give the corrected conclusion and justify the input/output mapping.

Besides naming two axis reflections, what single rigid motion describes $g(x) = -f(-x)$? Diagnose this student claim: The two axis reflections cancel, leaving every point fixed. Use the actual rule and supplied data to give the corrected conclusion and justify the input/output mapping.

Can one fixed pair a, b map $(1,2), (-2,5), (4,-1)$ to $(3,8), (-6,15), (12,-4)$, respectively, under $g(x) = af(bx)$? Diagnose this student claim: The matching horizontal ratios alone establish one common pair a, b. Use the actual rule and supplied data to give the corrected conclusion and justify the input/output mapping.

Function Reflections, Stretches, and Compressions

Full Worked Solutions - excerpt

Worked example

Problem. Function f has domain $(-2, 6)$ and constant range $\{4\}$. Find both sets for $g(x) = -5f(2x)$.

Answer. Domain $(-1, 3)$; range $\{-20\}$.

Explanation and check.

Divide source inputs by 2 and multiply the sole output by -5.

Worked example

Problem. If f has domain $[2, 10]$ and range $[-4, 6]$, find the domain and range of $g(x) = 3f(2x)$.

Answer. Domain $[1, 5]$; range $[-12, 18]$.

Explanation and check.

Require $2x$ in $[2, 10]$, giving x in $[1, 5]$. Multiplying outputs by 3 gives $[-12, 18]$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Combined Function Transformations

Combined Function Transformations | APPC-U01-P18

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for determinate or not, g from f, two maps, coordinate rules, equivalence and equivalence proof.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Combined Function Transformations

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Normalize $g(x) = 3f(6x + 15) - 2$ and state h. Bank label: ____	Normalize the inside of $g(x) = (\frac{3}{2})f(-0.4x - 2) + 1$ and state b, h. Bank label: ____
Double all x-coordinates of f, then shift the current graph left 3. Write $g(x)$. Bank label: ____	Function f has domain $[-3, 9)$ and range $(0, 8]$. Find both sets for $g(x) = (\frac{1}{2})f(\frac{x-7}{4}) - 3$. Bank label: ____

AP Precalculus

Combined Function Transformations

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Starting from f, shift the graph right 3 units and then double every x-coordinate. Write the final rule $g(x)$.	Function f has vertex (0,-2) and domain [-4,4]. Construct g(x)=a f(b(x-h))+k with a>0,b>0 so the vertex becomes (3,5) and domain becomes [1,5].
Let $f(u) = u + 1$ for $u < 0$ and $f(u) = u^2$ for $u \geq 0$. Write $g(x) = 2f(3(x-4)) - 5$ as a piecewise function of x.	Normalize $g(x) = -4f(-2x-6) + 1$ and give a, b, h, k and the coordinate map.

AP Precalculus

Combined Function Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(3, 0)$

Under (u, v) to $(h + \frac{u}{b}, k + av)$ with $a, b > 0$, source points $(0, 1), (4, 3)$ map to $(2, -1), (4, 5)$. Find a, b, h, k.

Complete response required: a, b, h, k.

2	−1
4	5

0	1
4	3

Your response: ______________

Incoming: Domain $[-20, -8)$ union $\{-2\}$ union $(8, 16]$; range $[-8, 4]$ union (16,infinity).

The graph of f has vertical asymptote $x = -2$. Find the corresponding vertical asymptote of $g(x) = 3f(2(x - 5)) + 4$.

Your response: ______________

Incoming: y = 3.

Under (u, v) to $(h + \frac{u}{b}, k + av)$ with $a, b > 0, (-3, 2), (5, 6)$ map to $(1, 7), (3, 15)$. Find a, b, h, k.

Complete response required: a, b, h, k.

1	7
3	15

−3	2
5	6

Your response: ______________

Incoming: a = $2, b = -3, h = 5, k = -2$.

Under an affine graph map, $(0, 0)$ maps to $(-2, 5)$ and $(6, 4)$ maps to $(1, -3)$. Find a, b, h, k.

Complete response required: a, b, h, k.

−2	5
1	−3

0	0
6	4

Your response: ______________

Incoming: a = $-2, b = 2, h = -2, k = 5$.

Rewrite $g(x) = 2f(3(x + 1)) + 5$ if $f(u) = 4$ for $u < 2$ and $f(u) = u - 1$ for $u \geq 2$.

Your response: ______________

Incoming: $g(x) = 13$ for $x < -\frac{1}{3}$; $g(x) = 6x + 9$ for $x \geq -\frac{1}{3}$.

Under an affine graph map, $(0, 2), (10, -3)$ map to $(4, 1), (9, 11)$. Find a, b, h, k.

Complete response required: a, b, h, k.

4	1
9	11

0	2
10	−3

Your response: ______________

AP Precalculus

Combined Function Transformations

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Shift f down 2, reflect the current graph across the x-axis, then triple outputs. Write $g(x)$. A student reports $g(x) = -3f(x) - 6$. Locate the sign or order-of-operations error, write the corrected intermediate calculation or coordinate map, and give the correct complete result.

Shift f left 2, triple current x-coordinates, reflect across the y-axis, then shift right 1. Write $g(x)$. A student reports $g(x) = f(\frac{-x-5}{3})$. Locate the sign or order-of-operations error, write the corrected intermediate calculation or coordinate map, and give the correct complete result.

Double outputs of f, reflect the current graph across the x-axis, then shift up 7. Write $g(x)$. A student reports $g(x) = -2f(x) - 7$. Locate the sign or order-of-operations error, write the corrected intermediate calculation or coordinate map, and give the correct complete result.

Halve x-coordinates of f, shift the current graph right 4, then reflect across the y-axis. Write $g(x)$. A student reports $g(x) = f(-2x + 8)$. Locate the sign or order-of-operations error, write the corrected intermediate calculation or coordinate map, and give the correct complete result.

Combined Function Transformations

Full Worked Solutions - excerpt

Worked example

Problem. For nonzero q, normalize g(x)=A f(qx-r)+K as A f(q(x-h))+K.

Answer. h=r/q, so g(x)=A f(q(x-r/q))+K.

Explanation and check.

Expand q(x-r/q)=qx-r.

Worked example

Problem. Function $f(d)$ gives euros d days after Jan 1. Define $g(w)$ for cents w weeks after Jan 8, using 110 cents per euro.

Answer. $g(w) = 110f(7 + 7w)$.

Explanation and check.

Jan 8 is day 7, and each new unit adds 7 days.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Building and Selecting Function Models

Building and Selecting Function Models | APPC-U01-P19

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for conclusion, claim, yes or no, decision, compatible model and model description.

8 PDFs and a README; 56 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Building and Selecting Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

A trapezoid has parallel bases x and $x + 4$ meters and height 6 meters. Select its area model. Bank label: ______	Which polynomial candidate has both tails falling? Bank label: ______
Admission costs \$12 plus \$4 for every hour, with no rate change. Does the fee require multiple operating branches? Bank label: ______	A square has side length s centimeters. Choose the candidate that models its area in square centimeters. Bank label: ______
Which formula has the two observed zeros? Candidate formulas: $(x + 2)(x - 5)$; $(x - 2)(x + 5)$; $x^2 + 10$. Observed: x-intercepts -2 and 5. Bank label: ______	A service charges \$15 once and \$9 for every month with no discounts. Is a piecewise model needed? Bank label: ______
A banner is x meters long and 7 meters wide. Which candidate represents area? Bank label: ______	Evaluate both models at the two requested inputs. Inputs: 1; 4. Models: $A(x) = 2x + 5$; $B(x) = x^2 + 1$. Bank label: ______

AP Precalculus

Building and Selecting Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

For equally spaced inputs, the outputs are $11, 17, 23, 29$. Which supplied rule is supported, and what evidence is decisive? Candidate formulas: $3x + 11$; $x^2 + 11$; $\frac{11}{x+1}$.

Complete response required: model.

x	y
0	11
2	17
4	23
6	29

A parcel costs \$8 through $5kg$, then each kilogram beyond 5 adds \$3. Should the supplied model be one global line or a threshold piecewise rule?

The products xy are constant in the table. Select the supported candidate and state what the finite table does not prove. Candidate formulas: $\frac{24}{x}$; $24 - 3x$; $3x^2$.

Complete response required: model.

x	y
1	24
2	12
4	6
8	3

Inputs are not equally spaced. Which candidate exactly matches all four values, and why would raw first differences be misleading? Candidate formulas: $2 + 3x$; $2 + x^2$; $2 + x + x^2$.

Complete response required: model.

x	y
0	2
1	4
3	14
6	44

AP Precalculus

Building and Selecting Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: x = 2; $A(2) = 7$ and $B(2) = 12$.

A triangle has base $3xcm$ and perpendicular height $x + 2cm$. Select the area model.

Your response: ______________

Incoming: Input B; its prediction gap 1.2 exceeds twice the 0.3 uncertainty.

A circular ring has inner radius r and outer radius $r + 2$. Select the area of material.

Your response: ______________

Incoming: $\frac{x+2}{x+3}$.

Equal input steps give first differences $-5, -3, -1, 1$. Which supplied candidate is supported? Candidate formulas: $x^2 - 2x - 1$; -$x - 1$; $\frac{4}{x+3}$.

Complete response required: model.

x	**y**
-2	7
-1	2
0	-1
1	-2
2	-1

Your response: ______________

Incoming: Piecewise rising segments separated by downward jumps at replacement times.

A1-by-1 square is removed from a rectangle measuring $x + 3$ by $x + 1$. Which expression models remaining area?

Your response: ______________

AP Precalculus

Building and Selecting Function Models

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Given the explicit linear-family assumption, do the two distinct data points determine a unique candidate? Diagnose the student claim: Even within the stated linear family, the two distinct input-output points leave many different linear models. Use the exact supplied assumptions and data to provide a counterexample or decisive calculation and state the strongest justified conclusion.

Complete response required: model.

1	5
4	14

Exhibit two distinct polynomial models matching both data points. Diagnose the student claim: Only one polynomial can pass through these two data points. Use the exact supplied assumptions and data to provide a counterexample or decisive calculation and state the strongest justified conclusion.

Complete response required: two models.

0	2
1	5

Building and Selecting Function Models

Full Worked Solutions - excerpt

Worked example

Problem. A triangle has base $3xcm$ and perpendicular height $x + 2cm$. Select the area model.

Answer. $A(x) = 1.5x(x + 2)$.

Explanation and check.

Triangle area is one half the product of base and perpendicular height.

Worked example

Problem. A circular ring has inner radius r and outer radius $r + 2$. Select the area of material.

Answer. $\pi((r + 2)^2 - r^2)$.

Explanation and check.

Subtract the inner disk area from the outer disk area; both terms have square-length units.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Model Domains and Validity

Model Domains and Validity | APPC-U01-P20

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for assumption failure, boundary behavior, claim, status, feasibility and domains.

8 PDFs and a README; 47 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Model Domains and Validity

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

State the mathematical, observed, and contextual domains. The contextual range is $x \geq 0$. Use the model $\frac{1}{x+1}$. Observations cover $[0, 2]$. Bank label: ______	Where is the mass prediction physically feasible? The setting is mass. The physical constraint is $M \geq 0$. Use the model $M(t)$ = 36-t^2. Assume $t \geq 0$. Bank label: ______
Name the three domains. The setting is seats. Allowed inputs are integers 0 through 40. The formula is defined for all real n. Observations cover 10, 20, 30. Bank label: ______	State formula, observed, and shift domains. The setting is hours. The formula is defined for all real. Observations cover $[2, 7]$. The work shift spans $[0, 8]$. Bank label: ______
Separate formula, observation, and device-supported inputs. The setting is time. The device operates with $0 \leq t \leq 12$. The formula is defined for all real. Observations cover $2 \leq t \leq 8$. Bank label: ______	When does the hourly-rate assumption stop? The setting is parking. The cap is \$14/day. The rate is \$2/hour. Bank label: ______
List formula, observed, and trip domains. The setting is distance. The formula is defined for all real time. Observations cover 1 to 3 hours. The trip lasts 0 to 5 hours. Bank label: ______	For $t \geq 0$, when is volume feasible? The setting is storage. The physical constraint is $0 \leq V \leq 70$. Use the model $V(t) = 70 - 4t$. Bank label: ______

AP Precalculus

Model Domains and Validity

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

State the mathematical, observed, and physically allowed input domains. The contextual range is time 0 through 10. Use the model $p(t) = t^2 + 1$. Observations cover $0 \le t \le 6$.	For $t \ge 0$, when does the model satisfy the population constraint? The setting is population. The physical constraint is $P \ge 0$. Use the model $P(t) = 80 - 5t$.
Classify a model prediction at input 4 relative to the data window. The observed inputs are $0, 2, 5, 8$. The requested input is 4.	Explain why the fixed-rate model should stop at 12 kg. The setting is agriculture. Use the model yield rises 4 units per kg. The trigger is soil nutrient exhausted after 12 kg.
Distinguish the formula domain from the observed and contextual count domains. The setting is inventory. The contextual range is 0 to 20 items. Observed n values are $1, 2, 3, 4$. Use the model $C(n) = 7n + 4$.	Can each policy use the rule at $x = 2.5$? The setting is tickets. Policy A requires real $0 \le x \le 8$. Policy B requires integer $0 \le x \le 8$. The requested input is 2.5. Use the rule $R(x) = 12x$.

AP Precalculus

Model Domains and Validity

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Day 55 is extrapolation beyond the observed days 20 to 50; the observations alone do not establish that the model remains valid there.

For which nonnegative x is the output a valid probability? The setting is probability. Use the model $p(x) = 0.2 + 0.1x$. Assume $x \geq 0$.

Your response: ________________

Incoming: $t > 3$.

State the three domains. The contextual range is $[0, 8]$. Use the model $\frac{2}{x-5}$. Observations cover $[0, 3]$.

Your response: ________________

Incoming: Rent rises by exactly \$25 each month from an initial \$900.

What must remain constant for this inverse relation? The setting is geometry. Use the model length=60/width.

Your response: ________________

Incoming: It is interpolation by location but unsupported by the pre-change protocol beyond dose 10.

State the three input domains. The setting is time. The contextual range is $t \geq 0$. Use the model t^3-t. Observations cover $[1, 4]$.

Your response: ________________

Incoming: All reals except 6; $[0, 4]$; $[0, 6)$ union $(6, 12]$.

Does the model ever violate nonnegativity for $t \geq 0$? The setting is inventory. Use the model $I(t) = \frac{30}{t+2}$. Assume $t \geq 0$.

Your response: ________________

Incoming: All real inputs; $\{2, 4, 6\}$; $\{0, 2, 4, 6, 8, 10\}$.

Separate formula, observed, and device domains. The setting is devices. Allowed inputs are integers 0 to 6. The formula is defined for real x. Observations cover $1, 2, 4$.

Your response: ________________

AP Precalculus

Model Domains and Validity

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Replace the causal claim with one supported by the model fit. Diagnose this claim: A fitted line proves that study hours cause scores. Identify the unsupported inference or missing condition, give a defensible replacement, and state the evidence or model change needed. The setting is education. A student claims: study hours cause score because a line fits.

Give a defensible replacement and a useful test. Diagnose this claim: Four observed years prove that the trend continues forever. Identify the unsupported inference or missing condition, give a defensible replacement, and state the evidence or model change needed. The setting is climate. A student claims: trend continues forever. The supplied data are: four years.

Rewrite the conclusion and identify needed evidence. Diagnose this claim: Data from weeks 1 through 6 prove that week 20 will be exactly 74.3. Identify the unsupported inference or missing condition, give a defensible replacement, and state the evidence or model change needed. The setting is health. A student claims: week 20 will be exactly 74.3. Observations cover weeks 1 to 6.

Correct the reporting claim. Diagnose this claim: Measurements rounded to the nearest 10 units support the exact prediction 247.381. Identify the unsupported inference or missing condition, give a defensible replacement, and state the evidence or model change needed. The setting is production. A student claims: prediction exactly 247.381. Measurements are rounded to the nearest 10 units.

Model Domains and Validity

Full Worked Solutions - excerpt

Worked example

Problem. State the conserved quantity and a test of the model. The setting is chemistry. Use the model $c(V) = \frac{24}{V}$.

Answer. Solute amount stays 24 units; check whether measured products Vc remain near 24 as volume changes.

Explanation and check.

Dilution creates inverse concentration only when solute is neither added nor lost.

Worked example

Problem. State the model's change assumption. The setting is rent. Use the model $R(m) = 900 + 25m$.

Answer. Rent rises by exactly $\$25$ each month from an initial $\$900$.

Explanation and check.

The slope is presumed constant over modeled months.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polynomial Modeling from Data

Polynomial Modeling from Data | APPC-U01-P21

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for regression model, uniqueness conclusion, family of models, attendance model, demand model and factored model.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polynomial Modeling from Data

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Record the calculator's quadratic model using approximate language. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The setting is manufacturing. The supplied hypothetical fitted model is $q(x) = 1.05x^2 - 0.19x + 2.01$. The inputs have one unit spacing. The outputs are nearest tenth. Treat the supplied fitted expression as a hypothetical model to interpret; it is not specified as the unconstrained least-squares fit to every displayed row.

Given domain/context: manufacturing

inputs	outputs
0	2.0
1	2.9
2	6.2
3	10.8

Bank label: ______

Translate the arch prototype to the stated peak and give its operating domain. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The setting is bridge arch. Work on the interval $0 \leq t \leq 8$. Use the prototype -x^2. The vertex is $(4, 10)$.

Bank label: ______

Write the area polynomial and its positive-side restriction. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The setting is rectangular sign. The diagram dimensions are length $x + 3$ width $x - 1$. The length is $x + 3$. The width is $x - 1$.

Bank label: ______

Write a continuous two-stage charge model. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The setting is battery test. For the first stage, use charge remains 100 through hour 1. For the second stage, then loses $4(t - 1)^2$ through hour 5.

Bank label: ______

AP Precalculus

Polynomial Modeling from Data

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Build a height model using the turning point and measurement, and state its time domain. The setting is fountain. The maximum occurs at the point $(3, 20)$. The graph passes through $(1, 12)$. Restrict time to $0 \le t \le 6$.

Translate the labeled net into a polynomial volume model and its positive-length domain. The setting is packaging. Remove squares of side x at four corners. The diagram dimensions are base $20 - 2x$ by $14 - 2x$ height x. The base measures $20 - 2x$ by $14 - 2x$. The height is x. The sheet measures 20 by 14.

State the quadratic regression model and whether it passes through every measurement exactly. The setting is calibration. The supplied hypothetical fitted model is $y = 1.02x^2 + 0.01x + 0.04$. The inputs have one unit spacing. The outputs are nearest tenth. Treat the supplied fitted expression as a hypothetical model to interpret; it is not specified as the unconstrained least-squares fit to every displayed row.

Given domain/context: calibration

inputs	**outputs**
-2	4.1
-1	1.2
0	0.0
1	1.1
2	4.2

Write a two-stage model with an unambiguous rule at the transition. The setting is production. Restrict the input to $0 \le t \le 5$. For the first stage, use output grows as $4t$ before $t = 3$. For the second stage, after $t = 3$ add $(t - 3)^2$ to the accumulated 12.

AP Precalculus

Polynomial Modeling from Data

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $S(x) = 2(\frac{x-1}{2})^3 - 2$, equivalently $\frac{(x-1)^3}{4} - 2$.

Use the single-turn trend to report an approximate polynomial model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The setting is traffic noise. The inputs have one hour spacing. The outputs are nearest tenth. The supplied hypothetical fitted model is $n(t) = -1.03t^2 + 5.08t + 7.99$. Treat the supplied fitted expression as a hypothetical model to interpret; it is not specified as the unconstrained least-squares fit to every displayed row.

Given domain/context: traffic noise

inputs	outputs
0	8.0
1	12.1
2	14.2
3	13.9
4	11.8

Your response: ____________

Incoming: $v(x)$ is approximately $0.98x^4 + 0.04x^2 + 1.06$ over the sampled inputs.

Use factored form to build the profit model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The setting is profit. A measured point is $(1, 16)$. The zeros are -3 and 5.

Your response: ____________

Incoming: $M(t) = 4t$ for $0 \le t < 3$, and $M(t) = 15 + 2(t-3)^2$ for $3 \le t \le 6$.

Construct the triangle area polynomial and impose positive dimensions. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The setting is triangular garden. The diagram dimensions are base x height $18 - 2x$. The base measures x. The height is $18 - 2x$.

Your response: ____________

START

Apply the described translations to the cubic prototype. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The setting is response index. Use the operating domain $-4 \le x \le 2$. Use the prototype x^3. Translate the prototype 2 units left and 1 unit down.

Your response: ____________

AP Precalculus

Polynomial Modeling from Data

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Construct an area polynomial from the dependent legs and restrict x physically. Student work: A(x)=(1/2)x(12-x), with 0<=x<12. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. The setting is triangular display. The diagram dimensions are horizontal leg x vertical leg 12-x. The horizontal leg has length x. The vertical leg has length 12-x. The two legs form a right angle.

Write a continuous fee model and assign $h = 2$ to the later rule. Student work: $C(h) = 5 + 3h$ for $0 \leq h < 2$, and $C(h) = 11 + 2(h-2)^2$ for $2 \leq h < 5$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. The setting is parking fee. Restrict the input to $0 \leq h \leq 5$. For the first stage, use $5 + 3h$ through $h = 2$. For the second stage, begin at 11 and add $2(h-2)^2$.

Polynomial Modeling from Data

Full Worked Solutions - excerpt

Worked example

Problem. Use factored form to build the profit model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The setting is profit. A measured point is $(1, 16)$. The zeros are -3 and 5.

Answer. $P(x) = -(x+3)(x-5)$.

Explanation and check.

At $x = 1$ the two factors multiply to negative sixteen, so a scale of negative one gives the measured profit.

Worked example

Problem. Find k and write the continuous piecewise model. The setting is signal. The first branch is $3t + 2$ for $t < 4$. The model must be continuous at $t = 4$. The second branch is $(t-4)^2 + k$ for $t \geq 4$.

Answer. k = 14; use $3t + 2$ for $t < 4$ and $(t-4)^2 + 14$ for $t \geq 4$.

Explanation and check.

Both pieces must have boundary value 14, which directly fixes the second piece's offset.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Rational Function Word Problems

Rational Function Word Problems | APPC-U01-P22

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for intensity model, model and invariant, model, concentrate fraction, concentration and domain and concentration model.

8 PDFs and a README; 54 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Rational Function Word Problems

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

<table>
<tr><td>Express width in terms of a nonnegative design input x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: rectangle. Area: 99. Dimensions: Length: $x+2$; Width: w.

Bank label: ______</td><td>Model the juice fraction after adding only water. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: juice. Initial juice: 8. Initial volume: 20. Water added: x liters.

Bank label: ______</td></tr>
<tr><td>Construct average printing cost per ticket. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: tickets. Per ticket: 2. Quantity: positive tickets n. Setup: 500.

Bank label: ______</td><td>Model frequency from the fixed speed relation. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: waves. Relationship: fixed wave speed. Input: wavelength lambda>0. Speed: 340.

Bank label: ______</td></tr>
<tr><td>Express height for a nonnegative parameter x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: triangle. Area: 54. Dimensions: Base: $x+3$; Height: h.

Bank label: ______</td><td>Model concentration after adding pure water. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: cleaner. Initial solute: 15. Initial volume: 60. Water added: x.

Bank label: ______</td></tr>
<tr><td>Express the second side as a function of positive x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: rectangle. Area: 144. Dimensions: Other side: y; Side: x.

Bank label: ______</td><td>Express height as a function of positive base. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: parallelogram. Area: 88. Dimensions: Base: b; Height: h.

Bank label: ______</td></tr>
</table>

AP Precalculus

Rational Function Word Problems

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Track salt and total liquid separately to build the concentration. Context: saline. Added concentration: 0.2. Added volume: x liters. Initial salt: 6. Initial volume: 30.	Construct cost per poster and state the admissible inputs. Context: printing. Fixed cost: 240. Quantity: n positive posters. Variable cost: 3.
Model the completion time when both constant-rate pumps work together. Context: pumps. Relationship: parallel workers. Pump a time: 6. Pump b time: $b > 0$ hours.	Audit the proposed blue concentration using units and feasible range. Context: paint. Blue: 4. Proposed: $\frac{10}{4}$. Total: 10.
Construct average cost and identify which term prevents a finite horizontal baseline. Context: manufacturing. Quantity: $q > 0$. Total cost: $C(q) = 500 + 4q + 0.02q^2$.	Express height in terms of radius for the fixed-volume cylinder. Context: cylinder. Dimensions: Height: h; Radius: r. Volume: 200.

AP Precalculus

Rational Function Word Problems

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START Build the simplest zero-baseline saturating ratio. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context: advertising. Half limit input: 5. Limit: 80. Your response: ______________________	**Incoming:** $A(n) = \frac{2500}{n} + 30$ for integers $1 \le n \le 90$; the average remains above 30. Model concentration after dilution. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context: soap. Initial volume: 44. Soap: 11. Water: x liters. Your response: ______________________
Incoming: $L(d) = \frac{288}{d^2}$ for $d > 0$. Construct cost per badge. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context: badges. Fixed: 360. Per badge: 1.5. Quantity: positive whole n. Your response: ______________________	**Incoming:** $h(x) = \frac{180}{(x-3)(x+1)}$ for $x > 3$. Express length for $x \ge 0$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context: rectangle. Area: 168. Width: $x + 4$. Your response: ______________________
Incoming: $h(x) = \frac{84}{x-1}$ for $x > 1$. Model the equal amount per participant. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context: shared budget. Participants: positive whole n. Total: 750. Your response: ______________________	**Incoming:** $I(d) = \frac{576}{d^2}$ for $3 \le d \le 18$. Construct a zero-start learning model with the specified half-saturation input. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context: learning. Half limit practice: 8. Limit: 100. Your response: ______________________

AP Precalculus

Rational Function Word Problems

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Express length as a function of positive width. Student work: $L(w) = \frac{72}{w}$ for $w \geq 0$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. Context: rectangle. Area: 72. Dimensions: Length: L; Width: w.

Model travel time from the fixed-distance invariant. Student work: $T(v) = \frac{180}{v}$ for $v \geq 0$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. Context: travel. Distance: 180. Input: speed $v > 0$.

Identify the conservation mistake in the constant proposal. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information. Context: dye. Initial dye: 5. Initial volume: 25. Proposed: $\frac{5}{25}$. Water added: x.

Rational Function Word Problems

Full Worked Solutions - excerpt

Worked example

Problem. Track salt and total liquid separately to build the concentration. Context: saline. Added concentration: 0.2. Added volume: x liters. Initial salt: 6. Initial volume: 30.

Answer. $C(x) = \frac{6+0.2x}{30+x}$ for $x \geq 0$.

Explanation and check.

Added solution contributes both $0.2x$ salt and x total liters.

Worked example

Problem. Model concentration after dilution. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context: soap. Initial volume: 44. Soap: 11. Water: x liters.

Answer. $C(x) = \frac{11}{44+x}$ for $x \geq 0$.

Explanation and check.

Water increases only the total volume.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Finding Parameters in Function Models

Finding Parameters in Function Models | APPC-U01-P23

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for existence, free family, freedom, parameter, parameters and check and parameters.

8 PDFs and a README; 59 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Finding Parameters in Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find k using the allowed input. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use the function $f(x) = \frac{k}{x}$. The graph passes through $(4, 7)$.

Bank label: ______

Read the shifts and calibrate the scale. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use the function $a(x - h)^2 + k$. The graph passes through $(0, 5)$. The vertex is $(-1, 2)$.

Bank label: ______

State the input-output feature encoded by 7 and 12. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use the model $y = a(x - 7)^2 + 12$.

Bank label: ______

Use the two observations to recover the slope and intercept. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given domain/context: motion

0	4
3	19

Bank label: ______

AP Precalculus

Finding Parameters in Function Models

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Are a and b identifiable? Use the function $f(x) = ax + b$.

Complete response required: freedom.

1	4
1	4

Give units for a, b, and c. height is measured in meters. Use the model $h(t) = at^2 + bt + c$. time is measured in seconds.

Audit the fitted denominator. The calibration input is 2. A student claims $b = 2$. Use the function $\frac{a}{x-b}$.

Interpret $100, 500$, and a. Use the model $C(q) = a(q - 100)^2 + 500$.

Explain the effect of the small slope difference at $x = 100$. Both models have intercept 1. Evaluate at input 100. Their slopes are $(2.0, 2.03)$.

Find the first observation the claimed fit violates. A student claims $(2, 1)$. Use the function $ax + b$.

Given domain/context: linear fit

0	1
1	3
2	6

AP Precalculus

Finding Parameters in Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Calibrate k at the allowed input. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the function $\frac{k}{x-1}$. The graph passes through $(4, 6)$.

Your response: ______________

Incoming: a has liters per hour cubed, $t0$ has hours, and $R0$ has liters per hour.

Determine a and c. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the function $ax^2 + c$.

Given domain/context: quadratic

0	3
2	15

Your response: ______________

Incoming: a = 2 and $b = 2$.

Decide uniqueness and recover k. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the function $10\frac{x}{x+k}$. The graph passes through $(2, 5)$.

Your response: ______________

Incoming: a = 3 and $b = 1$; the third row is consistent but adds no freedom.

Recover the limiting level and inverse offset from the two features. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the function $a + \frac{b}{x+1}$. The horizontal limiting value is 4. The output at input zero is 10.

Your response: ______________

Incoming: k is pressure-volume in kilopascal-liters.

Recover slope and baseline from the two measurements. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given domain/context: linear

0	-2
4	10

Your response: ______________

Incoming: a = 3 and $b = -2$.

Decide whether k is identifiable and solve it. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the function $\frac{k}{x+2}$. The graph passes through $(1, 7)$.

Your response: ______________

AP Precalculus

Finding Parameters in Function Models

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

What do 6 and 18 represent? Student work: At reference time $t = 6$, the modeled temperature is 17. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. Use the model $T(t) = m(t - 6) + 18$.

Correct the claimed slope. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information. A student claims $a = 2$. Use the function $ax + 2$. The graph passes through $(3, 11)$.

Finding Parameters in Function Models

Full Worked Solutions - excerpt

Worked example

Problem. Audit the fitted denominator. The calibration input is 2. A student claims $b = 2$. Use the function $\frac{a}{x-b}$.

Answer. The fit is invalid because x = 2 is undefined when $b = 2$.

Explanation and check.

A calibration equation cannot be cleared at a forbidden input.

Worked example

Problem. Interpret $100, 500$, and a. Use the model $C(q) = a(q - 100)^2 + 500$.

Answer. The reference quantity is 100, cost there is 500, and a controls curvature away from that quantity.

Explanation and check.

Shifted form attaches parameters to the vertex rather than input zero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Interpreting and Comparing Function Models

Interpreting and Comparing Function Models | APPC-U01-P24

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for direction and changing-rate conclusion, growth conclusion, existence conclusion, rate and interpretation, rate and meaning and rate and sign meaning.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Interpreting and Comparing Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the modeled tank volume after 4 hours and state its unit. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: water storage.

Work on the domain $0 \le t \le 10$.

Evaluate at the input 4.

Measure the input in hours.

Use the model $V(t) = 480 + 35t - 2t^2$.

Measure the output in liters.

Bank label: ______

At what time does the graph predict exactly 20 customers in line? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: queue length.

the horizontal axis measures minutes; the vertical axis measures customers.

The curve relation is below $y = 20$ for $t < 18$ and above $y = 20$ for $t > 18$.

Work on the domain $0 \le t \le 30$.

The intersection is $(18, 20)$.

The scale is 2 minutes horizontally and 5 customers vertically.

The target is 20 customers.

Bank label: ______

Where is the modeled arch height positive? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: playground arch.

The constraint is $H(x) > 0$.

Work on the domain $0 \le x \le 12$.

Measure the input in feet.

Use the model $H(x) = -(x + 2)(x - 10)$.

Measure the output in feet.

Bank label: ______

Find the modeled factory output on day 3. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context: factory output.

Evaluate at the input 3.

Measure the input in days.

Use the model $Q(d) = 90 + 24d - 3d^2$.

Measure the output in units.

Bank label: ______

AP Precalculus

Interpreting and Comparing Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

At what feasible times does the model predict 22 degrees Celsius?

Context: greenhouse temperature.

Work on the domain $0 \leq h \leq 6$.

Measure the input in hours after sunrise.

Use the model $T(h) = 18 + 5h - h^2$.

Measure the output in degrees Celsius.

The target is 22.

Determine the mileage ranges on which each service is cheaper, including the break-even distance.

Context: delivery pricing.

Work on the domain $0 \leq d \leq 25$.

Measure the input in miles.

The model A is $A(d) = 18 + 1.6d$.

The model B is $B(d) = 30 + 0.8d$.

Measure the output in dollars.

The criterion is lower cost.

Find where along the span the clearance is at least 9 meters.

Context: arch clearance.

The constraint is $H(x) \geq 9$.

Work on the domain $0 \leq x \leq 8$.

Measure the input in meters from left support.

Use the model $H(x) = 12 - 0.75(x - 4)^2$.

Measure the output in meters.

The threshold is 9.

Compute the model's average rate of change from day 2 to day 6 and interpret its sign.

Context: reservoir depth.

Measure the input in days.

The interval is $(2, 6)$.

The model assumption is The listed values come from one continuous depth model.

Measure the output in centimeters.

Context/domain: reservoir depth

Input (days)	Output (centimeters)
2	74
6	58

AP Precalculus

Interpreting and Comparing Function Models

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

<table>
<tr>
<td>

START

Find the average download rate over the indicated interval. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: page downloads.

Measure the input in minutes.

The interval is $(2, 7)$.

Use the model $D(t) = 180 + 45t$.

Measure the output in downloads.

Your response: ____________

</td>
<td>

Incoming: At t = 2 and $t = 6$ hours; the maximum is 48 milligrams per liter.

Use the supplied rounded model to predict temperature at 7 minutes. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: coffee cooling.

The coefficient note is 0.18 is rounded to two decimal places.

Evaluate at the input 7.

Measure the input in minutes.

Use the model $T(t) = 21 + \frac{68}{1+0.18t}$.

Measure the output in degrees Celsius.

The report precision is nearest tenth.

Your response: ____________

</td>
</tr>
<tr>
<td>

Incoming: 25.5 kilograms and 7 containers.

At what allowed price does the model predict 132 rentals? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: rental demand.

Work on the domain $8 \le p \le 14$.

Measure the input in dollars.

Use the model $D(p) = 180 - 12(p - 8)^2$.

Measure the output in rentals.

The target is 132.

Your response: ____________

</td>
<td>

Incoming: $-5 \le x < 3$ valve turns.

Find when the modeled noise level satisfies the 65-decibel limit. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: warehouse noise.

The constraint is $N(t) \le 65$.

Work on the domain $0 \le t \le 10$.

Measure the input in hours.

Use the model $N(t) = 65 + (t - 2)(t - 5)(t - 8)$.

Measure the output in decibels.

Your response: ____________

</td>
</tr>
</table>

AP Precalculus

Interpreting and Comparing Function Models

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

State when the modeled lake temperature is at least 18 degrees Celsius. Student work: $3 < m \leq 8$ months after January. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Context: lake temperature.

the horizontal axis measures months after January; the vertical axis measures degrees Celsius.

The crossings is $(3, 8)$.

The curve relation is below $y = 18$ before $m = 3$, on or above $y = 18$ from $m = 3$ through $m = 8$, below afterward.

Work on the domain $0 \leq m \leq 11$.

The endpoints is included.

The threshold is at least 18 degrees.

Interpreting and Comparing Function Models

Full Worked Solutions - excerpt

Worked example

Problem. Determine the feasible width that maximizes modeled area and give the maximum area.

Context: garden area.

Work on the domain $4 \le w \le 18$.

Measure the input in meters.

Use the model $A(w) = 48w - 2w^2$.

Measure the output in square meters.

Answer. w = 12 meters, with maximum area 288 square meters.

Explanation and check.

$A(w) = -2(w - 12)^2 + 288$, and the interior vertex lies in the allowed interval.

Worked example

Problem. Describe the long-run prediction of the model as impressions increase.

Context: advertising response.

Work on the domain $x \ge 0$.

Measure the input in thousands of impressions.

Use the model $R(x) = 72\frac{x}{x+18}$.

Measure the output in percent awareness.

Answer. Predicted awareness approaches 72 percent and does not exceed it under the model.

Explanation and check.

The numerator and denominator have the same degree, so the horizontal asymptote is $R = 72$ percent.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Residuals and Model Validation

Residuals and Model Validation | APPC-U01-P25

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for compliance and violations, acceptability, qualified adequacy, evidence claim, consistency judgment and correction and correction and meaning.

8 PDFs and a README; 105 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Residuals and Model Validation

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the residual and interpret it.

Given: domain: concert revenue; payload: observed: 18400; predicted: 19000; unit: dollars

Bank label: ______

Does the timing model meet the requirement?

Given: domain: package timing; payload: requirement: each error magnitude no greater than 2 minutes; residuals: $-1, 2, -3, 1$; unit: minutes

Bank label: ______

Find the observed tree mass.

Given: domain: tree mass; payload: definition: residual=observed-predicted; predicted: 48.2; residual: -1.6; unit: kilograms

Bank label: ______

State the four residual-plot coordinates.

Given: domain: room temperature; payload: input unit: hours; inputs: $0, 2, 5, 8$; residual unit: degrees Celsius; residuals: $-1, 0, 2, -2$

Construct the requested table/graph using the given axes, units and domain.

inputs	**residuals**
0	-1
2	0
5	2
8	-2

Bank label: ______

AP Precalculus

Residuals and Model Validation

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compare the models and explain why signed totals are misleading.

Given: domain: daily sales; payload: criterion: sum of absolute residuals; model A residuals: $6, -6, 5, -5$; model B residuals: $2, 2, -2, -2$; unit: items

Give the points for the residual plot against week and specify the reference line.

Given: domain: plant height; payload: height unit: centimeters; input spacing: unequal; rows: (observed: 4; predicted: 5; week: 0); (observed: 11; predicted: 10; week: 2); (observed: 23; predicted: 21; week: 5); (observed: 36; predicted: 39; week: 9)

Construct the requested table/graph using the given axes, units and domain.

observed	predicted	week
4	5	0
11	10	2
23	21	5
36	39	9

Choose the more reasonable predictive model and address the perfect training fit.

Given: domain: traffic volume; payload: structures: model P: degree-7 polynomial; model Q: quadratic; training absolute errors: model P: $0, 0, 0, 0$; model Q: $3, 2, 2, 3$; unit: vehicles per minute; validation absolute errors: model P: $28, 35, 41$; model Q: $5, 6, 4$

Recover the prediction and explain the negative residual.

Given: domain: reservoir depth; payload: definition: residual=observed-predicted; observed: 42.3; residual: -1.8; unit: meters

AP Precalculus

Residuals and Model Validation

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Case A has 5% error and case B has 0.25%; the same 0.2-meter residual is much more consequential at the smaller scale.

Use the model to produce the residual-plot points.

Given: domain: warehouse demand; payload: columns: day, observed orders; model: $P(t) = 50 + 8t$; rows: $(0, 51)$; $(2, 64)$; $(5, 93)$; $(9, 119)$

Construct the requested table/graph using the given axes, units and domain.

day	observed orders
0	51
2	64
5	93
9	119

Your response: ______________

Incoming: Inputs $1 - 6$ form a positive-negative-positive curve, indicating model-shape mismatch; input 7 adds a12-kilowatt outlier beyond tolerance.

Calculate the residual-plot points.

Given: domain: sound level; payload: columns: distance meters, observed decibels; model: $P(x) = 80 - 3x$; rows: $(1, 76)$; $(3, 72)$; $(6, 64)$; $(10, 47)$

Construct the requested table/graph using the given axes, units and domain.

distance meters	observed decibels
1	76
3	72
6	64
10	47

Your response: ______________

Incoming: No. The positive residual +6 is an underprediction exceeding the five-unit limit; -7, $+4$, and -3 satisfy their direction-specific limits.

Does the model satisfy the temperature requirement?

Given: domain: food temperature; payload: requirement: each magnitude at most 1 degree; residuals: $-1.2, 0.8, 1.5, -0.4$; unit: degrees Celsius

Your response: ______________

Incoming: Model B is more reasonable there; its errors stay within 3 decibels, while A's spread expands to 15 despite its smaller training errors.

List the residual-plot points against day.

Construct the requested table/graph using the given axes, units and domain.

day	observed centimeters	predicted centimeters
0	12	11
3	19	21
8	27	24
14	31	35

Your response: ______________

AP Precalculus

Residuals and Model Validation

Name: ________________ **Date:** ________________ **Class:** ________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Identify the logical gap and replace the claim with a defensible conclusion. Diagnose the extension from three exact historical fits to thirty future years, including the changed migration mechanism.

Given: domain: population forecast; payload: claim: The model fits three observed years exactly, so it will predict the next 30 years exactly.; evidence: future domain: 3 through 30; mechanism change: a migration policy begins in year 3; residuals: $0, 0, 0$; sample years: $0, 1, 2$

Residuals and Model Validation

Full Worked Solutions - excerpt

Worked example

Problem. Judge the model for each intended use.

Given: domain: map distance; payload: residual: 0.12; unit: kilometer; uses: hiking map limit: 0.25; survey limit: 0.02

Answer. The 0.12-kilometer error is acceptable for the hiking map but not the survey, whose tolerance is only 0.02 kilometer.

Explanation and check.

The same error is compared with different decision thresholds.

Worked example

Problem. Recover the prediction and explain the negative residual.

Given: domain: reservoir depth; payload: definition: residual=observed-predicted; observed: 42.3; residual: -1.8; unit: meters

Answer. 44.1 meters; the prediction is 1.8 meters above the observation.

Explanation and check.

predicted=observed-residual= $42.3 - (-1.8) = 44.1$, consistent with overprediction.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Arithmetic vs Geometric Sequences

Arithmetic vs Geometric Sequences | APPC-U02-P01

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for terms, safe explicit form, and exponential limitation, explicit rule, signs, and real-domain limitation, all ratios and sequences, possible completions, two valid completions and conclusion and classification with failure evidence.

8 PDFs and a README; 51 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Arithmetic vs Geometric Sequences

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Classify the consecutive terms $1, 4, 9, 16$ as arithmetic, geometric, or neither. For the answer bank, record the classification. Complete and justify all parts of the original task in your work. Bank label: _______	Determine the sequence type for $-4, -9, -14, -19$ and state its step change. For the answer bank, record the common difference. Complete and justify all parts of the original task in your work. Bank label: _______
Translate $a_0=12$ and $a_n=a_{n-1}-7$ into a direct rule for whole-number n. For the answer bank, record the expression for a_n. Complete and justify all parts of the original task in your work. Bank label: _______	Convert $b_n=125(0.2^{n+1}), n = 0, 1, 2,...,$ to a recurrence with the correct zeroth term. For the answer bank, record the zeroth term b_0. Complete and justify all parts of the original task in your work. Bank label: _______
Classify $7, 14, 28, 56$ and state the common change. For the answer bank, record the common ratio. Complete and justify all parts of the original task in your work. Bank label: _______	Write an explicit formula for $b_1=100$ and $b_n=b_{n-1}-15$ for $n \geq 2$. For the answer bank, record the expression for b_n. Complete and justify all parts of the original task in your work. Bank label: _______
For $a_n=\frac{7-2n}{5}$ at whole-number $n \geq 0$, find a_0 and an exact recurrence. For the answer bank, record the zeroth term a_0. Complete and justify all parts of the original task in your work. Bank label: _______	An arithmetic sequence has $a_{10}=1$ and decreases by 2 each step. Write a_n from $n = 1$ onward and find a_1. For the answer bank, record the first term a_1. Complete and justify all parts of the original task in your work. Bank label: _______

AP Precalculus

Arithmetic vs Geometric Sequences

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For indices 1 through 4, the terms are $11, 11, 11, 11$. Classify every arithmetic or geometric description that applies and state its invariant.	The terms at $n = 0, 1, 2, 3$ are all 0. If a geometric classification requires defined consecutive ratios, classify the sequence and explain the zero case.
For a_n=$5 - 3(n-1), n = 1, 2,...$, give the necessary initial term and a recurrence.	An account begins with \$120, and each completed deposit adds \$25. Let n count completed deposits. Write the balance sequence and list its first three graph points.
Complete the arithmetic sequence a_1=4, a_2=?, a_3=16, a_4=22, and justify the missing term.	Convert g_1=48 and g_n=$(\frac{1}{2})g_{n-1}$ for $n \geq 2$ into an explicit sequence rule.

AP Precalculus

Arithmetic vs Geometric Sequences

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Given $a_0=7$ and $a_n=a_{n-1}+4$ for $n \geq 1$, write a direct rule for whole-number n.

Your response: ______________

Incoming: d = 5 and $a_2=-1$.

An arithmetic sequence has $a_3=14$ and $a_8=34$. Find its common difference and a_1.

Context/domain: sequence

3	14
8	34

Your response: ______________

Incoming: r = 3 and $a_1=-9$.

A geometric sequence with positive ratio satisfies $a_2=12$ and $a_5=324$. Recover r and a_1.

Your response: ______________

Incoming: d = −3 and $a_1=31$.

An arithmetic sequence satisfies $a_6=-5$ and $a_{10}=19$. Determine d and extrapolate backward to a_1.

Your response: ______________

Incoming: r = −3, uniquely.

For an arithmetic sequence indexed from 0, $a_2=5$ and $a_7=-10$. Find the common difference and a_0.

Context/domain: sequence

2	5
7	−10

Your response: ______________

Incoming: Geometric with ratio 0.8.

A positive-ratio geometric sequence has $a_4=54$ and $a_7=2$. Recover the common ratio and a_1.

Your response: ______________

AP Precalculus

Arithmetic vs Geometric Sequences

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The exact terms are $2, 10, 50, 245$. A student calls them geometric with ratio 5. Evaluate the claim and state the fourth term that would make it true. Student work/claim: The first two ratios equal 5, so all four given terms form a geometric sequence. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A student labels $10, 7, 4, 1, -3$ arithmetic with $d = -3$. Decide whether the complete exact list supports the claim and repair the last term if needed. Student work/claim: Every difference in the given list is -3. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Can the blank in $4, ?, 14, 19, 25$ be filled so the entire exact list is arithmetic? Explain which evidence is decisive. Student work/claim: Filling the blank with 9 makes the entire list arithmetic. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Arithmetic vs Geometric Sequences

Full Worked Solutions - excerpt

Worked example

Problem. Given $a_0 = 7$ and $a_n = a_{n-1} + 4$ for $n \geq 1$, write a direct rule for whole-number n.

Answer. $a_n = 7 + 4n$ for $n = 0, 1, 2, \ldots$

Explanation and check.

From index 0 to index n there are n additions of 4, so the term is $7 + 4n$.

Worked example

Problem. Starting at $a_3 = 81$, each later term is one third of the previous term. Write the explicit rule for $n \geq 3$.

Answer. $a_n = 81(\frac{1}{3})^{n-3}$.

Explanation and check.

The exponent is zero at the first allowed index and counts the subsequent divisions by 3.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sequences to Linear and Exponential Models

Sequences to Linear and Exponential Models | APPC-U02-P02

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for differences, ratios, interpretation, domain judgment, domain distinction, two compatible models, existence and reason and class and amount.

8 PDFs and a README; 62 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Sequences to Linear and Exponential Models

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Extend $a(n) = -4 + 1.5n$ from whole-number indices to all real x and evaluate $f(-2)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	An account begins with \$140 and receives exactly \$25 each week with no interest. Is the weekly pattern linear or exponential, and what is the balance after one week? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
A sequence starts at 18 and increases by 6 per index. If the constant rate is extended between indices, what is its value at index 2.5? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The starting value is the value at index 0. Bank label: ______	A material starts at $250g$ and loses 12% of current mass per hour. Give its factor and model. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______

AP Precalculus

Sequences to Linear and Exponential Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

The sequence rule is $a(n) = 3 + 2n$ for whole-number n at least 0. Extend the same linear rule to real x at least 0, then find $f(2.5)$.

Exact samples at $x = 0, 1, 2, 3$ are $5, 8, 11, 14$. Which model class is consistent, and what does the finite table not prove?

Context/domain: real-input data

0	5
1	8
2	11
3	14

A balance starts at \$200 and rises each hour by 10% of the initial \$200, not its current value. Construct a real-time rule and classify it.

For exact points $(0, 1)$, $(2, 7)$, and $(5, 16)$, compare change per input unit rather than raw differences. Which class is supported?

Context/domain: real-input data

0	1
2	7
5	16

$L(x) = 4 + 16x$ and $E(x) = 4(3^x)$ share $(0, 4)$ and $(2, 36)$. Compare their values at $x = 1$.

Context/domain: matched models

0	4
2	36

A linear table gives $10, 15, 20$ and an exponential table gives $10, 15, 22.5$ at $x = 0, 1, 2$. After the shared $+5$, what differs?

AP Precalculus

Sequences to Linear and Exponential Models

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Production starts at 12 units and adds 6 units per minute. Write the model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ____________

Incoming: Both give factor 0.8: $\sqrt{0.64} = 0.8$ and cube root of $0.512 = 0.8$.

A sequence has value -2 at index 0 and adds 5 at each next whole-number index. Extend its constant-rate rule to real x and evaluate it at $x = -1.5$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ____________

Incoming: L keeps +5 while ratios fall; E keeps ratio 2 while increments grow $5, 10, 20$.

At consecutive inputs $-2, -1, 0, 1$, the exact outputs are $-7, -2, 3, 8$. Classify the data from their change pattern. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context/domain: integer samples

-2	-7
-1	-2
0	3
1	8

Your response: ____________

Incoming: Linear: 105. Exponential: 15.

A population starts at 90 and grows 7% of current size per year. Write the model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ____________

AP Precalculus

Sequences to Linear and Exponential Models

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A display begins with 46 items and staff add exactly 7 items each day. Classify the model and find the count after four days. Student work: exponential; 74 items. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use $L(x) = 12 + 6x$ and $E(x) = 12(2^x)$ to refute “same starting value means same change.” Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

Pallet count is $p(n) = 40 + 6n$ after n completed trucks. Does $p(2.5) = 55$ describe a sequence term in this context? Student work: No. The algebraic line gives 55, but 2.0 completed trucks is outside the stated event domain. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

An amount starts at $400mg$ and loses 5% of its current amount each hour. Classify the change and find the amount after two hours. Student work: linear decay; $361mg$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Sequences to Linear and Exponential Models

Full Worked Solutions - excerpt

Worked example

Problem. A tank starts with $500L$ and drains at a fixed $30L$ per day. Write the model and name its change type.

Answer. $V(t) = 500 - 30t$, a linear decrease while $V(t) \geq 0$.

Explanation and check.

A fixed amount per time unit gives slope -30 rather than a percentage.

Worked example

Problem. An investment starts at $\$600$ and grows 5% of its current balance per year. State its factor and continuous-time rule.

Answer. The factor is 1.05, so $B(t) = 600(1.05^t)$.

Explanation and check.

Current-value growth preserves $a1.05$ multiplier while dollar increments change.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Exponential Parameters and Reference Points

Exponential Parameters and Reference Points | APPC-U02-P03

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for adjusted ratios and limitation, adjusted sequences and conclusion, actual constant, base sign and consequence, degenerate class and failed condition and domain failures.

8 PDFs and a README; 57 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential Parameters and Reference Points

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Classify $h(x) = 4(1^{x-7}) - 2$ and give its actual value for every x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	Does $f(x) = (\frac{1}{2})(\frac{5}{4})^x$ define a real nonconstant exponential on all real inputs? Explain. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
Is $f(x) = -8(6^x)$ a real nonconstant exponential on all real x? Explain. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	For $N(t) = 40(0.9^{t-6}) + 12$, what happens at day 6, and what does 40 measure there? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
Translate the base 0.875 in $q(x) = 90(0.875^x)$ into a percent change per unit x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	For which real p is $g(x) = 7(5 - 2p)^x$ a real nonconstant exponential on all real inputs? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______

AP Precalculus

Exponential Parameters and Reference Points

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For which real m does $f(x) = 4(m+2)^x$ define a real nonconstant exponential for every real x?	For $f(x) = 7(2^x) + 5$, what ratio stays constant when x increases by 1?
A student says $P(t) = 800(1.04^t)$ grows by 1.04% per year. Correct the statement.	Does $g(x) = 2(-3)^x$ define a real exponential for every real x? Give a decisive input.
For $f(x) = 6(2^{x-3})$, identify the input where the coefficient 6 appears as an output, then find $f(0)$.	Interpret the base 0.73 in $A(t) = 320(0.73^t)$ as a percent change per unit time.

AP Precalculus

Exponential Parameters and Reference Points

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $f(x) = 9(2^x) + 4$, find $f(0)$ and interpret the coefficient 9 relative to the baseline. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________

Incoming: $f(0) = 13$; the coefficient 9 is the displacement $13 - 4$ from baseline 4.

For $g(x) = -11(3^x)$, find $g(0)$ and interpret the coefficient. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________

Incoming: $g(0) = -11$; the coefficient is the zero-input output.

Is $q(x) = 7((2 - \sqrt{3})^x)$ a valid real nonconstant exponential for all real x? Justify the base bounds. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________

Incoming: Yes. Since $1 < \sqrt{3} < 2$, the base $2 - \sqrt{3}$ lies strictly between 0 and 1 and is not 1.

For $f(x) = 4(3^x) - 10$, state the constant ratio involving displacement from the baseline. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________

Incoming: $\frac{f(x+1)+10}{f(x)+10} = 3$.

For $D(t) = 22(0.75^{t-3}) - 5$, find $D(3)$ and express $D(0)$. What does 22 represent? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________

Incoming: $D(3) = 17$ and $D(0) = 22(\frac{4}{3})^3 - 5$; 22 is displacement above baseline -5 at day 3.

Compare $f(x) = 12(0.6^x)$ and $g(x) = 6(0.6^x)$. How does the coefficient change affect every output? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________

AP Precalculus

Exponential Parameters and Reference Points

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Compare $f(x) = 8(1.2^{x-2}) + 5$ and $g(x) = 8(0.7^{x-2}) + 5$. Give their shared reference point and describe their directions to its right. Student work: Both pass through $(3, 13)$. To the right, f's displacement grows while g's displacement decays. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

A table of integer inputs makes $u(x) = 5(-2)^x$ look defined. Why is it not a real exponential on all real inputs? Student work: Fractional inputs such as $x = \frac{1}{2}$ produce nonreal values, so integer evidence does satisfy the all-real domain requirement. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Interpret factors 1.12 and 0.12 as percent changes of a positive quantity. Distinguish percent of the prior value from percent increase or decrease. Student work: Factor 1.12 means 112% remains and $a12\%$ increase. Factor 0.12 means 12% remains and an 87% decrease. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

For $f(x) = 3(4^x) - 2$, compare raw output ratios and baseline-adjusted ratios at $x = 0, 1, 2$. Student work: Outputs $1, 10, 46$ have raw ratios 10 and 4.6. Adding 2 gives displacements $3, 12, 47$ with ratio 4. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Exponential Parameters and Reference Points

Full Worked Solutions - excerpt

Worked example

Problem. For $h(x) = 5(4^{x-1}) - 3$, compute $h(1)$ and $h(0)$. Which quantity is represented directly by the coefficient 5?

Answer. $h(1) = 2$ and $h(0) = -\frac{7}{4}$. The coefficient 5 is the displacement $h(1) - (-3)$, not either displayed output by itself.

Explanation and check.

At $x = 1$ the factor is 1, then the baseline -3 must still be added.

Worked example

Problem. For which real c is $h(x) = (c - 5)(2^x)$ a nonconstant exponential?

Answer. All real c except c = 5.

Explanation and check.

The base is already valid; only the coefficient must be nonzero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Features and Concavity

Exponential Features and Concavity | APPC-U02-P04

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for monotonicity and concavity, monotonicity, concavity, rate trend, monotonicity, concavity, slope trend, concavity and differences, differences and concavity and differences, direction, concavity.

8 PDFs and a README; 99 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential Features and Concavity

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

State the range and horizontal asymptote of $f(x) = -3(2^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	Find the absolute minimum and maximum of $f(x) = 4(\frac{1}{2})^x$ on $[1, 3]$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
Is $f(x) = 6(1.2^x) + 3$ increasing or decreasing? Describe the exponential displacement from 3. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	State the range of $g(x) = 8(\frac{1}{3})^x$ and its limit as $x \to \infty$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
Compare $f(x) = 3(1.4^x)$ and $g(x) = 12(1.4^x)$ for all real x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	A nonconstant $f(x) = a(2^{cx}) + k$ stays below $y = k$ and decreases. What signs must a and c have? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______

AP Precalculus

Exponential Features and Concavity

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A known exponential has exact outputs $3, 6, 12, 24$ at $x = 0, 1, 2, 3$. Use first differences to describe its concavity.

Precision: Given values are exact

x	f(x)
0	3
1	6
2	12
3	24

AP Precalculus

Exponential Features and Concavity

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Is $h(x) = -10(0.8^x) - 2$ increasing or decreasing, and how does it approach its baseline? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ________________

Incoming: It increases toward -2 from below.

On $[2, \infty)$, determine the minimum and whether a maximum exists for $f(x) = 3(2^x) - 5$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ________________

Incoming: Minimum 7 at $x = 2$; no maximum.

Find the range and both tail limits of $g(x) = 10 - 7(0.2^x)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ________________

Incoming: Range $(-\infty, 10)$; $g(x) \to -\infty$ as $x \to -\infty$ and $g(x) \to 10$ from below as $x \to \infty$.

Describe the monotonicity and concavity of $f(x) = 5(1.5^x) - 20$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ________________

Incoming: It is increasing and concave up.

Find the effective one-unit factor and monotonicity of $f(x) = 10(2^{-3x}) + 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ________________

Incoming: The effective factor is $\frac{1}{8}$, so f decreases toward 1 from above.

Find the absolute extrema of $f(x) = 5 - 2(\frac{1}{2})^x$ on $[-1, 2]$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ________________

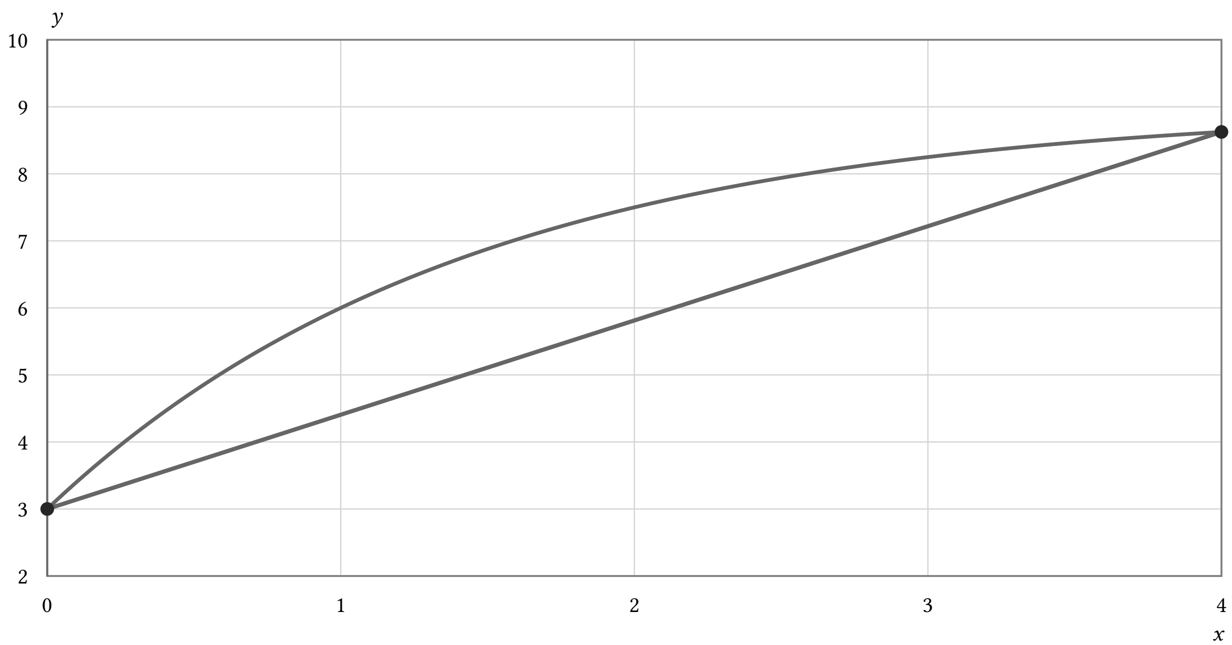
AP Precalculus

Exponential Features and Concavity

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $f(x) = 9 - 6(0.5^x)$, a chord joins $x = 0$ and $x = 4$. Does the chord overestimate or underestimate interior values? Explain. Student work: It underestimates them because f is concave up and lies above its chord in the interior. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.



Exponential Features and Concavity

Full Worked Solutions - excerpt

Worked example

Problem. A calculator rounds $f(12)$ to 0 for $f(x) = 0.1^x$. Explain why this does not mean the graph reaches its asymptote.

Answer. $f(12) = 10^{-12} > 0$. The value may round to 0, but no finite input makes a positive-base exponential equal 0.

Explanation and check.

Display precision is not exact attainment.

Worked example

Problem. Find where $f(x) = 2^x + 5$ and $g(x) = 3(2^x) - 1$ intersect, then state which is larger on each side.

Answer. Equality gives $2^x = 3$, so $x = \log$ base 2 of 3. Before that $f > g$; after that $g > f$.

Explanation and check.

Different baselines and amplitudes create one order reversal.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Rewriting Exponential Expressions

Rewriting Exponential Expressions | APPC-U02-P05

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for general proof and counterargument, normalized formulas and graph distinction, vertical multiplier and interpretation, exact multiplier, multiplier and interpretation and decision and normalized comparison.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Rewriting Exponential Expressions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Rewrite $3(2^{x+2})$ in the form $A(B^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	Write 2^{-x} with x as a positive exponent. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
Put $2(5^{x-1})$ in the form $A(B^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	Write 10^{-x} using a positive exponent x and a base below 1. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
Rewrite $7(3^{x+1})$ as $A(B^x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	Express (cube root of $125)^x$ without a radical. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
For $f(x) = 3^x$, express $f(x+2)$ as a constant multiple of $f(x)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	Normalize $5(3^{2x-1})$ into $A(B^x)$, with an exact coefficient. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______

AP Precalculus

Rewriting Exponential Expressions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A student writes $3^{x+2} = 3^x + 9$. Identify the failed exponent law and give the correct rewrite.	Show that shifting $f(x) = 2^x$ right 3 units produces the same graph as a vertical scaling. State the scale factor.

AP Precalculus

Rewriting Exponential Expressions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START Normalize $\left(\frac{1}{7}\right)^{2-x}$ as $A(B^x)$ with $B > 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Your response: ______________	**Incoming:** $(\frac{1}{49})(7^x)$ Convert $256^{3\frac{x}{8}}$ to a single exponential with an integer base. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Your response: ______________
Incoming: 8^x Express $f(x+\frac{1}{3})$ as a multiple of $f(x)$ when $f(x) = 8^x$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Your response: ______________	**Incoming:** $f(x+\frac{1}{3}) = 2f(x)$. Write $(\frac{1}{2})(81^{\frac{x}{4}+\frac{1}{2}})$ as $A(B^x)$ with B an integer. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Your response: ______________
Incoming: $(\frac{9}{2})(3^x)$ Find a so that $a(2^{x+1}) = 14(2^x)$ for all real x. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Your response: ______________	**Incoming:** a = 7 Express 3^{2-x} as $A(B^x)$ with $0 < B < 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Your response: ______________

AP Precalculus

Rewriting Exponential Expressions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Find h so that $2^{x+h} = 8(2^x)$ for every real x. Student work: $h = 4$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

The table shows $f(x) = 4^x + 2$ and $g(x) = 2^x + 2$ agree at $x = 0$. Are they identical? Use the normalized bases to explain. Student work: No. They share the translated anchor at $x = 0$, but bases 5 and 2 give different values for other inputs. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Precision: Given values are exact

x	f(x)	g(x)
-1	2.25	2.5
0	3	3
1	6	4

Rewriting Exponential Expressions

Full Worked Solutions - excerpt

Worked example

Problem. A student writes $3^{x+2} = 3^x + 9$. Identify the failed exponent law and give the correct rewrite.

Answer. Exponents add when same-base powers are multiplied, not added. The correct identity is $3^{x+2} = 9(3^x)$.

Explanation and check.

Use $3^{x+2} = 3^x \cdot 3^2$.

Worked example

Problem. Show that shifting $f(x) = 2^x$ right 3 units produces the same graph as a vertical scaling. State the scale factor.

Answer. $f(x-3) = 2^{x-3} = (\frac{1}{8})2^x$, so the right shift equals multiplying outputs by $\frac{1}{8}$.

Explanation and check.

The constant exponent -3 factors out as 2^{-3}.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Compound Growth and Decay Rates

Compound Growth and Decay Rates | APPC-U02-P06

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for three-year factors and ranking, one-year and five-year comparisons, model and exact factor, model and remaining factor, model and target factor and hourly and daily factors with percent interpretation.

8 PDFs and a README; 52 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Compound Growth and Decay Rates

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An exponential process multiplies by 2.5 over six months. Determine its monthly factor and approximate monthly percent increase. For the answer bank, record the exact monthly factor. Complete and justify all parts of the original task in your work. Bank label: ______	A quantity retains 96% every five days. Find its ten-day factor and cumulative percent decrease. For the answer bank, record the ten-day factor. Complete and justify all parts of the original task in your work. Bank label: ______
An index grows by factor 1.03 each quarter. Find its 18-month factor and total percent increase. For the answer bank, record the exact 18-month factor. Complete and justify all parts of the original task in your work. Bank label: ______	An exponential process has factor 9 over two days. Find the positive daily factor and daily percent increase. For the answer bank, record the positive daily factor. Complete and justify all parts of the original task in your work. Bank label: ______
A quantity retains 80% each week. What factor and total percent decrease apply after two weeks? For the answer bank, record the two-week factor. Complete and justify all parts of the original task in your work. Bank label: ______	A quantity has a3-year half-life. Write its model and give the factor remaining after 6 years. For the answer bank, record the fraction remaining after 6 years. Complete and justify all parts of the original task in your work. Bank label: ______
For $A(t) = A_0 e^{-0.05t}$, where t is in days, find the ten-day multiplier and percent decrease. For the answer bank, record the exact ten-day factor. Complete and justify all parts of the original task in your work. Bank label: ______	A process multiplies by 1.12 every three days. Determine its nine-day factor and percent increase. For the answer bank, record the nine-day factor. Complete and justify all parts of the original task in your work. Bank label: ______

AP Precalculus

Compound Growth and Decay Rates

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A quantity is multiplied by 1.20 each year. What factor and percent change apply over two years?	An exponential process has factor 4 over two hours. Find its positive hourly factor and hourly percent change.
The model $P(h) = 500(1.02^h)$ uses hours. Rewrite the same process using d days.	A population doubles every 5 days. Write its factor model and state the factor after 10 days.
Process A grows by factor 1.20 every two years; process B grows by factor 1.09 each year. Which grows faster after normalizing to two years?	For $Q(t) = Q_0 e^{0.04t}$, with t in years, find the five-year multiplier and percent increase.

AP Precalculus

Compound Growth and Decay Rates

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

The model $T(s) = 18(0.97^{\frac{s}{15}})$ uses seconds and a15-second factor. Rewrite it with m measured in minutes. For the matching card, record the expression for $T(m)$. Complete and justify all parts of the original task in your work.

Your response: ______________

Incoming: 1.2

Rewrite $C(d) = 60(1.5^d)$, with d in days, using h measured in hours. For the matching card, record the expression for $C(h)$. Complete and justify all parts of the original task in your work.

Your response: ______________

Incoming: $40(1.3)^{\frac{m}{60}}$

The model $F(m) = 200(1.03^m)$ uses months. Rewrite the same process with y in years. For the matching card, record the expression for $F(y)$. Complete and justify all parts of the original task in your work.

Your response: ______________

Incoming: $(\exp(-0.09), \exp(-0.36))$

A population has a monthly growth factor of 1.1. What is its growth factor over three months? For the matching card, record the three-month growth factor. Complete and justify all parts of the original task in your work.

Your response: ______________

Incoming: 27

A medication level is multiplied by 0.6561 every four hours. Find the equivalent hourly multiplier. For the matching card, record the hourly multiplier. Complete and justify all parts of the original task in your work.

Your response: ______________

Incoming: $0.4^{\frac{7}{2}}$

The model $Q(w) = 80(0.95^w)$ uses weeks. Rewrite it as a function of elapsed days d. For the matching card, record the expression for $Q(d)$. Complete and justify all parts of the original task in your work.

Your response: ______________

AP Precalculus

Compound Growth and Decay Rates

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student adds two 20% decreases and reports $a40\%$ loss. Show the correct compounded result and explain the discrepancy. Student work/claim: Two losses of 20% sum to $a40\%$ total loss. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A student divides $a30\%$ two-interval increase by 2 and reports 15% per interval. Find the correct repeated factor and quantify what two 15% increases would produce. Student work/claim: A two-interval gain of 30% means 15% in each interval. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A report says that $a6\%$ decrease every half-year means $a12\%$ decrease in one year. Determine the actual annual percent decrease and explain the discrepancy. Student work/claim: Two half-year losses of 6% make an annual loss of 12%. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

A two-year investment gain is 21%. A student divides by 2 and reports 10.5% per year. Find the constant annual percent rate that actually compounds to the observed gain, and verify it. Student work/claim: Divide the two-year gain 21% by two to obtain the annual compound rate. Identify the first invalid step or inference in the displayed work. Correct it and complete the original task.

Compound Growth and Decay Rates

Full Worked Solutions - excerpt

Worked example

Problem. A balance grows by factor 1.08 every two weeks. What factor and total percent increase apply over six weeks?

Answer. The six-week factor is $1.08^3 = 1.259712$, giving $a25.9712\%$ increase.

Explanation and check.

Six weeks contains three of the stated two-week intervals.

Worked example

Problem. An exponential process has factor 4 over two hours. Find its positive hourly factor and hourly percent change.

Answer. The hourly factor is $\sqrt{4} = 2$, corresponding to $a100\%$ hourly increase.

Explanation and check.

The positive unit factor must square to the stated two-hour multiplier.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Growth and Decay Applications

Exponential Growth and Decay Applications | APPC-U02-P07

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for day-based formula, formula and day-fourteen value, formula and earlier value, formula and hour-four value, formula and modeled initial value and formula and prediction.

8 PDFs and a README; 54 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential Growth and Decay Applications

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An account starts with \$50 and increases by 12% of its current balance each year. Write its balance after t years. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	A subscriber count is 500 at week 6 and grows by factor 1.04 each week. Write a model $S(w)$ anchored at the known week. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
A count starts at 50 and doubles weekly through week 3. Thereafter it grows by factor 1.1 per week. Write a piecewise model with no jump. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	A fish population begins at 3500 and approaches a habitat capacity of 5000. Each year, 80% of the current difference from capacity remains. Write $P(t)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
Create a value table for $A(n) = 500(1.1)^n$ at $n = 0, 1, 2, 3$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	At week 5 a quantity is 70, and afterward its weekly multiplier is 0.8. Write an anchor-form model using week w. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______
A fund starts at 100, grows by factor 1.03 annually through year 5, and then retains factor 0.97 annually. Write a continuous piecewise model. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______	A temperature starts at 90°F and approaches a65°F environment. After each hour, 30% of its excess remains. Write $T(h)$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Bank label: ______

AP Precalculus

Exponential Growth and Decay Applications

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

For $P(t) = 40(1.5)^t$, where t is a whole number of days from 0 through 4, construct a value table and state the initial value.	A room is 80°F when cooling begins toward a surrounding temperature of 20°F. Every ten minutes, half of the remaining temperature difference is retained. Build a model and find the temperature after thirty minutes.
A culture begins at 100, grows by factor 1.1 per day through day 3, and then decays by factor 0.8 per day. Construct a continuous-at-switch piecewise model through day 5 and evaluate day 5.	A quantity is 300 at $t = 4$ and grows by factor 1.25 every 2 time units. Audit the proposal $Q(t) = 300(1.25)^{\frac{t}{2}}$, repair it, and verify the anchor and one interval later.
A filter leaves 85% of the current impurity after each hour. If 200mg is present initially, write a model and state the percent removed per hour.	A process has value 250 at $t = 3$ and factor 1.2 per four time units. Construct its anchor form. The value is also reported as 300 at $t = 7$; determine whether that report is consistent and, if so, write the equivalent $t = 7$ anchor form.

AP Precalculus

Exponential Growth and Decay Applications

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $M(t) = 100 + 40(0.7)^t$.

A measured level starts at 30 and approaches a floor of 12. Each week, 40% of its excess above 12 remains. Write a model $L(w)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________________

Incoming: $Q(t) = 250(1.008)^{\frac{t}{60}}$.

A quantity equals 243 at $t = 5$ and triples every two time units. Find its value at $t = 1$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________________

Incoming: $A(t) = 625(0.8)^{\frac{t-8}{4}}$.

A machine worth \$1200 loses 7% of its current value each quarter. Write its value after q quarters. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________________

Incoming: $A(t) = 96(\frac{2}{3})^{\frac{t-6}{4}}$, and $A(14) = 96(\frac{2}{3})^2 = 42\frac{2}{3}$.

At day 20 a quantity is 81 and is multiplied by $\frac{1}{3}$ every ten days. Write an anchored exponential formula. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________________

Incoming: $V(q) = 1200(0.93)^q$.

An investment starts at 250, grows by factor 1.1 per quarter through quarter 4, and then by factor 1.03 per quarter. Write a continuous piecewise model. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________________

Incoming: 11

A measurement starts at 140 and approaches baseline 100. Each day, 70% of its excess above baseline remains. Write $M(t)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ______________________

AP Precalculus

Exponential Growth and Decay Applications

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

List the table values of $Q(n) = 30(2)^n$ for $n = 0, 1, 2, 3$. Student work: $(0, 30), (1, 60), (2, 120)$, and $(3, 239)$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

An asset starts at 400 and loses 12% of its current value each day. Audit the proposed model $V(t) = 400(0.12)^t$ and repair it. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

An account starts at 600 and gains 15% every three months. With t measured in months, audit $A(t) = 600(1.15)^{3t}$, repair it, and find the correct month-6 value. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

A score begins at 15 and approaches a ceiling of 75. Every two weeks, 60% of its remaining signed difference from the ceiling is retained. Build the model and evaluate week 6. Student work: $S(w) = 75 + 60(0.6)^{\frac{w}{2}}$, and $S(6) = 62.04$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Exponential Growth and Decay Applications

Full Worked Solutions - excerpt

Worked example

Problem. A quantity is 96 at month 15 and has factor 1.2 every three months. Find its modeled value at month 6. Use the complete final response to identify the next Match $\&$ Move card; show the reasoning required by the original task.

Answer. The value is $\frac{96}{1.2^3} = 55\frac{5}{9} \approx 55.56$.

Explanation and check.

Month 6 is three three-month intervals earlier, so undo three factors of 1.2.

Worked example

Problem. A value begins at 260 and approaches baseline 200, retaining 90% of its excess each week. Write $V(w)$. Use the complete final response to identify the next Match $\&$ Move card; show the reasoning required by the original task.

Answer. $V(w) = 200 + 60(0.9)^w$.

Explanation and check.

The initial excess is 60; apply the weekly factor to that excess and add 200.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential Functions from Two Points

Exponential Functions from Two Points | APPC-U02-P08

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for parameters and model, sufficiency decision and parameters, uniqueness and parameters, a and model, base and model and consistency and information decision.

8 PDFs and a README; 92 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential Functions from Two Points

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Can any exponential function fit exact rows $(1, 4)$ and $(1, 5)$? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Points:

x	f(x)
1	4
1	5

Bank label: ______

Relative to baseline 5, the exact outputs at $x = 0$ and $x = 1$ are 45 and 25. Determine the scale and positive base of their exponential displacement. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Points:

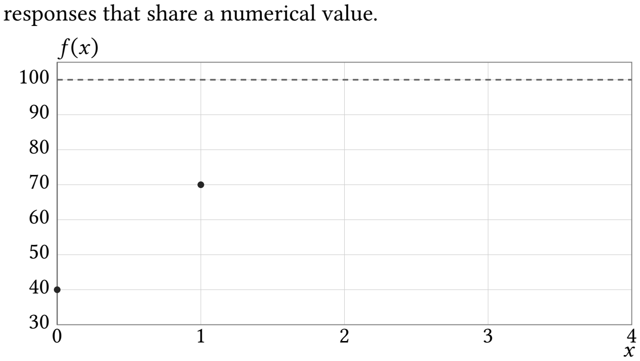
x	f(x)
0	45
1	25

Bank label: ______

For $f(x) = 10 + a(0.5)^x$, is $f(2) = 15$ sufficient to determine the model? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Bank label: ______

A graph on x: 0 to 4 by 1 and y: 30 to 105 by 10 has exact asymptote $y = 100$ and exact marks $(0, 40)$ and $(1, 70)$. Determine $f(x) = 100 + ab^x$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.



x	exact marked f(x)
0	40
1	70

Bank label: ______

AP Precalculus

Exponential Functions from Two Points

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Can one nonconstant real model $f(x) = ab^x$ with $b > 0$ pass through the exact points $(0, 5)$ and $(2, -20)$? Explain.

Points:

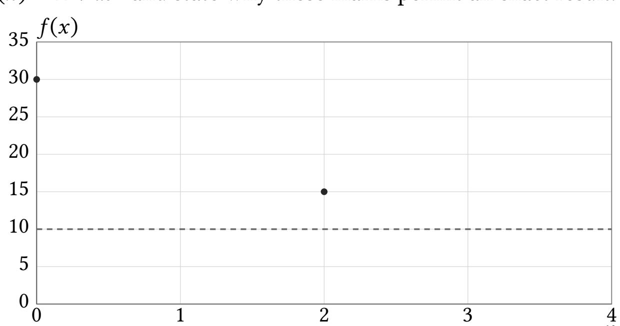
x	f(x)
0	5
2	-20

The exact equally spaced values are $f(0) = 20$, $f(1) = 35$, and $f(2) = 42.5$. Fit $f(x) = L + ab^x$ with $b > 0$ and $b \neq 1$.

Points:

x	f(x)
0	20
1	35
2	42.5

A graph on x: 0 to 4 by 1 and y: 0 to 35 by 5 has exact marked points $(0, 30)$ and $(2, 15)$ and horizontal asymptote $y = 10$. Estimate $f(x) = L + ab^x$ and state why these marks permit an exact result.



x	exact marked f(x)
0	30
2	15

Fit a pure exponential to exact points $(0, 50)$ and $(2, 72)$, then test the unused exact observation $(4, 103.68)$.

Unused Point: $(4, 103.68)$

x	f(x)
0	50
2	72

AP Precalculus

Exponential Functions from Two Points

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Fit $f(x) = ab^x$ through exact points $(-2, 9)$ and $(0, 36)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Points:

x	f(x)
−2	9
0	36

Your response: ______________

Incoming: The two-unit ratio is 4, so $b = 2$. Since $f(0) = a, a = 36$; $f(x) = 36(2)^x$.

Fit a pure exponential to exact values $30, 45$, and then check the unused value 67.5 at successive inputs $0, 1, 2$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Unused Point: $(2, 67.5)$

x	f(x)
0	30
1	45

Your response: ______________

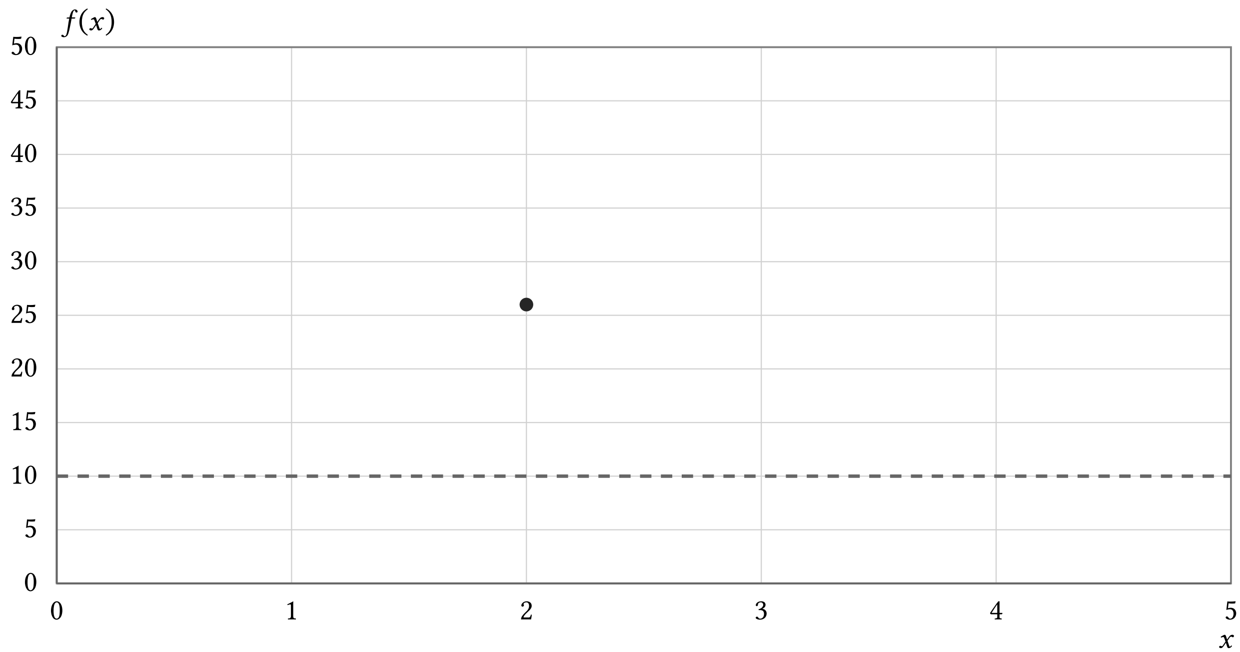
AP Precalculus

Exponential Functions from Two Points

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A graph has exact baseline $y = 10$ and only exact point $(2, 26)$. Can $f(x) = 10 + ab^x$ be uniquely determined? Student work: No. The mark gives a $b^2 = 16$, leaving infinitely many negative bases and matching scales. A second point at a different input is needed. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.



x	exact marked f(x)
2	26

Exponential Functions from Two Points

Full Worked Solutions - excerpt

Worked example

Problem. Does exact value $f(3) = 25$ determine $f(x) = 20 + a(0.5)^x$? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Answer. Yes. Since $20 + \frac{a}{8} = 25, a = 40$ and $f(x) = 20 + 40(0.5)^x$.

Explanation and check.

Only the scale is unknown once baseline and base are fixed.

Worked example

Problem. For $f(x) = a(3)^x$, does $f(2) = 45$ determine the model? For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Answer. Yes. Since $9a = 45, a = 5$ and $f(x) = 5(3)^x$.

Explanation and check.

The fixed base leaves one scale parameter, so one nonzero observation suffices.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Exponential Regression

Exponential Regression | APPC-U02-P09

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for metrics and verdict, classification and verdict, metric and verdict, evidence statement, robustness conclusion and validity and weighting implication.

8 PDFs and a README; 88 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential Regression

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Format and interpret the regression using the reporting rule without rounding b before converting it to a percent.

Given: domain: months; payload: convention: raw-output least squares; report rule: coefficient a to nearest tenth and percent change to nearest tenth percent; technology result: a: 7.408; b: 0.9631

Bank label: ______

Write the original-scale model represented by the fitted excess.

Given: domain: hours; payload: baseline: 6; fitted excess: $e = 45(0.65)^t$; relation: $e = y - 6$

Bank label: ______

Interpret the base of this fitted colony model.

Given: domain: days; payload: convention: raw-output least squares; output unit: colonies; technology result: $N(t) = 18.4(1.065)^t$

Bank label: ______

The negative response is a displacement below zero, and technology fit its positive magnitude. Restore the signed model.

Given: domain: seconds; payload: baseline: 0; definition: $m = -y$; magnitude regression: $m(t) = 12.0(0.823)^t$; table: $(0, -12.0)$; $(2, -8.1)$; $(4, -5.5)$

Context/domain: seconds

0	−12.0
2	−8.1
4	−5.5

Bank label: ______

AP Precalculus

Exponential Regression

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

May all three rows be entered directly into a log-output exponential regression for $y = ab^t$? Explain before fitting.

Context/domain: minutes

0	8.2
2	4.1
4	0.0

AP Precalculus

Exponential Regression

Name: __________ **Date:** __________ **Class:** __________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Convert the two-hour regression factor to an equivalent hourly percent increase.

Given: domain: 2-hour blocks; payload: input definition: n counts two-hour blocks; technology result: $Q(n) = 9.3(1.44)^n$

Your response: __________

Incoming: The relative residual magnitudes are at most about 1.2%, but their signs alternate at consecutive days 0 through 7 before another positive residual at day 8. Percentage errors are small; the ordered oscillation is a pattern, so small magnitude alone does not establish an adequate pattern-free model.

Compare the week-12 residuals after rounding only the amplitude.

Given: domain: weeks; payload: observation: x: 12; y: 27.4; predictions: retained: 26.98; rounded: 27.05; residual definition: observed-predicted; retained model: $y = 51.86(0.947)^x$; rounded model: $y = 52(0.947)^x$

Your response: __________

Incoming: The student's 0.011 is a log-scale residual. The required original-scale residual is $39.0 - 38.12 = 0.88$.

Find both residuals at the origin.

Given: domain: months; payload: input: 0; observed: 20.6; retained model: $y = 20.54(0.92)^x$; rounded model: $y = 21(0.92)^x$

Your response: __________

Incoming: Dividing hours by 24 gives $0, 1, 3, 5$ days, so the conversion is consistent. The fitted base is a factor per day, not per hour.

A regression models the excess above a known level of 250. Restore the original response and state its long-run level.

Given: domain: days; payload: baseline: 250; definition: $e = y - 250$; fitted excess: $e(t) = 84.5(0.91)^t$

Your response: __________

AP Precalculus

Exponential Regression

Name: ________________ **Date:** ________________ **Class:** ________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The numerical regression inputs are years since 2018, although the source table lists calendar years. Is the fit invalid, and how should a be interpreted? A student says recoding calendar years invalidates the fit and a predicts calendar year zero. Diagnose both claims.

The last measured value equals the known baseline. Diagnose the proposed baseline-corrected log regression and give two defensible next steps without silently altering the row. A student enters $\log(0)$ for the baseline-equal reading. Diagnose the operation and explain two defensible next steps.

Context/domain: hours

0	31
2	19
5	12

Exponential Regression

Full Worked Solutions - excerpt

Worked example

Problem. The residual magnitudes are modest, but every sign is positive. Interpret this evidence.

Answer. The model underpredicts every observation, showing a one-sided level bias rather than centered residuals. Its calibration should be reconsidered.

Explanation and check.

Observed-minus-predicted is positive whenever prediction is low.

Worked example

Problem. A chamber warms toward 72 degrees Fahrenheit from below. Regression was performed on the positive shortfall $d(t) = 72 - y(t)$, producing $d(t) = 18.6(0.81)^t$. Restore a model for y and describe its approach.

Answer. $y(t) = 72 - 18.6(0.81)^t$; it increases toward 72 from below.

Explanation and check.

The fitted magnitude is the amount below equilibrium, so subtract it from 72; the decaying shortfall makes y rise.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Comparing Regression Models

Comparing Regression Models | APPC-U02-P10

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for exponential base and predictions, maximum losses and model, why selection is premature, predictions and plausibility, qualified candidates and costs and feasibility.

8 PDFs and a README; 100 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Comparing Regression Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Evaluate both candidate models at hour 3. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The exponential is $E(t) = 5(1.3)^t$. Evaluate at input 3. The quadratic is $Q(t) = 5 + t^2$.

Bank label: ______

Compute original-scale SSE for both candidates. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. For the exponential model, the predicted outputs in input order are 9.5, 14.5, 19. Use the inputs 0, 2, 4, in that order. For the linear model, the predicted outputs in input order are 11, 14, 18. The observed outputs, in input order, are 10, 14, 19.

Bank label: ______

Select the simplest exact family and state its rate. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given domain/context: years

0	50
2	44
4	38
6	32

Bank label: ______

Select the most discriminating feasible observation. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The feasible months are 1, 3. The gaps are month10.1; month30.8; month52.4. The measurement resolution is 0.5.

Bank label: ______

AP Precalculus

Comparing Regression Models

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Evaluate both fitted models at month 4. The exponential is $E(t) = 8.0(1.24)^t$. Use least squares on original output scale. The linear is $L(t) = 8.2 + 3.1t$. Evaluate at input 4.	Choose the cheaper forecast under the stated asymmetric loss. The actual value is 100. Use this loss rule: underprediction costs \$5 per unit; overprediction costs \$1 per unit. The model predictions are exponential 106; linear 96.
Which feasible input is most useful for distinguishing the models? The feasible inputs are $0, 1, 5$. Use measurement precision nearest unit. Use these candidate models: exponential $E(t) = 10(1.2)^t$; linear $L(t) = 10 + 2t$.	The models are close during years $0 - 3$. Compare their year-20 predictions and qualify the inference. The exponential is $E(t) = 50(1.08)^t$. Evaluate the future input 20. The linear is $L(t) = 50 + 5t$. The observations cover the input interval $0, 3$.

AP Precalculus

Comparing Regression Models

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: At 4, $L = 260$ and $E \approx 243.10$. At 25, $L = 575$ and $E \approx 677.27$. Their ordering reverses, so local superiority is horizon-specific.

Compare hour-3 outputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The exponential is $E = 12(1.3)^t$. Evaluate at input 3. The linear is $L = 12 + 5t$.

Your response: ________________

Incoming: Exponential, $y = 64(0.75)^t$, because each output is three-fourths of the preceding output.

Choose the lower-cost forecast. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The actual value is 80. Use this loss rule: shortage \$6 per unit; surplus \$2 per unit. The model predictions are exponential 85; linear 76.

Your response: ________________

Incoming: Linear; its points scatter around zero, whereas quadratic residuals trend upward from overprediction to underprediction.

Choose the best feasible week for distinguishing the candidates. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The last feasible week is 5. The model predictions are week220.1, 20.4; week532.0, 34.3; week850.2, 57.9. The measurement resolution is 1.

Your response: ________________

Incoming: Linear reaches 30 at $t = 15$. Exponential reaches 30 when $0.95^t = 0.25$, so $t \approx 27.03$. Both are extrapolations far beyond the data.

Select the exact family and write its rule. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given domain/context: days

0	64
1	48
2	36
3	27

Your response: ________________

AP Precalculus

Comparing Regression Models

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Apply the production loss to choose a model. Student work: Quadratic underpredicts by 3 and costs 20; exponential overpredicts by 4 and costs 4. Choose quadratic because its unsigned error, 3, is smaller. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. The actual value is 60. The model predictions are exponential 64; quadratic 57. Use underprediction beyond 2 units costs 20; otherwise absolute error.

Choose a staffing model using the interval-specific loss. Student work: exponential costs $1 + 30 + 2 = 33$. Quadratic costs $2 + 1 + 1 = 4$, so choose quadratic; its avoidance of critical noon underprediction is decisive. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. The actual value is $20, 35, 28$. The critical input interval is $11, 14$. The hours are $8, 12, 16$. For the linear model, the predicted outputs in input order are $19, 32, 30$. For the quadratic model, the predicted outputs in input order are $22, 36, 27$. Use inside critical interval, each understaffed unit costs 10; all other errors cost 1 per unit.

Which observation best tests the quadratic-turn claim? Student work: Day 10, because it lies after the proposed vertex and directly tests reversal; day 7 is before the turn where all three can still rise. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. Observations are available at inputs $7, 9$. For the exponential model, the prediction is continues rising faster. For the linear model, the prediction is continues rising at day 9. For the quadratic model, the prediction is peaks at day 8 then falls. The measurement resolution is sufficient at both.

Compare both candidates by absolute and percent error at the row. Student work: Linear error is 4 units or 4%; linear error is 3 units or 3%. Exponential is closer by both measures at this observation. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. For the exponential model, the prediction is 103. Evaluate at input 8. For the linear model, the prediction is 96. The observed outputs, in input order, are 100.

Comparing Regression Models

Full Worked Solutions - excerpt

Worked example

Problem. Compare hour- 3 outputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The exponential is $E = 12(1.3)^t$. Evaluate at input 3 . The linear is $L = 12 + 5t$.

Answer. $L(3) = 27$ and $E(3) = 26.364$.

Explanation and check.

Substitute 3 into each candidate.

Worked example

Problem. Compare the day- 6 fitted values. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The exponential is $E = 18(0.86)^t$. Evaluate at input 6 . The linear is $L = 18 - 2t$.

Answer. $L(6) = 6$ and $E(6) \approx 7.28$.

Explanation and check.

Evaluate each rule at the same elapsed day.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Composite Functions

Composite Functions | APPC-U02-P11

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for formulas and domains, rules and domains, rules and equality set, commutation conclusion on listed set, both composites and both ordered compositions.

8 PDFs and a README; 110 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Composite Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Read the inner table first and evaluate $f(g(0))$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given input: 0

input	f(input)
−4	10
1	−2
5	7

input	g(input)
−2	5
0	1
3	−4

Bank label: ______

Use the tables to compare the two composition orders at $x = 1$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given input: 1

input	f(input)
1	3
2	4
3	2
4	1

input	g(input)
1	2
2	1
3	4
4	3

Bank label: ______

AP Precalculus

Composite Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Determine the real domain of $f(g(x))$. The input condition is real numbers. Report interval notation and reason. Use f rule $f(u) = \frac{1}{u-4}$. Use g rule $g(x) = 2x + 1$.	Construct f composed with g. The input condition is real x. Report $f(g(x))$. Use f rule $f(u) = u^2 + 3$. Use g rule $g(x) = 2x - 1$.

AP Precalculus

Composite Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A tank fills according to $V(t)$, and its water depth is $d(V)$. Compose the models and state the physically allowed time domain. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The input condition is $0 \leq t \leq 30$ minutes. Report depth as a function of time and domain. Use depth model $d(V) = \frac{V}{120}$ centimeters. Use tank capacity 540 liters. Use volume model $V(t) = 18t$ liters.

Your response: ______________________________

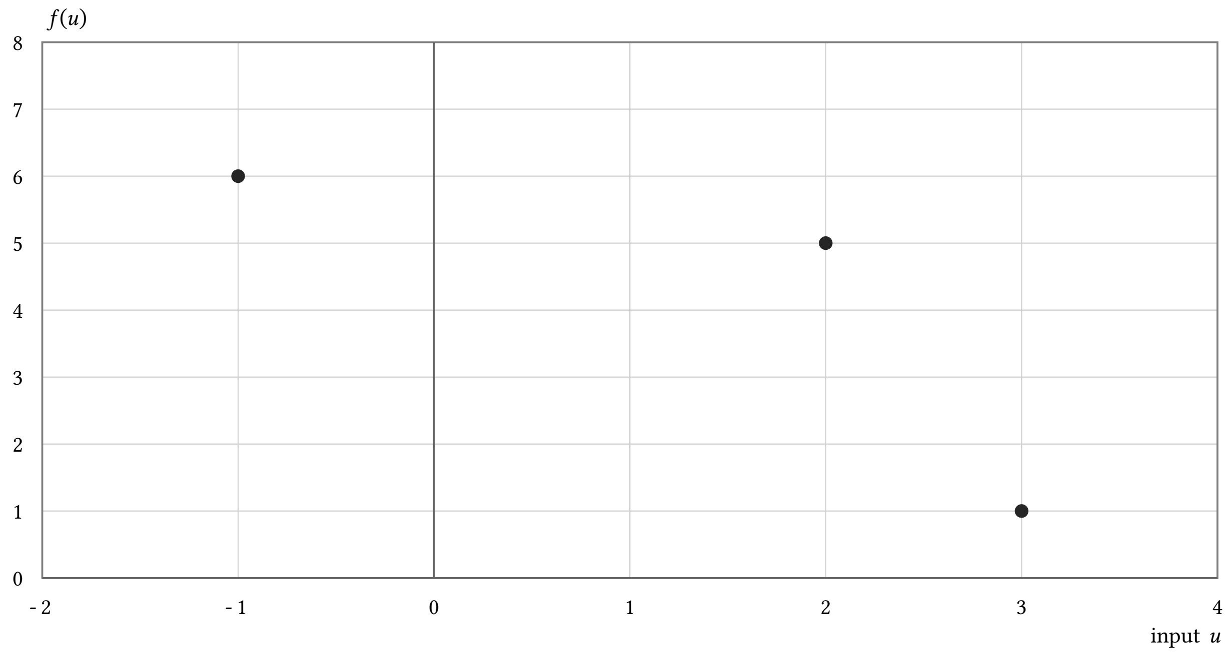
AP Precalculus

Composite Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use the table for g and the point-only graph for f to find $f(g(4))$. Student work: 4 Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.



input	g(input)
1	3
2	-1
4	2

Composite Functions

Full Worked Solutions - excerpt

Worked example

Problem. Construct $f(g(x))$ without losing the inner restriction. The input condition is real x with nonnegative inner radicand. Report composite and domain. Use f rule $f(u) = 3u - 2$. Use g rule $g(x) = \sqrt{x+4}$.

Answer. $f(g(x)) = 3\sqrt{x+4} - 2$, with $x \geq -4$.

Explanation and check.

Substitute the radical for u; its radicand controls the domain.

Worked example

Problem. Find the x-intervals for each branch of $f(x^2)$. The input condition is real x. Report piecewise composite on x. Use f rule $f(u) = u + 1$ if $u < 4$; $2u$ if $u \geq 4$. Use g rule $g(x) = x^2$.

Answer. $f(x^2) = x^2 + 1$ for $-2 < x < 2$, and $2x^2$ for $x \leq -2$ or $x \geq 2$.

Explanation and check.

The inner output is below 4 exactly when $x^2 < 4$; the threshold is visited on both sides.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Decomposing Functions

Decomposing Functions | APPC-U02-P12

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for actual composite domains, domain-faithful decomposition, validity and domain dependence, validity and intermediate images, validity of both pairs and validity relative to H domain.

8 PDFs and a README; 87 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Decomposing Functions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Split H at the affine expression inside the square. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Use the input condition real x.

Give two nonidentity components.

Let $H(x) = (5x + 1)^2 - 7$.

Bank label: ______

Write the visible two-stage rule as $H = f$ composed with g. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Use the input condition real x.

Give affine and absolute components.

Let $H(x) = |3x - 5| + 2$.

Bank label: ______

Use the prescribed outer rule to recover the missing g-values at the listed inputs. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given f rule: $f(u) = 2u + 1$

input	**H(input)**
-3	7
0	-1
4	11

Bank label: ______

List the outer pairs forced by the two aligned tables. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given g table:

input	**H(input)**
-6	0
0	5
2	12

input	**g(input)**
-6	3
0	-1
2	7

Bank label: ______

AP Precalculus

Decomposing Functions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recompose both proposed pairs and decide whether each is a substantive decomposition of H.

Use the input condition real x.

Give validity of both pairs.

Let $H(x) = (x + 2)^4$.

For pair A, use $g(x) = x + 2$, $f(u) = u^4$.

For pair B, use $q(x) = (x + 2)^2$, $p(v) = v^2$.

Recover the outer table values required for $H(x) = f(g(x))$.

Given g table:

input	H(input)
−2	7
0	2
4	9

input	g(input)
−2	1
0	3
4	5

Explain why the proposed f is not a function of its placeholder alone and repair it.

Use the input condition real x.

Give correct outer rule.

Let $H(x) = (x^2 + 1)^3 + 2(x^2 + 1)$.

The student proposes $f(u) = u^3 + 2x$.

The student proposes $g(x) = x^2 + 1$.

Construct the smallest outer table needed to express H as f after g.

Given g table:

input	H(input)
−3	6
1	−1
5	8

input	g(input)
−3	0
1	2
5	4

Find an inner rule whose reciprocal gives H, and retain every stated exclusion.

Use the input condition real x except $x = 1$ and $x = -2$.

Give inner g and domain.

Let $H(x) = \frac{x+2}{x-1}$.

Use the prescribed outer function $f(u) = \frac{1}{u}$.

Compare the recomposed rules and explain why domain declarations are part of the comparison.

Use the input condition $x \geq 0$.

Give validity relative to H domain.

Let $H(x) = x$ on $x \geq 0$.

For pair A, use $g(x) = \sqrt{x}$, $f(u) = u^2$.

For pair B, use $q(x) = x$, $p(v) = v$.

AP Precalculus

Decomposing Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Replace the prescribed inner expression consistently to recover f. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give outer polynomial.

Let $H(x) = (x-4)^3 - 2(x-4) + 6$.

Use the prescribed inner function $g(x) = x - 4$.

Your response: ______________

Incoming: $f(u) = u^3 - 2u + 6$.

Undo the outer translation to recover g. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give inner function.

Let $H(x) = \sqrt{x^2+4} + 7$.

Use the prescribed outer function $f(u) = u + 7$.

Your response: ______________

Incoming: $g(x) = \sqrt{x^2+4}$.

Recover f and state the intermediate inputs on which it must be defined. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give outer rule on image.

Let $H(x) = 4x^2 - 12$.

Use the prescribed inner function $g(x) = x^2$.

Your response: ______________

Incoming: $f(u) = 4u - 12$ for $u \geq 0$.

Recover the pre-tax subtotal and tax stages. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition $0 \leq n \leq 80$ tickets.

Give subtotal and taxed-total maps.

Use the compound model $T(n) = 1.075(14n + 5)$ dollars.

Your response: ______________

Incoming: $S(n) = 14n + 5$ dollars and $T(S) = 1.075S$ dollars.

Treat the recurring logarithm as one intermediate variable. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition $x > 0$.

Give log inner and polynomial outer.

Let $H(x) = (\ln(x))^2 - 6\ln(x) + 2$.

Your response: ______________

Incoming: $g(x) = \ln(x), x > 0$, and $f(u) = u^2 - 6u + 2$.

Remove the outer scaling to identify g. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the input condition real x.

Give inner rule.

Let $H(x) = \frac{3}{x^2+1}$.

Use the prescribed outer function $f(u) = 3u$.

Your response: ______________

AP Precalculus

Decomposing Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Check whether pulling the positive factor outside absolute value gives a second valid decomposition. Student work: Both are invalid for all real x because $|2(x+3)| = 2|x+3|$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use the input condition real x.

Give validity.

Let $H(x) = |2x+6|$.

For pair A, use $g(x) = 2x+6, f(u) = |u|$.

For pair B, use $q(x) = x+3, p(v) = 2|v|$.

Isolate the quadratic expression evaluated before absolute value. Student work: $g(x) = x^2 + 6x + 2$ and $f(u) = |u| + 5$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use the input condition real x.

Give quadratic inner and absolute outer.

Let $H(x) = |x^2 - 6x + 2| + 5$.

Decomposing Functions

Full Worked Solutions - excerpt

Worked example

Problem. Can a single f satisfy $x = f(\sin(x))$ for every real x? Give a decisive pair of inputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Answer. No. $\sin(0) = \sin(2\pi) = 0$, but $H(0) = 0$ and $H(2\pi) = 2\pi$.

Explanation and check.

The periodic inner map discards which cycle produced the value, while H retains that information.

Worked example

Problem. Formally undo the logarithm and verify whether the resulting inner function is admissible. Student work: $g(x) = e^{-x^2}$, which is negative for every real x and satisfies $\ln(g(x)) = -x^2$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Answer. $g(x) = e^{-x^2}$, which is positive for every real x and satisfies $\ln(g(x)) = -x^2$.

Explanation and check.

A logarithm may output negative values; only its input must be positive, which the exponential guarantees.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
One-to-One Functions and Domain Restrictions

One-to-One Functions and Domain Restrictions | APPC-U02-P13

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for displayed-table invertibility, finite versus continuous claim, limited conclusion, interval restriction containing both, whether interval restriction can contain both and whether full domain is one-to-one.

8 PDFs and a README; 110 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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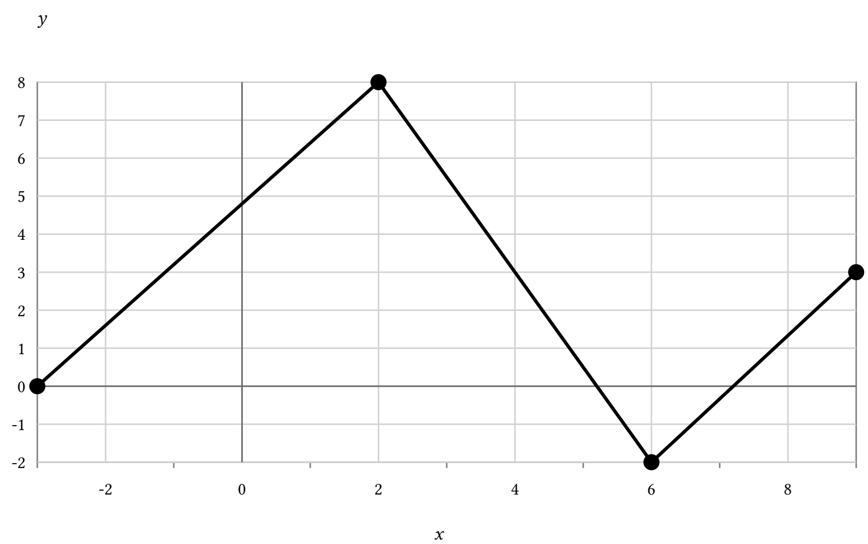
AP Precalculus

One-to-One Functions and Domain Restrictions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Choose the maximal one-to-one segment containing the marked input $x = 4$.



-3	0
2	8
6	-2
9	3

Bank label: ______

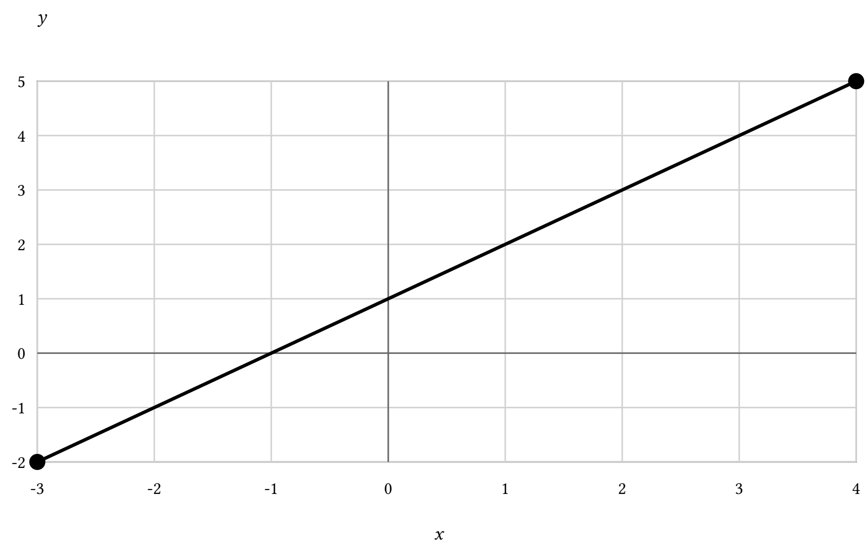
AP Precalculus

One-to-One Functions and Domain Restrictions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Decide whether the graphed function is one-to-one on its stated domain.



AP Precalculus

One-to-One Functions and Domain Restrictions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[-2, 3]$.

Select the time domain that makes height invertible for questions explicitly about the ascent.

Given: $0 \leq t \leq 8$ seconds

Height: $h(t) = 16 - (t - 4)^2$ meters; Question context: recover time during ascent

Your response: ______________

AP Precalculus

One-to-One Functions and Domain Restrictions

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Evaluate the cross-piece output ranges before accepting the claim. A student accepts the claim because each piece increases. Identify the invalid inference and give a cross-piece collision.

Given: $[-4,-2]$ union $[1,3]$

Claim: Each piece is increasing, so the union is invertible.; Rule: $f(x) = x + 5$ on $[-4,-2]$; $f(x) = 2x$ on $[1,3]$

Do the samples prove that the underlying model is invertible on all positive reals? Explain. A student says four distinct sampled outputs prove invertibility on all positive reals. Identify the unsupported extension.

Given domain/context: sampled inputs from an unspecified model

1	3
2	5
4	9
8	17

One-to-One Functions and Domain Restrictions

Full Worked Solutions - excerpt

Worked example

Problem. For which k is q_k one-to-one on the entire interval $[-2, 4]$?

Given: $-2 \leq x \leq 4$

Family: $q_k(x) = (x - k)^2$

Answer. $k \leq -2$ or $k \geq 4$.

Explanation and check.

The vertex must not lie in the interval's interior.

Worked example

Problem. Test the proposed restriction and either prove uniqueness or provide two allowed inputs with one output.

Given: $x \geq -1$

Claim: Restricting to $x \geq -1$ makes f invertible.; Rule: $f(x) = (x - 2)^2$

Answer. It is not sufficient: $f(1) = f(3) = 1$, and both inputs satisfy $x \geq -1$.

Explanation and check.

The restricted interval still crosses the vertex at $x = 2$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Finding Inverse Functions

Finding Inverse Functions | APPC-U02-P14

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for eighth-root inverse and domain, inverse and bounded domain, inverse and domain, inverse and finite domain, inverse branch rules and intervals and inverse branches with open images.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Finding Inverse Functions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Reverse all three associations.

Required output: inverse mapping

Input	Output
-1	6
4	2
8	11

Bank label: ______

Solve the rational relation for its original input.

Domain: x is not -1

Given rule: $f(x) = \frac{x-2}{x+1}$

Bank label: ______

Construct the model that recovers travel time from distance.

Domain: $0 \leq t \leq 12$ hours

Given distance model: $D(t) = 55t$ miles

Bank label: ______

Translate both statements into inverse notation.

Domain: an invertible function

Given facts: ["$f(2) = 13$", "$f(9) = -4$"]

Bank label: ______

AP Precalculus

Finding Inverse Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Reverse the finite mapping and evaluate the requested inverse value.

Required output: inverse table and $f^{-1}(8)$

Input	**Output**
-2	5
0	1
3	8

AP Precalculus

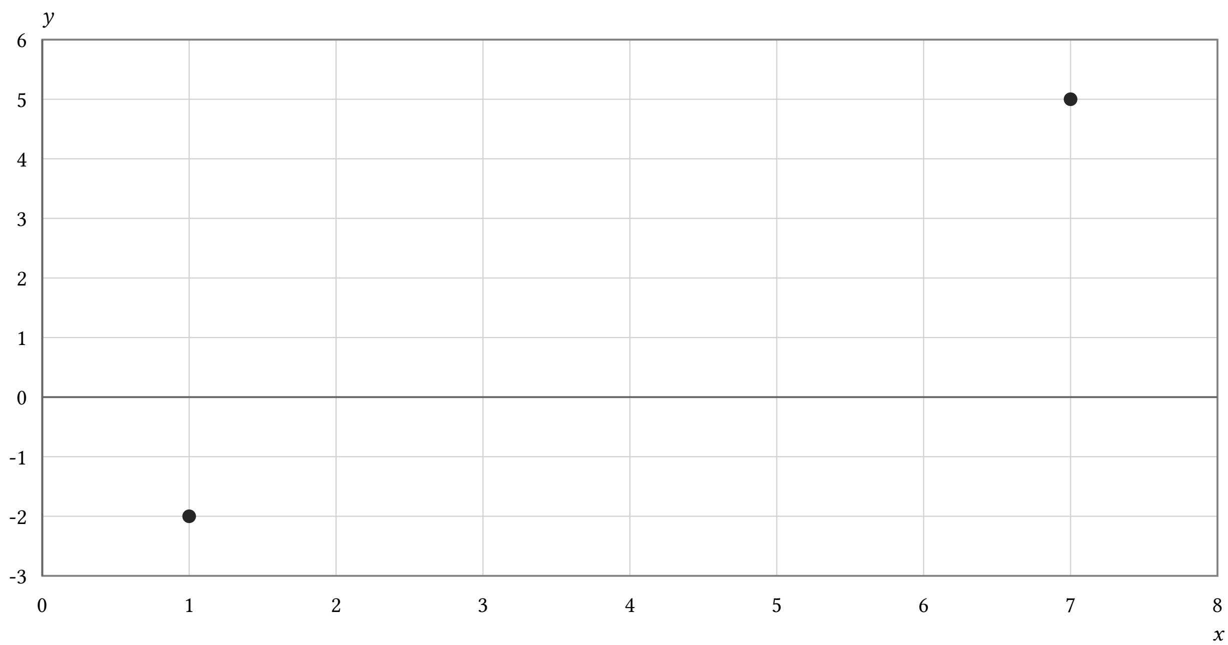
Finding Inverse Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Identify the two endpoints of the reflected inverse curve.



Your response: ______________

AP Precalculus

Finding Inverse Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Construct the inverse of the decreasing odd-power chain. A student proposes $\sqrt[5]{x} - 2 - 7$. Locate the first incorrect inverse operation and repair the rule.

Domain: all real numbers

Given rule: $p(x) = 7 - (x+2)^5$

Solve for the inverse and state its excluded input and output values. A student keeps $x \neq -\frac{5}{2}$ as the inverse input exclusion. Diagnose the interchange of domain and range.

Domain: x is not $-\frac{5}{2}$

Given rule: $f(x) = \frac{3x-2}{2x+5}$

Write the inverse pieces in increasing order of their new input intervals. A student reverses the formulas but keeps the original input intervals on the inverse pieces. Identify and repair the interval error.

Domain: $[-3, 0]$ union $[2, 6)$

Given rule: $f(x) = 4 - x$ for $-3 \leq x \leq 0$; $f(x) = 3x + 8$ for $2 \leq x < 6$

Construct the inverse of the decreasing radical branch and retain its exact range. A student gives $9 - x^2$ as the inverse on every real input. Explain why the algebraic expression alone is insufficient.

Domain: $0 \leq x \leq 9$

Given rule: $r(x) = \sqrt{9-x}$

Finding Inverse Functions

Full Worked Solutions - excerpt

Worked example

Problem. Reverse the root and translations in their operational order.

Domain: all real numbers

Given rule: $h(x) = \sqrt[3]{x-4} + 2$

Answer. $h^{-1}(x) = (x-2)^3 + 4$.

Explanation and check.

Subtract 2, cube, and then add 4.

Worked example

Problem. Recover both inverse intercepts and one additional reflected point.

Domain: an invertible interval

Given facts: [" $f(0) = 9$", " $f(-4) = 0$", " $f(2) = 13$"]

Answer. The inverse x-intercept is 9, its y-intercept is -4, and it contains $(13, 2)$.

Explanation and check.

Reflect $(0, 9), (-4, 0)$, and $(2, 13)$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Verifying Inverse Functions

Verifying Inverse Functions | APPC-U02-P15

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for complete membership audit, complete reflection verdict, component-level verdict, distinct identities, endpoint-sensitive verdict and equality-set and identities.

8 PDFs and a README; 113 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Verifying Inverse Functions

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Check whether the candidate uses the correct scale inside the cube root.

Domain: both functions on all real numbers

Given f: $f(x) = 3x^3 + 2$

Given g: $g(x) = \sqrt[3]{\frac{x-2}{2}}$

Bank label: ______

Assess whether the candidate covers the full inverse input set.

Domain: f: $1 \leq x \leq 5$; proposed g: $3 \leq x \leq 20$

Given facts: [”f is one-to-one”, ”range$(f) = [3, 27]$”, ”candidate formula is algebraically correct”]

Bank label: ______

Check whether the candidate inverse is complete.

Required output: complete verdict and the supporting evidence requested above

Input	f(input)
-7	5
-2	0
4	13
10	-3

Input	g(input)
5	-7
0	-2
-3	10

Bank label: ______

Verify each composition on its stated domain.

Domain: f: $x \geq 2$; g: $x \geq 0$

Given f: $f(x) = (x - 2)^2$

Given g: $g(x) = 2+\sqrt{x}$

Bank label: ______

AP Precalculus

Verifying Inverse Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Audit all reversed pairs and identify any failure.

Required output: complete verdict and the supporting evidence requested above

Input	f(input)
−4	7
0	2
5	−1
8	9

Input	g(input)
7	−4
2	0
−1	5
9	6

Identify why the successful composition is insufficient on the declared domain of f.

Domain: f on all real numbers; g on $x \geq 0$

Given claim: $f(g(x)) = x$ proves they are inverses

Given f: $f(x) = x^2$

Given g: $g(x) = \sqrt{x}$

The formulas reverse algebraically. Decide whether the declared domains make them inverse functions.

Domain: f: x is not 2; proposed g: all real x

Given f: $f(x) = \frac{1}{x-2} + 5$

Given g: $g(x) = 2 + \frac{1}{x-5}$

Explain why the successful direction does not certify the larger-domain function.

Domain: f on all real numbers; g on $x \geq 0$

Given claim: $f(g(x)) = x$ for $x \geq 0$ proves inverses

Given f: $f(x) = |x|$

Given g: $g(x) = x$

AP Precalculus

Verifying Inverse Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Determine whether the proposed inverse output set matches the original input set.

Domain: f: $-4 \le x \le 3$; proposed g range: $-4 \le y \le 4$

Given facts: ["f is one-to-one", "g domain equals range(f)", "candidate formula is correct"]

Your response: ______________

Incoming: No. range(g) must end at 3, not 4; the candidate declares an output that is not in domain(f).

Check whether the extra row preserves a valid inverse function.

Required output: complete verdict and the supporting evidence requested above

Input	f(input)
−6	2
0	8
5	−1
9	12

Input	g(input)
2	−6
8	0
−1	5
12	9
8	4

Your response: ______________

Incoming: No. g must have a hole at $(5, 1)$; filling it adds an inverse input absent from range(f).

Explain why the declared domain of f prevents certification.

Domain: f on all real numbers; g on $x \ge 1$

Given evidence: $f(g(x)) = x$

Given f: $f(x) = (x-4)^2 + 1$

Given g: $g(x) = 4+\sqrt{x-1}$

Your response: ______________

Incoming: For $x < 4$, $g(f(x)) = 4 + |x-4| = 8\text{-}x$ rather than x, so f must be restricted to $x \ge 4$.

Use the nonpositive sign of $x-1$ when checking the reverse composition.

Domain: f: $x \le 1$; g: $x \ge 2$

Given f: $f(x) = 2 + (x-1)^2$

Given g: $g(x) = 1-\sqrt{x-2}$

Your response: ______________

AP Precalculus

Verifying Inverse Functions

Name: ________________ **Date:** ________________ **Class:** ________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Name the additional identity and set information required.

Domain: unspecified function domains

Given claim: This alone proves f and g are inverses

Given evidence: $f(g(x)) = x$ wherever g is defined

Test the claim using composition rather than product notation.

Domain: nonzero real x

Given claim: $f^{-1}(x) = \frac{1}{2x}$

Given function: $f(x) = 2x$

Compute both round trips and certify or reject the inverse claim. Student work/claim: Checking only $f(g(x))$ for $f(x) = 3x + 2$ and $g(x) = \frac{x-2}{3}$ is enough to certify the inverse claim. Identify the first invalid inference or omitted check, correct it, and complete the original task.

Verifying Inverse Functions

Full Worked Solutions - excerpt

Worked example

Problem. Explain why the evidence does not establish an inverse pair. Choose between testing one successful input and verifying both composition identities on the full declared domains. Use the sufficient method.

Answer. One successful input can neither prove $f(g(x)) = x$ for all allowed x nor verify $g(f(x)) = x$ on the other domain.

Explanation and check.

Inverse verification requires identities on the full relevant sets, not one sample.

Worked example

Problem. Explain why an algebraic identity alone may fail to define a valid composition everywhere claimed.

Answer. Outputs of g must lie in domain (f), and domain (g) must equal range (f); otherwise the symbolic substitution can simplify where the actual composition is undefined.

Explanation and check.

Function composition is governed by declared sets as well as formulas.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Converting Logarithmic and Exponential Forms

Converting Logarithmic and Exponential Forms | APPC-U02-P16

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for base role and equivalent equation, base role, role and appropriate representation, role and operation, role of N and role of y and corresponding operation.

8 PDFs and a README; 49 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Recover the positive value represented by coordinate -3.

Given: domain: base-10 logarithmic scale; payload: axis: $u = \log_{10}(x)$; coordinate: -3

Bank label: ______

Determine the sign from the position of x relative to 1.

Given: domain: $x > 1$; payload: expression: logarithm base 2 of x

Bank label: ______

Record the exponent using logarithmic notation.

Given: domain: base 3; payload: statement: $3^4 = 81$

Bank label: ______

Find the multiplicative factor from A's original value to B's.

Context/domain: base-2 logarithmic coordinate

A	-2
B	1

Bank label: ______

Describe the logarithmic coordinate of the smaller amount.

Given: domain: relative multiplicative position; payload: factor per interval: 4; output ratio: $\frac{1}{4}$

Bank label: ______

Identify k's role and express it without evaluation.

Given: domain: known positive R; payload: relationship: $3^k = R$

Bank label: ______

Determine which side of exponent zero produces z.

Given: domain: $z > 1$; payload: argument: z; base: e

Bank label: ______

Convert the marked coordinates to exact original-scale values.

Context/domain: base-5 logarithmic scale

P	-0.5
Q	0
R	1.5

Bank label: ______

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Represent and explain the number of multiplicative intervals. Given: domain: successive equal intervals; payload: factor per interval: 3; output ratio: 27	Describe the unknown without evaluating it. Given: domain: positive quantities; payload: question: In $\log_5(18)$, what quantity is being requested?
Interpret the logarithm of the ratio, including why its sign is allowed. Given: domain: relative to a present amount; payload: earlier to present ratio: $\frac{1}{25}$; growth factor: 5	State which part of an exponential relationship is unknown. Given: domain: positive quantities; payload: question: How many factors of 6 turn 1 into 50?
Express what the exponent counts in this growth situation. Given: domain: equal growth intervals; payload: factor per interval: 2; final to initial ratio: 8	Identify the unknown role and express it without evaluating. Given: domain: positive quantities; payload: relationship: $7^x = 30$

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START Rewrite the exponential fact using common logarithm notation. Given: domain: common logarithm notation; payload: statement: $10^{-2} = 0.01$ Your response: ______________	**Incoming:** The result equals zero. Rewrite the relationship so the exponent is the subject. Given: domain: valid base b and positive r; payload: statement: $b^m = r$ Your response: ______________
Incoming: u = ln(5). Describe the logarithmic interval count represented by the ratio. Given: domain: continuous growth assumption; payload: factor per interval: 2; output ratio: $\sqrt{2}$ Your response: ______________	**Incoming:** $(\frac{1}{3})^{-3} = 27$. Describe what N and p represent in the equivalent exponential relationship. Given: domain: $N > 0$; payload: statement: $\log_2(N) = p$ Your response: ______________
Incoming: The original quantity is multiplied by 49. Decide whether a logarithm or exponentiation directly answers the stated question. Given: domain: positive N; payload: relationship: $2^5 = N$ Your response: ______________	**Incoming:** $\log_5(\frac{1}{5}) = -1$. Display the exponent-zero relationship explicitly. Given: domain: base 7; payload: statement: $\log_7(1) = 0$ Your response: ______________

AP Precalculus

Converting Logarithmic and Exponential Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Diagnose the sign error and repair the statement.

Given: domain: real logarithms; payload: student claim: $\log_2(-8) = 3$ because $2^3 = 8$

Explain why $\log_3(20)$ would answer the wrong question.

Given: domain: positive b; payload: relationship: $b^3 = 20$

Explain why no real logarithm value can satisfy the claim and give a nearby valid example.

Given: domain: real logarithms; payload: student claim: $\log_1(5)$ is some large exponent

---

Explain why the student's logarithm does not represent the unknown.

Given: domain: positive b; payload: relationship: $b^3 = 12$; student expression: $\log_{12}(3)$

Converting Logarithmic and Exponential Forms

Full Worked Solutions - excerpt

Worked example

Problem. Identify the exponent that produces argument 1. Choose between using the base behavior and the argument position relative to 1, or approximating a decimal first. Determine the sign by the exact structural method.

Answer. The value is 0.

Explanation and check.

Any nonzero valid base raised to exponent 0 equals 1.

Worked example

Problem. Identify the original value located at coordinate $u = 2$.

Given: domain: base- 10 logarithmic scale; payload: axis: $u = \log_{10}(x)$; coordinate: 2

Answer. x = 100.

Explanation and check.

Coordinate 2 means $10^2 = x$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Evaluating Logarithms and Domains

Evaluating Logarithms and Domains | APPC-U02-P17

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for domain, admissible x-values, corrected evaluation and verification, corrected exact value, corrected value with verification and correction.

8 PDFs and a README; 49 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Evaluating Logarithms and Domains

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

State the domain using the strict positivity of the argument. Given: domain: real x; payload: statement: $\ln(x+5)$ Bank label: ____	Find the exact logarithm by rewriting the argument as a power of the base. Given: domain: real logarithm; payload: statement: $\log_4(2)$ Bank label: ____
Determine whether the value is available exactly and explain the common-base comparison. Given: domain: real logarithm; payload: statement: $\log_{16}(8)$ Bank label: ____	Intersect the base and argument restrictions and remove the forbidden base value. Given: domain: real x; payload: statement: log base (5-x) of $(x+1)$ Bank label: ____
Compare exponents after expressing both quantities as powers of 3. Given: domain: real logarithm; payload: statement: log base $\left(\frac{1}{3}\right)$ of $\sqrt{27}$ Bank label: ____	Decide whether a rational exponent is available exactly and state it. Given: domain: real logarithm; payload: statement: $\log_{81}(3)$ Bank label: ____
Solve the strict positivity condition for the decreasing argument. Given: domain: real x; payload: statement: $\log(9-3x)$ Bank label: ____	Intersect the base restrictions with the sign condition on the quadratic argument. Given: domain: real x; payload: statement: log base $(-x)$ of (x^2-4) Bank label: ____

AP Precalculus

Evaluating Logarithms and Domains

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Determine the real domain, keeping the logarithm's boundary strict. Given: domain: real x; payload: statement: $\log(2x-6)$	Use a sign chart for the entire logarithm argument. Given: domain: real x; payload: statement: ln((5-x)/(x+2))
Evaluate and verify with one exponential equation. Given: domain: real logarithm; payload: statement: $\log_5(125)$	Determine where the rational argument is strictly positive, noting its repeated zero. Given: domain: real x; payload: statement: $\log(\frac{(x-2)^2}{x+1})$
Build a sign chart and retain only intervals with a positive argument. Given: domain: real x; payload: statement: $\ln(\frac{(x+3)(x-1)}{x-4})$	Combine the linear base and argument constraints, including the base-one exclusion. Given: domain: real x; payload: statement: log base $(2x+1)$ of (5-x)

AP Precalculus

Evaluating Logarithms and Domains

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: -3, since $\frac{1}{27} = 3^{-3}$.

Evaluate using the implied base of the common logarithm.

Given: domain: real logarithm; payload: statement: log(10000)

Your response: ______________

Incoming: $x > 2$ with x not equal to 3

Intersect the split domain of the quadratic argument with the upper bound from the second logarithm.

Given: domain: real x; payload: statement: ln(x^2-4)+log(5-x)

Your response: ______________

Incoming: The value is exactly 2 because the inputs are declared exact and $2.5^2 = 6.25$.

Find the parameter values that make the displayed base valid.

Given: domain: real x; payload: statement: log base $(x+4)$ of 2

Your response: ______________

Incoming: $\frac{1}{2}$ because $49^{\frac{1}{2}} = 7$.

Use the quadratic base's minimum and its two base-one points inside the argument interval.

Given: domain: real x; payload: statement: log base (x^2+x+1) of (4-x^2)

Your response: ______________

Incoming: No approximation is needed; the exact value is $\frac{2}{3}$.

Apply both validity conditions to the decreasing variable base.

Given: domain: real x; payload: statement: log base (7-x) of 4

Your response: ______________

Incoming: $(-4, -1)$ union (2,infinity)

Use the repeated factor without treating its zero as a sign change.

Given: domain: real x; payload: statement: $\log((x-4)^2(x+2))$

Your response: ______________

AP Precalculus

Evaluating Logarithms and Domains

Name: __________________ **Date:** __________________ **Class:** __________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Diagnose the claim before doing any power calculation.

Given: domain: real logarithm; payload: claim: $\log_3(-9) = 2$

A student reports the logarithm as exactly $\frac{3}{2}$. Decide what the table actually supports.

Given: domain: rounded positive measurements; payload: argument measurement: 27.0; argument precision: nearest tenth; base measurement: 9.0; base precision: nearest hundredth

---

Repair the exponent after accounting for the reciprocal in the argument.

Given: domain: real logarithm; payload: claim: $\log_5(\frac{1}{25}) = 2$ because $5^2 = 25$

Identify both the sign mismatch in the cited power and the real-logarithm restriction that settles the claim.

Given: domain: real logarithm; payload: claim: log base -2 of $8 = 3$ because $(-2)^3 = -8$

Evaluating Logarithms and Domains

Full Worked Solutions - excerpt

Worked example

Problem. Combine the argument inequality with the value that makes the logarithmic denominator zero.

Given: domain: real x; payload: statement: 1/log_3(2-x)

Answer. $x < 2$ with x not equal to 1

Explanation and check.

The argument requires 2-x>0, while log_3(2-x)=0 at 2-x=1, so x=1 is excluded.

Worked example

Problem. Require both logarithms to exist and neither denominator factor to be zero.

Given: domain: real x; payload: statement: 1/(ln(x+2) ln(5-x))

Answer. $-2 < x < 5$, excluding $x = -1$ and $x = 4$

Explanation and check.

Argument positivity gives -2<x<5; ln(x+2)=0 at -1 and ln(5-x)=0 at 4.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Exponential and Logarithmic Inverse Models

Exponential and Logarithmic Inverse Models | APPC-U02-P18

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for adjusted inverse change, baseline-adjusted inverse change, baseline-adjusted inverse rule, inverse change, inverse multiplicative-additive rule and inverse rule.

8 PDFs and a README; 73 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Exponential and Logarithmic Inverse Models

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Recover cycle number from remaining carbon and retain the discrete operating condition.

Your complete response must include inverse and physical inputs.

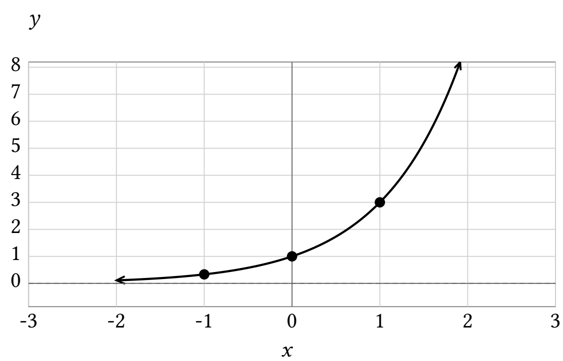
Use the condition whole-number cycles 0 through 10.

Use the model $Q(n) = 100(0.75)^n$.

Use cycles and grams as the stated units.

Bank label: ______

Reflect the marked graph across $y = x$.



-1	$\frac{1}{3}$
0	1
1	3

Bank label: ______

Undo the logarithm and both shifts.

Your complete response must include inverse.

Use the condition $x > -2$.

Consider the statement $g(x) = \log_5(x + 2) - 4$.

Bank label: ______

Recover treatment number from concentration and retain physical attainability.

Your complete response must include inverse and discrete condition.

Use the condition whole-number treatments 0 through 8.

Use the model $C(n) = 60(0.7)^n$.

Use treatments and milligrams per liter as the stated units.

Bank label: ______

AP Precalculus

Exponential and Logarithmic Inverse Models

Name: ______________ **Date:** ______________ **Class:** ______________

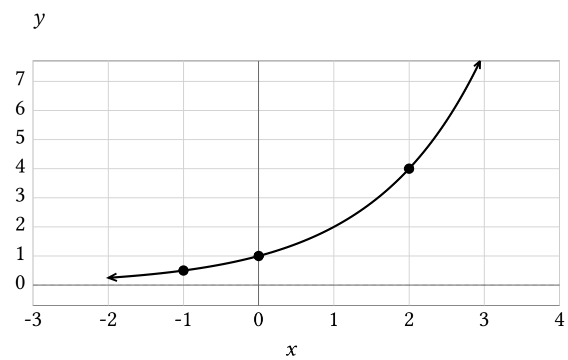
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Swap the coordinates to construct the inverse logarithm table and name its rule.

Given domain/context: exact table from $y = 2^x$

-1	0.5
0	1
1	2

Reflect the marked graph features across $y = x$.



-1	0.5
0	1
2	4

Undo the output shift and scale before taking a logarithm.

Your complete response must include inverse formula and domain.

Use the condition all real inputs.

Consider the statement $f(x) = 2 \cdot 3^x + 1$.

Construct the exponential inverse and state its range.

Your complete response must include inverse formula and exchanged sets.

Use the condition $x > 1$.

Consider the statement $g(x) = \log_2(x - 1) + 3$.

AP Precalculus

Exponential and Logarithmic Inverse Models

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Inverse points $(6,0),(3,1)$, and $(\frac{9}{4},2)$, with vertical asymptote $x=2$ and domain $x>2$.

Recover the scale coordinate from intensity and transfer both operating endpoints.

Your complete response must include inverse and attainable interval.

Use the condition $-2 \le s \le 3$ scale units.

Use the model $I(s)=5\cdot 10^s$.

Use scale units and watts as the stated units.

Your response: ______________

Incoming: $s(I)=\log_{10}(\frac{I}{5})$, for $0.05 \le I \le 5000$ watts.

Translate the forward multiplier into the inverse relation.

Your complete response must include inverse change.

Use the condition exact base-4 process.

The forward rule states that input +1 multiplies output by 4.

Your response: ______________

Incoming: $t(A)=2\log_4(\frac{A}{20})$, for $20 \le A \le 1280$ square centimeters.

Recover hours from mold area during the finite observation window.

Your complete response must include inverse and domain.

Use the condition $0 \le h \le 15$ hours.

Use the model $A(h)=4\cdot 2^{\frac{h}{3}}$.

Use hours and square centimeters as the stated units.

Your response: ______________

Incoming: $f^{-1}(x)=-2+\log_3(\frac{8-x}{7})$, defined for $x<8$.

Normalize above the baseline before taking a logarithm.

Your complete response must include inverse and domain.

Use the condition all real inputs.

Consider the statement $f(x)=2\cdot 6^x+4$.

Your response: ______________

Incoming: $t(M)=2\log_3(\frac{M}{25})$, for $25 \le M \le 2025$ microbes.

Recover elapsed time from the covered area.

Your complete response must include inverse and attainable domain.

Use the condition $0 \le t \le 6$ hours.

Use the model $A(t)=20\cdot 4^{\frac{t}{2}}$.

Use hours and square centimeters as the stated units.

Your response: ______________

Incoming: $g^{-1}(x)=5+2^{4-x}$, with range $x>5$.

Recover time from a count over the stated experiment.

Your complete response must include inverse and interval.

Use the condition $0 \le t \le 8$ hours.

Use the model $M(t)=25\cdot 3^{\frac{t}{2}}$.

Use hours and microbes as the stated units.

Your response: ______________

AP Precalculus

Exponential and Logarithmic Inverse Models

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Test the proposed inverse and repair the shift. Identify the wrong shift in the proposed inverse and repair it. Verify your correction by composition.

Your complete response must include correction.

Use the condition real exponential inputs.

Let $f(x) = 2^{x+3}$.

The proposed inverse is $\log_2(x) + 3$.

Assess the formulas and the claimed inverse domain separately. Diagnose the claimed inverse domain separately from the two formulas.

Your complete response must include domain repair.

Use the condition real exponential inputs.

Evaluate the claim g has domain $x > 0$.

Let $f(x) = 2^x + 1$.

Let $g(x) = \log_2(x - 1)$.

Exponential and Logarithmic Inverse Models

Full Worked Solutions - excerpt

Worked example

Problem. Translate the forward multiplier into the inverse relation.

Your complete response must include inverse change.

Use the condition exact base- 4 process.

The forward rule states that input $+1$ multiplies output by 4.

Answer. Multiplying an inverse input by 4 increases its recovered output by 1.

Explanation and check.

The former output ratio becomes the inverse input ratio.

Worked example

Problem. Normalize above the baseline before taking a logarithm.

Your complete response must include inverse and domain.

Use the condition all real inputs.

Consider the statement $f(x) = 2 \cdot 6^x + 4$.

Answer. $f^{-1}(x) = \log_6(\frac{x-4}{2})$, for $x > 4$.

Explanation and check.

Subtract 4 and divide by 2 to expose the positive power.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Logarithmic Function Features

Logarithmic Function Features | APPC-U02-P19

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for boundary and domain, domain and asymptote, domain and boundary side, domain and branch location, domain and graph boundary and domain and side.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Logarithmic Function Features

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the strict positivity of the affine argument to identify the boundary side. The input condition is real x. Report domain and asymptote. Use rule $y = \log_7(9 - 3x)$.

Bank label: ______

Identify where the real logarithmic branch begins. The input condition is real x. Report boundary and domain. Use rule $y = \log_5(x + 6)$.

Bank label: ______

Locate the excluded boundary and the graph's allowed side. The input condition is real x. Report domain and asymptote. Use rule $y = \log_3(x - 8)$.

Bank label: ______

Determine the vertical scale from the table's ratio-to-difference pattern.

Given model family: $y = a\log_5(x)$

x	y
1	0
5	2
25	4

Bank label: ______

AP Precalculus

Logarithmic Function Features

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

State which quantities have the constant ratio and why the outputs are equally spaced.

Given model: $y = \log_5(x - 2)$

x	y
3	0
7	1
27	2

Specify three exact anchor points and the boundary needed for a structured sketch. The input condition is $x > 0$. Report anchors, asymptote, and shape. Use axes unit linear scales. Use rule $y = \log_2(x)$.

Find both attained output extrema on the restricted interval. The input condition is $1 \leq x \leq 8$. Report attained extrema. Use rule $y = \log_2(x)$.

State the multiplicative input change paired with each unit output change.

Given model: $y = \log_{10}(x)$

x	y
0.1	-1
1	0
10	1

Use argument values $1, 3$, and $\frac{1}{3}$ to specify a structured sketch. The input condition is $x > 1$. Report exact graph features. Use axes unit linear scales. Use rule $y = 2\log_3(x - 1) + 4$.

Solve the argument inequality and identify the graph's side of the boundary. The input condition is real x. Report domain, asymptote, and side. Use rule y=ln(5-x).

AP Precalculus

Logarithmic Function Features

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

State both limiting outputs and note whether the vertical shift changes them. The input condition is $x < 3$. Report end behavior. Use rule $y = \log_2(3 - x) + 4$.

Your response: ____________

Incoming: As x approaches 3 from the left, y approaches -infinity; as x heads to -infinity, y approaches infinity.

Use reciprocal e-based inputs to organize a precise sketch. The input condition is $x > 0$. Report exact points and shape. Use axes unit linear scales. Use rule $y = -\ln(x)$.

Your response: ____________

Incoming: Plot $(\frac{1}{e}, 1), (1, 0)$, and $(e, -1)$; asymptote $x = 0$; the graph decreases and is concave up.

Track the outer negative sign at both ends of the domain. The input condition is $x > -3$. Report signed limiting behavior. Use rule $y = -\log$ base $\frac{1}{2}$ of $(x + 3)$.

Your response: ____________

Incoming: The output tends to -infinity near -3 from the right and to infinity as x grows without bound.

Give the signs of unbounded behavior for a base below one. The input condition is $x > 1$. Report both end limits. Use rule $y = \log$ base $\frac{1}{4}$ of $(x - 1)$.

Your response: ____________

Incoming: At x = 1 from the right, y tends to infinity; as x tends to infinity, y tends to -infinity.

Solve the argument inequality containing a fractional coefficient. The input condition is real x. Report domain and boundary. Use rule $y = \log_7(3 - \frac{x}{2})$.

Your response: ____________

Incoming: The graph has asymptote x = 6 and domain $x < 6$.

Describe what happens near $x = 4$ and far to the left. The input condition is $x < 4$. Report allowed-side limits. Use rule $y = \ln(8 - 2x)$.

Your response: ____________

AP Precalculus

Logarithmic Function Features

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Correct the claimed effect of the output translation. Identify why adding 6 to the output cannot move the vertical asymptote. Correct the claimed boundary. The input condition is real logarithm. Report repaired feature claim. Use claim Adding 6 to $\log_2(x-4)$ moves its vertical asymptote from $x=4$ to $x=10$..

Repair the inference by separating monotonicity from concavity. Locate the invalid implication from decreasing to concave down, then give both correct properties. The input condition is real logarithm. Report corrected curvature claim. Use claim The graph of log base $\frac{1}{3}$ of x is concave down because it is decreasing..

Repair the statement by distinguishing input position from possible outputs. Identify the confusion between the input side and the output set. Repair both statements. The input condition is real logarithm. Report corrected domain and range. Use claim Since ln(4-x) is drawn left of x=4, its range is y<4..

Explain why an asymptote cannot be used as an anchor point here. Diagnose why the proposed asymptote point is not on the graph and give a valid zero anchor. The input condition is real logarithm. Report correct boundary interpretation. Use claim The vertical asymptote $x=2$ supplies the point $(2,0)$ on $y=\log_3(x-2)$..

Logarithmic Function Features

Full Worked Solutions - excerpt

Worked example

Problem. Separate the sign of an input from the sign of a logarithm output.

Answer. The range is all real numbers; for example $\log_2(\frac{1}{2}) = -1$ even though the argument $\frac{1}{2}$ is positive.

Explanation and check.

Argument positivity is a domain condition, not a restriction to positive outputs.

Worked example

Problem. Solve the strict positivity condition for the entire argument. The input condition is real x. Report domain, asymptote, and branch side. Use rule $y = \ln(-3(x-4))$.

Answer. Vertical asymptote x = 4; domain $x < 4$, so the branch is on the left.

Explanation and check.

The condition $-3(x-4) > 0$ reverses to $x < 4$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Logarithmic Transformations and Applications

Logarithmic Transformations and Applications | APPC-U02-P20

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for asymptotes and formulas, formulas and asymptotes, ordered results, resulting rules and relation, two expressions and offset and two formulas and asymptotes.

8 PDFs and a README; 71 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Logarithmic Transformations and Applications

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Describe the two vertical transformations and the unchanged asymptote. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given: $x > 0$

Parent: $f(x) = \log_3(x)$; Rule: $g(x) = -\log_3(x) + 2$

Bank label: ______

Evaluate the model at the given trial count and classify the prediction. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given: $2 \le n \le 40$ trials

Input: 27; Model: $R(n) = 15 + 6 \cdot \log_3(n)$; Output units: successful responses

Bank label: ______

Identify the row that prevents fitting the requested logarithmic regression to the full table. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context/domain: raw predictor must be positive

0	3.1
1	4.0
3	5.2
9	6.4

Bank label: ______

Map the three points and state what happens to the asymptote. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given: $x > 0$

Axes: unit linear scales; Map: $(x, y) \to (x, 2y + 1)$; Parent asymptote: $x = 0$

Context/domain: $x > 0$

1	0
2	1
4	2

Bank label: ______

AP Precalculus

Logarithmic Transformations and Applications

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Describe the transformation from f to g and locate the new vertical asymptote.

Given: $x > 3$

Parent: $f(x) = \log_2(x)$; Rule: $g(x) = \log_2(x-3) + 4$

Use the documented logarithmic fit to predict the held-out score and compute observed minus predicted.

Given: $x > 0$ hours

Documented fit: $y = 2 + 1.5 \cdot \ln(x)$; Held out row: $(4, 4.2)$; Output units: score points; Rounding: nearest hundredth

Context/domain: $x > 0$ hours

1	2.1
2	3.0
3	3.7
6	4.6

Construct a model that is zero at $x = 2$ and gains 3 whenever input doubles.

Map the three anchors and the vertical asymptote.

Given: $x > 2$

Axes: unit linear scales; Map: $(x, y) \to (x+2, y-1)$; Parent asymptote: $x = 0$

Context/domain: $x > 2$

1	0
2	1
0.5	−1

AP Precalculus

Logarithmic Transformations and Applications

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Determine whether the model supplies the requested prediction. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $q > 10$ minutes, calibrated for $12 \leq q \leq 50$

Input: 10; Model: $W(q) = 7 + \ln(q - 10)$; Output units: watts

Your response: ______________

Incoming: A gives $\log_3(3(x - 6))$ with asymptote 6; B gives $\log_3(3x - 6)$ with asymptote 2.

Map the full defining set to the translated graph. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $x > 4$

Axes: unit linear scales; Map: $(x, y) \to (x + 4, y - 2)$; Parent asymptote: $x = 0$

Context/domain: $x > 4$

1	0
3	1
$\frac{1}{3}$	-1

Your response: ______________

Incoming: k = 6 and $a = -4$, so $g(x) = -4 \cdot \log_2(-x - 3) + 6$; the branch increases.

Predict the held-out mass and report its residual with units. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $x > 0$ days

Documented fit: $y = 12 + 2.5\ln(x)$; Held out row: $(3, 14.5)$; Output units: grams; Rounding: nearest tenth

Context/domain: $x > 0$ days

1	12.1
2	13.8
5	15.9
9	17.4

Your response: ______________

Incoming: Prediction about $5.8g$; residual about $0.3g$, so the model underpredicts.

Write a logarithmic rule satisfying the zero and fivefold-change conditions. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Given: $x > 0$

Input ratio: 5; Normalization: use base 5; Output increment: 2; Zero input: 3

Your response: ______________

AP Precalculus

Logarithmic Transformations and Applications

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Decide whether the requested prediction exists and explain why. Student work: No real prediction exists at $x = 10$ because the logarithm argument is 0; the model requires $x \geq 10$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

error hunt

Given: $x > 10$ seconds, calibrated for $12 \leq x \leq 100$

Input: 10; Model: $V(x) = 8 + 3 \cdot \log_2(x - 10)$; Output units: meters per second

Decide whether a fitted residual can be computed for the held-out row. Student work/claim: The displayed fitted equation is an exact identity for every input, including values outside the observed data window. Locate the unsupported exact-fit/extrapolation step, then state the properly qualified model and justify the qualification from the displayed data.

error hunt

Given: $x > 0$ hours

Documented fit: $y = 6 + 1.4\ln(x)$; Held out row: $(0, 5.5)$; Output units: points

Context/domain: $x > 0$ hours

1	6.1
2	7.0
5	8.3

Logarithmic Transformations and Applications

Full Worked Solutions - excerpt

Worked example

Problem. Build a log curve through $(6, 0)$ whose output drops 4 whenever input triples.

Answer. A suitable rule is $y = -4 \cdot \log_3\left(\frac{x}{6}\right)$.

Explanation and check.

The normalized argument is 1 at $x = 6$; tripling x raises the logarithm by 1, so scale -4 creates the stated drop.

Worked example

Problem. Construct a model that is zero at $x = 2$ and gains 3 whenever input doubles.

Answer. One normalized model is $y = 3 \cdot \log_2\left(\frac{x}{2}\right)$.

Explanation and check.

The ratio $\frac{x}{2}$ anchors the zero, and $\log_2$ gains 1 per doubling, so the output scale is 3.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Log Properties and Change of Base

Log Properties and Change of Base | APPC-U02-P21

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for value to nearest thousandth, correction and denominator issue, valid subset and failing branch, correct interval, correct form and domain and source domain and extra condensed intervals.

8 PDFs and a README; 60 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Log Properties and Change of Base

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Rewrite the logarithm using natural logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_2(7)$. Bank label: ______	Combine the two terms and carry along the original domain. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_5(u) + \log_5(v)$. Bank label: ______
Separate the positive factors into individual base-five logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_5(7rs)$. Bank label: ______	Convert the expression to a quotient of common logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_5(11)$. Bank label: ______
Combine the difference into one base-eight logarithm and retain both source restrictions. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_8(p) - \log_8(q)$. Bank label: ______	Expand the product into separate logarithms of its constant and variable factors. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\ln(11at)$. Bank label: ______
Represent the value using only natural logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Start with $\log_6(13)$. Bank label: ______	Correct the proposed power-law rewrite so it is equivalent on the full domain of the original expression. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. The candidate expression is $2\ln(x)$. Start with $\ln(x^2)$. Bank label: ______

AP Precalculus

Log Properties and Change of Base

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Expand the logarithm completely, keeping the positive constant inside its own logarithm. Start with $\log_2(3xy)$.	Condense to one logarithm and retain the domain of the displayed expression. Start with $\log_3(x) + \log_3(y)$.
Decide whether the claim is an identity, and justify the decision with one exact counterexample if it is false. Evaluate the claim $\ln(u + v) = \ln(u) + \ln(v)$.	Identify the first invalid line and state what valid law could replace the mistaken idea. The student writes: $\ln(a + b) = \ln(a) + \ln(b) = \ln(ab)$.
Write the expression as a sum and difference of natural logarithms with no powers inside them. Start with $\ln(a^2 \cdot \frac{\sqrt{b}}{c})$.	Produce an equivalent single logarithm and report the original restrictions. Start with $2\ln(a) - \ln(b)$.

AP Precalculus

Log Properties and Change of Base

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Expand the quotient and simplify the logarithm of the numerical factor. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Start with $\log_6(\frac{x}{36yz})$.

Your response: ______________

Incoming: Yes; the equality holds for every allowed a and x.

Combine the shifted logarithms and derive the allowed x-values from the separate terms. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Start with $\log_7(x+2)+\log_7(x-4)$.

Your response: ______________

Incoming: k = $\frac{1}{2}$

Express the function using a base-three logarithm and simplify the coefficient. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use logarithms with base 3.

Start with $\log_9(y)$.

Your response: ______________

Incoming: 1.661

Pull the exponent in front without losing inputs less than the center. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Start with $\log_7((z-1)^2)$.

Your response: ______________

Incoming: $6\log_5|2x-1|$, with $x \neq \frac{1}{2}$

Test the proposed quotient rule and disprove it exactly if it is not an identity. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Evaluate the claim $\log_4(\frac{u}{v}) = \frac{\log_4(u)}{\log_4(v)}$.

Your response: ______________

Incoming: $\log_3(m^4 \cdot \frac{\sqrt[3]{p}}{n^2})$, with $m > 0, n > 0, p > 0$

Find the scaling constant that makes the two logarithmic functions identical. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Use the identity $K\log_3(x) = \log_{81}(x)$.

Your response: ______________

AP Precalculus

Log Properties and Change of Base

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Evaluate the identity claim and, if needed, refute it using powers of 2. Identify the precise error or unsupported step in the stated claim or method, then give the corrected conclusion and justify it from the supplied information.

Evaluate the claim $\log_2(uv) = \log_2(u)\log_2(v)$.

Use base 2 as the change-of-base reference to find the exact value. Student work: $-\frac{3}{1}$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

The reference logarithm base is 2.

Start with $\log_{\frac{1}{4}}(8)$.

Log Properties and Change of Base

Full Worked Solutions - excerpt

Worked example

Problem. Expand the logarithm completely, keeping the positive constant inside its own logarithm.
Start with $\log_2(3xy)$.

Answer. $\log_2(3) + \log_2(x) + \log_2(y)$

Explanation and check.

The argument is a product of the three positive factors $3, x$, and y, so the product law separates them.

Worked example

Problem. Decide whether the claim is an identity, and justify the decision with one exact counterexample if it is false.
Evaluate the claim $\ln(u + v) = \ln(u) + \ln(v)$.

Answer. False; $u = v = 1$ gives $\ln(2)$ on the left and 0 on the right.

Explanation and check.

Logarithms convert products, not sums, into sums; the positive pair $1, 1$ directly contradicts the claim.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Solving Exponential Equations

Solving Exponential Equations | APPC-U02-P22

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for real solution set, time-domain conclusion, time or impossibility, solution set, all real solutions and finite none or all.

8 PDFs and a README; 64 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Solving Exponential Equations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Rewrite the reciprocal as a power of 3 and equate exponents. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $3^{x-2} = \frac{1}{27}$. Bank label: _______	Solve the exponential equation exactly using logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $3^x = 10$. Bank label: _______
Use the indicated positive substitution and recover every admissible x-value. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $4^x - 5 \cdot 2^x + 4 = 0$; substitution: $u = 2^x$. Bank label: _______	Find the unique real input where the two exponential expressions are equal. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $2 \cdot 3^x = 6 \cdot 2^x$. Bank label: _______
Rewrite the reciprocal and the left side as powers of 2. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $4^{x-1} = \frac{1}{8}$. Bank label: _______	Give the exact solution using natural logarithms. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $7^x = 13$. Bank label: _______
Factor after substitution and invert both positive values of u. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $9^x - 5 \cdot 3^x + 4 = 0$; substitution: $u = 3^x$. Bank label: _______	Divide to obtain one base-two power and solve the crossover exactly. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context/domain: real x. equation: $2 \cdot 4^x = 16 \cdot 2^x$. Bank label: _______

AP Precalculus

Solving Exponential Equations

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Choose a so that $x = 2$ is a solution, then decide whether that equation has any other real solution. Context/domain: $a > 0$ and real x. equation: $a \cdot 3^x = 45$; required solution: $x = 2$.	Identify the first invalid logarithm step and solve the original equation correctly. Context/domain: real x. equation: $2^x + 3 = 11$; student work: $[\ln(2^x) + \ln(3) = \ln(11), x = \frac{\ln(11)}{\ln(2)+\ln(3)}]$.
Solve without dividing by an expression that could be zero, and classify the solution set. Context/domain: real x. equation: $5 \cdot 2^x = 7 \cdot 2^x$.	Choose k from the prescribed solution, then inspect whether $x = 3$ remains the only solution. Context/domain: $k > 0$ and real x. equation: $k \cdot 2^x = 4 \cdot 2^{x+1}$; required solution: $x = 3$.
Find the first point at which the student turns an inadmissible substitution root into an x-value. Context/domain: real x. equation: $4^x + 3 \cdot 2^x - 4 = 0$; student work: $[u = 2^x, (u-1)(u+4) = 0, x = 0$ or $x = 2]$.	Solve the level equation conceptually and determine whether its time belongs to the model's operating domain. Context/domain: $t \geq 0$ years. model: $A(t) = 200(1.1)^t$; target: 150 units.

AP Precalculus

Solving Exponential Equations

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Solve exactly without approximating the irrational exponent. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $11^x = 2$.

Your response: ____________

Incoming: $x \approx 1.771$, uniquely on $[-2, 3]$

Use the positive substitution and invert both factored roots. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $16^x - 5 \cdot 4^x + 4 = 0$; substitution: $u = 4^x$.

Your response: ____________

Incoming: n = 4

Express every factor as a power of 3 and solve. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $3 \cdot 9^x = 81 \cdot 3^x$.

Your response: ____________

Incoming: x = 4

Use the unique exponent that produces 1 for base 5, then solve the resulting algebraic equation. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $5^{x^2-3x} = 1$.

Your response: ____________

Incoming: x = $\frac{2}{3}$

Isolate the positive exponential quantity and state the exact real solution. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $7 \cdot 3^{2x} - 1 = 0$.

Your response: ____________

Incoming: x = 1 or $x = \frac{\ln(3)}{\ln(2)}$

Form one exponential ratio and solve for the crossover input. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Context/domain: real x. equation: $3 \cdot 5^x = 12 \cdot 2^x$.

Your response: ____________

AP Precalculus

Solving Exponential Equations

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Determine whether the declining model reaches the higher target after $t = 0$. Student work: No nonnegative time; 360 would require extending the growth model backward. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: $t \geq 0$ weeks. model: $A(t) = 300(0.9)^t$; target: 360 units.

Apply the repeated-exponential substitution and verify both recovered exponents. Student work: $x = 1$ or $x = 4$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: real x. equation: $9^x - 30 \cdot 3^x + 81 = 0$; substitution: $u = 3^x$.

Choose a to make $x = 3$ a solution and check whether the solution is unique. Student work: $a = 3$; strict increase makes $x = 3$ not unique Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: $a > 0$. equation: $a \cdot 4^{x-1} = 48$; required solution: $x = 3$.

Evaluate the x-independent exponential and decide whether the equation is an identity or contradiction. Student work: No x works because the left side is fixed at 171, not 169. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens. error hunt. Context/domain: real x. equation: $7 \cdot 5^{0x+2} - 4 = 170$.

Solving Exponential Equations

Full Worked Solutions - excerpt

Worked example

Problem. Use base 3, solve the resulting quadratic, and retain both real roots. Context/domain: real x. equation: $9^{x^2} = 3^{5x+6}$.

Answer. x = $\frac{5-\sqrt{73}}{4}$ or $x = \frac{5+\sqrt{73}}{4}$

Explanation and check.

Equating exponents gives $2x^2 - 5x - 6 = 0$, whose discriminant is 73.

Worked example

Problem. Express every factor as a power of 3 and solve. Use the complete final response to identify the next Match $\&$ Move card; show the reasoning required by the original task. Context/domain: real x. equation: $3 \cdot 9^x = 81 \cdot 3^x$.

Answer. x = 3 from $2x + 1 = x + 4$

Explanation and check.

The sides become 3^{2x+1} and 3^{x+4}, whose exponents agree only at $x = 3$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Solving Logarithmic Equations

Solving Logarithmic Equations | APPC-U02-P23

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for complete solution set, solution interval, solution intervals, solution set, correct root and method error and correct solution set and domain error.

8 PDFs and a README; 94 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Solving Logarithmic Equations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the intensity ratio represented by the logarithmic scale target. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Your complete response must include positive intensity ratio.

Use the condition $\frac{I}{I_0} > 0$.

Use the model $L = 10\log(\frac{I}{I_0})$.

The target is $L = 30$ decibels.

Bank label: _______

Solve the quadratic in the logarithmic quantity and exponentiate each real u-root. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Your complete response must include positive x-solutions.

Use the condition $x > 0$.

Solve the equation $(\log_2(x))^2 - 3\log_2(x) + 2 = 0$.

Use the substitution $u = \log_2(x)$.

Bank label: _______

Solve by exponentiating and verify the logarithm's original argument condition. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Your complete response must include real solution set.

Use the condition real x.

Solve the equation $\log_2(x - 1) = 3$.

Bank label: _______

Recover the concentration represented by the given pH target. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Your complete response must include positive hydrogen-ion concentration.

Use the condition $[H+] > 0$ mol/L.

Use the model pH=-log($[H+]$).

The target is pH=6.5.

Bank label: _______

AP Precalculus

Solving Logarithmic Equations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Check both algebraic candidates in the two original logarithm arguments. Your complete response must include valid solution set and rejection reason. Use the condition real x. Solve the equation $\log_2(x-3)+\log_2(x-5)=3$. The student proposes the candidate values 17.	Condense the logs, solve the quadratic, and enforce both starting restrictions. Your complete response must include valid roots. Use the condition real x. Solve the equation $\log_2(x-1)+\log_2(x+1)=3$.
Exponentiate, solve the quadratic, and test both roots in the original argument. Your complete response must include all real roots. Use the condition real x. Solve the equation $\log_3(x^2-5)=2$.	Use the supplied maximum fact to determine whether the curves ever meet. Your complete response must include solution set. Use the condition $x>0$. Solve the equation $\ln(x)=x$. Use the supplied evidence: For $x>0, \ln(x)-x$ has maximum -1 at $x=1$.
Recover the intensity ratio, then decide whether the calibrated model supports that target. Your complete response must include ratio and calibration status. Use the condition $1 \le \frac{I}{I_0} \le 10^8$ calibration range. Use the model $L=10\log(\frac{I}{I_0})$. The target is 100 decibels.	Explain why exponentiating to obtain 9 does not validate the student's answer in the real logarithm setting. Your complete response must include correct conclusion. Use the condition real x. Solve the equation $\log_{-3}(x)=2$. The student reports $x=9$.

AP Precalculus

Solving Logarithmic Equations

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: x = $2 + \sqrt{3}$ only

Factor the quadratic in u and invert the natural logarithm for both roots. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include positive x-solutions.

Use the condition $x > 0$.

Solve the equation $(\ln(x))^2 - 5\ln(x) + 6 = 0$.

Use the substitution $u = \ln(x)$.

Your response: ____________

Incoming: x = 10

Exponentiate the equation and verify positivity of the original argument. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include real root.

Use the condition real x.

Solve the equation $\log_7(2x - 5) = 2$.

Your response: ____________

Incoming: x = e^2 or $x = e^3$

Use both roots of the difference of squares and exponentiate each one. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include positive x-solutions.

Use the condition $x > 0$.

Solve the equation $(\log_4(x))^2 - 1 = 0$.

Use the substitution $u = \log_4(x)$.

Your response: ____________

Incoming: x = e is the only intersection

Determine the input count that reaches the stated logarithmic capacity target. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include positive device count.

Use the condition $n > 0$ devices.

Use the model $C(n) = 12\log_2(n)$.

The target is $C = 48$ units.

Your response: ____________

Incoming: $x \approx 2.763$, the sole intersection for $x > 1$

Intersect the argument restrictions and then equate the arguments. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include valid root.

Use the condition real x.

Solve the equation $\ln(2 - x) = \ln(x + 5)$.

Your response: ____________

Incoming: x = $2 + \sqrt{10}$ only

Condense the difference and retain the common positivity restriction. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your complete response must include valid real root.

Use the condition real x.

Solve the equation $\ln(x + 2) - \ln(x - 4) = \ln(3)$.

Your response: ____________

AP Precalculus

Solving Logarithmic Equations

Name: ________________ **Date:** ________________ **Class:** ________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use injectivity and check the result against both affine positivity conditions. Student work: $x = \frac{9}{3}$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Your complete response must include valid rational root.

Use the condition real x.

Solve the equation $\log_7(3x+2) = \log_7(11-x)$.

Factor in u and remember that both negative logarithmic values produce positive inputs. Student work: $x = e^{+1}$ or $x = e^{-5}$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Your complete response must include positive x-values.

Use the condition $x > 0$.

Solve the equation $(\ln(x))^2 + 6\ln(x) + 5 = 0$.

Use the substitution $u = \ln(x)$.

Solving Logarithmic Equations

Full Worked Solutions - excerpt

Worked example

Problem. Solve the condensed quadratic and screen its roots against each source logarithm.

Your complete response must include valid solution set.

Use the condition real x.

Solve the equation $\log_5(x) + \log_5(x+4) = 1$.

Answer. x = 1; discard $x = -5$

Explanation and check.

The source requires $x > 0$. Since $x(x+4) = 5$ has roots 1 and -5, only 1 remains.

Worked example

Problem. Equate the same-base arguments, then check positivity at each resulting root.

Your complete response must include all valid roots.

Use the condition real x.

Solve the equation $\log_3(x^2+2) = \log_3(5x-4)$.

Answer. x = 2 or $x = 3$; both arguments are positive

Explanation and check.

The equation $x^2 + 2 = 5x - 4$ factors as $(x-2)(x-3) = 0$, and both roots make $5x - 4$ positive.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exponential and Logarithmic Inequalities

Exponential and Logarithmic Inequalities | APPC-U02-P24

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for case intervals, solution by k region, solution intervals by base region, solutions by m sign, solutions by p sign and case solutions.

8 PDFs and a README; 69 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exponential and Logarithmic Inequalities

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Combine the logarithm domain with the transformed inequality. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: real x or the stated contextual schedule. Solve the inequality $\log_2(x-1) > 3$. Bank label: ______	Solve the quadratic inequality in u and map its positive interval back to x. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: real x or the stated contextual schedule. Solve the inequality $4^x - 5 \cdot 2^x + 4 < 0$. The substitution is $u = 2^x > 0$. Bank label: ______
Isolate the positive exponential and solve the strict threshold. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: real x or the stated contextual schedule. Solve the inequality $2^x + 1 > 9$. Bank label: ______	Translate the base-seven comparison directly into exponent order. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Context: real x. Solve the inequality $7^{x+2} \leq 7^5$. Bank label: ______

AP Precalculus

Exponential and Logarithmic Inequalities

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Use the intersection as a boundary and identify where the exponential is at or below the horizontal line. Context: real x or the stated contextual schedule. the horizontal axis measures input; the vertical axis measures output; the displayed window has $-2 \le x \le 4$ and $0 \le y \le 25$. The comparison is $e^x \le e^2$. Compare the curves $(y = e^x, y = e^2)$. The intersection is at $x = 2$; this input is included.	Separate increasing and decreasing valid bases before comparing exponents. Context: real x or the stated contextual schedule. Solve the inequality $a^x > a^2$. Assume $a > 0$ and $a \neq 1$.
Check the order direction for the reciprocal base. Context: real x or the stated contextual schedule. Solve the inequality $(\frac{1}{2})^x > (\frac{1}{2})^3$. Evaluate this student answer: $x > 3$.	Separate positive, negative, and zero scale factors before dividing. Context: real x or the stated contextual schedule. Solve the inequality $k \cdot 2^x \le 8k$. Assume real k.
First reduce the two positivity conditions, then compare the arguments and intersect the sign solution. Context: real x or the stated contextual schedule. Solve the inequality $\ln(x^2 - 4) > \ln(3x)$.	Partition the parameter by monotonicity and retain the positive argument domain in both cases. Context: real x or the stated contextual schedule. Solve the inequality $\log_a(x) > 1$. Assume $a > 0$ and $a \neq 1$.

AP Precalculus

Exponential and Logarithmic Inequalities

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Combine positivity with the nonpositive logarithm condition. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x.

Solve the inequality $\ln(x+5) \leq 0$.

Your response: ____________

Incoming: Week 4

Compare exponents of the increasing natural base. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x.

Solve the inequality $e^{2x-1} > e^3$.

Your response: ____________

Incoming: $[2, 8]$ within the displayed domain

Remove the factor 2 before using the base-five boundary. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x.

Solve the inequality $2 \cdot 5^x + 3 > 13$.

Your response: ____________

Incoming: $\log_{\frac{1}{2}}(3) < x < -1$

Separate coefficient signs before division. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Context: real x and real s.

Solve the inequality $s \cdot 2^x < 4s$.

Your response: ____________

AP Precalculus

Exponential and Logarithmic Inequalities

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use the crossing and graph order on either side. Student work: $(-1, \infty)$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Context: real x.

Use standard Cartesian axes; the horizontal axis measures input; the vertical axis measures function value.

The comparison is $e^x > 1$.

Compare the curves $(y = e^x, y = 1)$.

The intersection is at $x = 0$; this input is excluded.

The displayed window has $-3 \leq x \leq 3$ and $0 \leq y \leq 8$.

Use strict positivity of the exponential to classify the threshold. Student work: $x = 0$ is a solution because both sides of $2 - 5^x \geq 2$ are then nonnegative. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Exponential and Logarithmic Inequalities

Full Worked Solutions - excerpt

Worked example

Problem. Use strict positivity of the exponential to classify the threshold. Student work: $x = 0$ is a solution because both sides of $2 - 5^x \geq 2$ are then nonnegative. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Answer. No real solution

Explanation and check.

The inequality would require $-5^x \geq 0$, impossible because $5^x > 0$.

Worked example

Problem. Reverse when removing the negative factor, then intersect with strict argument positivity.

Context: real x.

Solve the inequality -$\log_2(x^2 - 4) \geq -1$.

Answer. $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

Explanation and check.

The conditions are $x^2 > 4$ and $x^2 - 4 \leq 2$, so $4 < x^2 \leq 6$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Linearizing Exponential Data

Linearizing Exponential Data | APPC-U02-P25

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for corrected line, least integer threshold, ratio and observed value, slope evolution, tail behavior and values and explanation.

8 PDFs and a README; 73 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Linearizing Exponential Data

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Linearize the decay model in base 3. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use $y = 9(\frac{1}{3})^x$. Use the transformation $Y = \log_3(y)$.

Bank label: ______

Read the original output at the marked height $x = 2$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use these axis scales: scale logarithmic y; linear x; x input; y original y on base-10 logarithmic scale. The marked point has $x2$; y label 100. The labeled levels are $1, 10, 100, 1000$.

Bank label: ______

Use the fitted table to predict the original output at $x = 4$. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Context/domain: data

x	**fitted Y=ln(y)**
0	0
2	1
4	2

Bank label: ______

Place the decay observations on a base-2 transformed table. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value. Use logarithm base 2. The raw observations are $(0, 8), (1, 4)$, and $(2, 2)$; only these observations are asserted. Construct the transformed output column yourself.

Context/domain: data

x	**raw y**
0	8
1	4
2	2

Bank label: ______

AP Precalculus

Linearizing Exponential Data

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Diagnose the parameter error and state the correct factor. The given line is $Y = 1 + 0.3x$. A student claims: The raw growth factor is 0.3. Use the transformation $Y = \log_{10}(y)$.	Write the transformed line and identify its intercept and slope. Use $y = 4(2^x)$. Use the transformation $Y = \log_2(y)$.
Correct the student's reading of the plotted point. Use these axis scales: scale linear-linear transformed plot; x input; $yY = \log_2(y)$. The point are $3, 5$. A student reads $y = 5$.	Express the semi-log line exactly and interpret the two coefficients. Use $y = 5(100^x)$. Use the transformation $Y = \log_{10}(y)$.
Compare the initial values and one-input growth factors. Use these axis scales: scale linear-linear transformed plot; x input unit; $yY = \log_{10}(y)$. The lines are (model P; rule $Y = 1 + 0.4x$), (model Q; rule $Y = 2 + 0.4x$).	Map all three raw observations to the base-4 plot. Context/domain: data <table><tr><td>0</td><td>2</td></tr><tr><td>1</td><td>8</td></tr><tr><td>3</td><td>128</td></tr></table>

AP Precalculus

Linearizing Exponential Data

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Recover the positive raw y-value represented by $Y = -2$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use these axis scales: scale linear in Y; x input; yY =$\log_{10}(y)$. The point are $-1, -2$.

Your response: ____________

Incoming: Y-values $\log_2(16)$=4, $\log_2(20)$, $\log_2(28)$; their increments are unequal.

Predict y at $x = 2$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The fitted relation is $Y = 1 + 0.5x$ where Y =$\log_{10}(y)$. Evaluate at input 2. The observations cover x in $0, 4$.

Your response: ____________

Incoming: x = 1 and common $y = 27$

State the original output at the plotted point $(3, 5)$. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use these axis scales: scale linear in Y; x stage; yY =$\log_2(y)$. The point are $3, 5$.

Your response: ____________

Incoming: Y = 4.5 and $y = 16\sqrt{2}$

Convert the fitted height at $x = 1$ to original units. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. The fitted relation is Y = 3-x where Y =$\log_2(y)$. Evaluate at input 1. The observations cover x in $0, 3$.

Your response: ____________

Incoming: The slope is initially negative but approaches 0 as Y approaches $\log_3(9)$=2.

What multiplicative change is encoded by moving one labeled vertical interval from 1 to 10? Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Your response: ____________

Incoming: y = $100(\frac{1}{\sqrt{5}})^x$

Translate the half-unit rise into a ratio of original outputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use these axis scales: scale linear in Y; x input; yY =$\log_4(y)$. The vertical displacement is 0.5.

Your response: ____________

AP Precalculus

Linearizing Exponential Data

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Judge the student's claim that equal-looking log increments prove $y = 10(1.5^x)$ exactly. Student work/claim: The displayed fitted equation is an exact identity for every input, including values outside the observed data window. Locate the unsupported exact-fit/extrapolation step, then state the properly qualified model and justify the qualification from the displayed data.

Context/domain: data

x	raw y	Y=log_base(y)
0	10.0	1.0
1	15.1	1.179
2	22.4	1.35
3	34.0	1.531

Construct every admissible transformed row and diagnose the remaining observation. Student work: $(0, 1.079)$ and $(3, 2.079)$ are admissible; $x = 1$ can be plotted because $y = 0$ Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Context/domain: data

x	raw y	Y=log_base(y)
0	12	1.079
1	0	_____
3	120	2.079

Linearizing Exponential Data

Full Worked Solutions - excerpt

Worked example

Problem. Write the transformed line and identify its intercept and slope. Use $y = 4(2^x)$. Use the transformation $Y = \log_2(y)$.

Answer. $\mathrm{Y} = 2 + x$; intercept 2 and slope 1

Explanation and check.

$\log_2(4) = 2$ and $\log_2(2) = 1$, so logarithms turn the product into $2 + x$.

Worked example

Problem. What multiplicative change is encoded by moving one labeled vertical interval from 1 to 10? Use the complete final response to identify the next Match $\&$ Move card; show the reasoning required by the original task.

Answer. a factor of 10

Explanation and check.

Consecutive decade levels differ by a ratio of 10 even though their numerical differences are not constant.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Periodic Functions, Radians, and the Unit Circle | APPC-U03-COMB-P01-P02

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for directed radian measure, supported periodicity claim, recurrence rates and range comparison, exact degrees, exact radians and exact ordering.

8 PDFs and a README; 131 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Express $-2\frac{\pi}{9}$ radians as an exact directed angle in degrees.

Bank label: ______

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

On a circle of radius 4 m, a point rotates through $\frac{3}{8}$ radians. Find the directed arc displacement s and state whether the motion is clockwise or counterclockwise.	For the directed angle $13\frac{\pi}{6}$, describe its terminal ray and list every coterminal representative in the interval $(\pi, 5\pi]$. Respect the endpoint symbols.
An exact periodic graph has period 9. A cycle marker occurs at $t = 5$; the graph reaches a specified decreasing phase 4 time units after every marker. Is $t = 38$ that same phase? Give an equivalent cycle window beginning at the marker.	For the directed angle $15\frac{\pi}{8}$, describe its terminal ray and list every coterminal representative in the interval $(0, 4\pi)$. Respect the endpoint symbols.

AP Precalculus

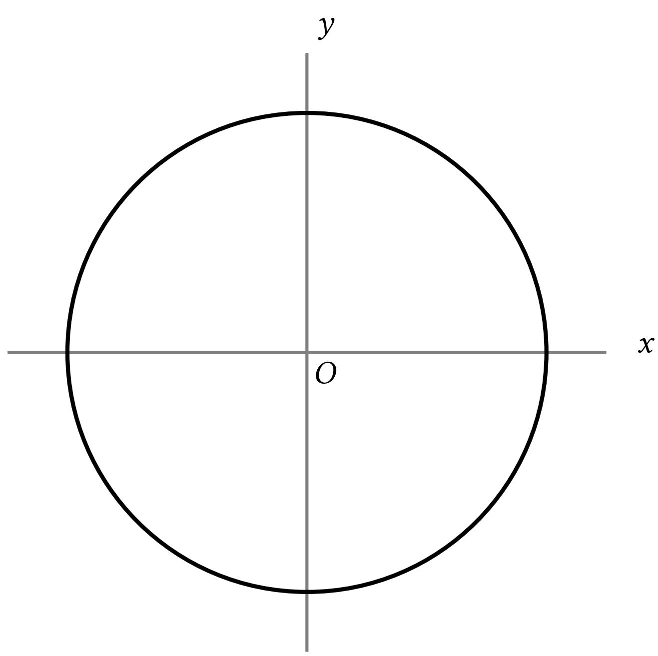
Periodic Functions, Radians, and the Unit Circle

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: 105°

Point $P = (\frac{8}{17}, y)$ lies on the unit circle and is below the x-axis. Find y and verify the point.



Your response: ______________

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims that the smallest period is 7 time units for these exact repeating (sample index, value) pairs: $(0,78)$; $(1,71)$; $(2,76)$; $(3,73)$; $(4,72)$; $(5,73)$; $(6,82)$; $(7,78)$; $(8,71)$; $(9,76)$; $(10,73)$; $(11,72)$; $(12,73)$; $(13,82)$. Consecutive sample indices are 3 time units apart. Diagnose the claim, state the corrected smallest period, and cite one decisive comparison.

Identify the error or first invalid claim, give the corrected conclusion, and justify the correction.

sample index	value
0	78
1	71
2	76
3	73
4	72
5	73
6	82

sample index	value
7	78
8	71
9	76
10	73
11	72
12	73
13	82

A circle centered at O has radius 18 cm. A point moves counterclockwise along the circle through an arc of length $\frac{162}{5}$ cm. Find the directed angle in radians and explain how the sign records the sweep direction. Before solving, choose and justify the valid method. Method A: Use radius divided by arc length. Method B: Use signed arc length divided by radius.

Periodic Functions, Radians, and the Unit Circle

Full Worked Solutions - excerpt

Worked example

Problem. Convert the directed rotation 225° to an exact radian measure. Preserve its direction.

Answer. $5\frac{\pi}{4}$ radians

Explanation and check.
Multiply by $\frac{\pi}{180}^\circ$: $225(\frac{\pi}{180}) = 5\frac{\pi}{4}$. The sign is preserved.

Worked example

Problem. Convert the directed rotation -24° to an exact radian measure. Preserve its direction.

Answer. $-2\frac{\pi}{15}$ radians

Explanation and check.
Multiply by $\frac{\pi}{180}^\circ$: $-24(\frac{\pi}{180}) = -2\frac{\pi}{15}$. The sign is preserved.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Trig Ratios from Triangles and Coordinates

Trig Ratios from Triangles and Coordinates | APPC-U03-P03

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for exact coordinate ratios, radius and three ratios, selected exact ratios, scale, ray relation, and ratio behavior, domain and ratio comparison and ray directions and quotient behavior.

8 PDFs and a README; 112 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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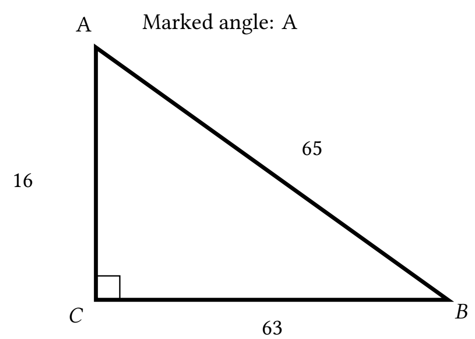
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: ______________ **Date:** ______________ **Class:** ______________

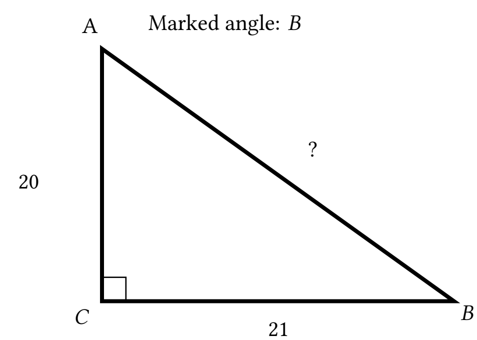
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

In right triangle ABC, $\angle C = 90^\circ$, AC = 16, BC = 63, and AB = 65. The marked angle is ∠A; the drawing may be rotated. Identify the opposite, adjacent, and hypotenuse sides relative to ∠A, then write its sine, cosine, and tangent.



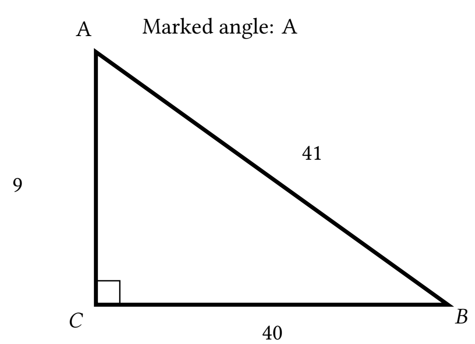
Bank label: ______

Right triangle ABC has $\angle C = 90^\circ$, AC = 20, BC = 21, and marked angle $\angle B$. First recover AB, then use the side roles relative to $\angle B$ to find sine, cosine, and tangent.



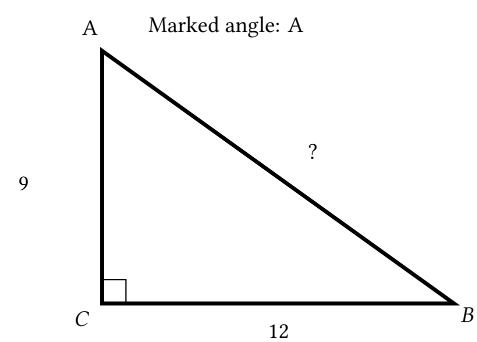
Bank label: ______

In right triangle ABC, $\angle C = 90^\circ$, AC = 9, BC = 40, and AB = 41. The marked angle is ∠A; the drawing may be rotated. Identify the opposite, adjacent, and hypotenuse sides relative to ∠A, then write its sine, cosine, and tangent.



Bank label: ______

Right triangle ABC has $\angle C = 90^\circ$, AC = 9, BC = 12, and marked angle ∠A. First recover AB, then use the side roles relative to ∠A to find sine, cosine, and tangent.



Bank label: ______

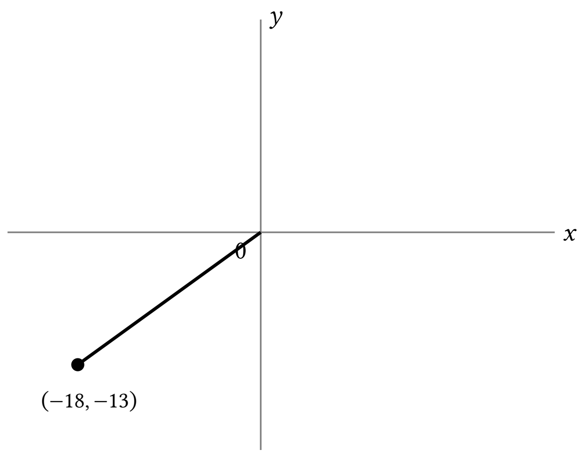
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A graphed ray starts at the origin and passes through $(-18, -13)$. Find its slope and interpret that quotient as $\tan\theta$ for the ray's directed angle.



AP Precalculus

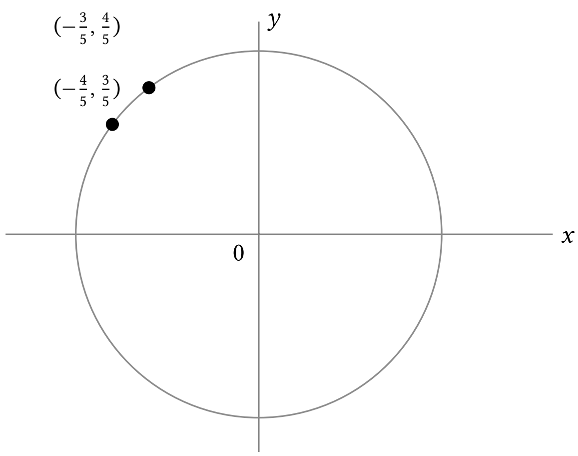
Trig Ratios from Triangles and Coordinates

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

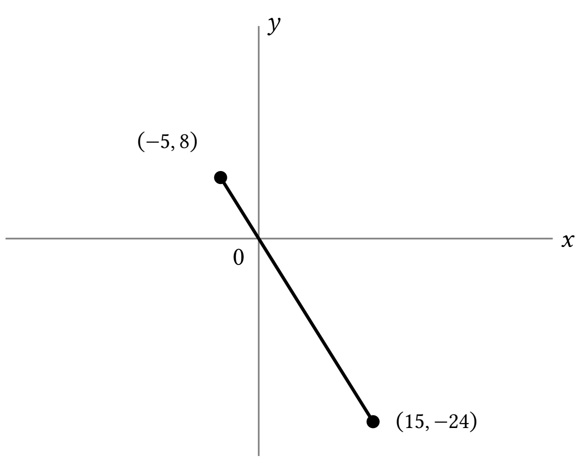
A unit-circle point moves along the shorter counterclockwise arc in the second quadrant from $(-\frac{3}{5}, \frac{4}{5})$ to $(-\frac{4}{5}, \frac{3}{5})$. Describe the endpoint changes in sine and cosine and compare the initial and final tangent ratios.



Your response: ______________

Incoming: sin: $\frac{4}{5} \to \frac{3}{5}$; cos: $-\frac{3}{5} \to -\frac{4}{5}$; tan: $-\frac{4}{3} \to -\frac{3}{4}$

Rays OP and OQ pass through $P = (-5, 8)$ and $Q = (15, -24)$. Compare their directions, slopes, and tangent values. Explain what slope loses that sine and cosine would retain.



Your response: ______________

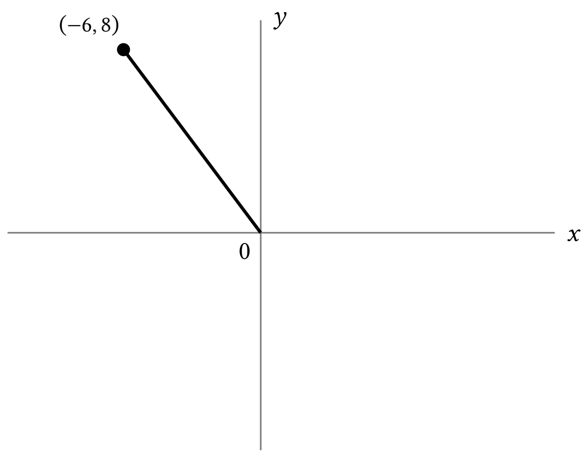
AP Precalculus

Trig Ratios from Triangles and Coordinates

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For ray point $(-6, 8)$, a student assigns a negative radius because a coordinate is negative. Diagnose the error, state the correct r, and repair sine and cosine.



Trig Ratios from Triangles and Coordinates

Full Worked Solutions - excerpt

Worked example

Problem. The only information about a nonzero terminal ray is $\tan\theta = -\frac{7}{4}$. Describe the ray or rays that are determined, give representative point (s), and explain the scale or sign ambiguity that remains.

Answer. Two opposite rays, represented by $(4, -7)$ and $(-4, 7)$; any positive scaling on either ray also works.

Explanation and check.

Tangent fixes $\frac{y}{x}$, so it fixes a line through the origin but not which of the two opposite rays or the radial scale.

Worked example

Problem. Can a real nonzero ray satisfy $\sin\theta = \frac{6}{5}$ under the coordinate definition? Decide and prove the conclusion using $|\text{coordinate}| \le r$.

Answer. No real ray satisfies the claim.

Explanation and check.

For every real point, the magnitude of the relevant coordinate is at most r, so $|\sin\theta| \le 1$. Here $\left|\frac{6}{5}\right| > 1$, a contradiction.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Exact Trig Values at Special Angles

Exact Trig Values at Special Angles | APPC-U03-P04

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for correct equality and approximation, mode diagnosis and exact value, valid exact and approximate relations, exact equality groups and ordering, all permitted angle labels and completed exact row.

8 PDFs and a README; 128 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Exact Trig Values at Special Angles

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A special-angle table row has $(\sin\theta, \cos\theta) = (1, 0)$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate.

Required response

Angle	sin θ	cos θ
______	1	0

Bank label: ______

On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos\theta = \frac{1}{2}$. Use the unit-circle line intersection and check both endpoint flags.

Bank label: ______

AP Precalculus

Exact Trig Values at Special Angles

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

The pair at $\frac{\pi}{3}$ is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. The target ray $\frac{\pi}{6}$ is related by reverse complementary transfer. Track coordinate exchange and any signs to infer its exact pair.

Audit this special-angle table row: $\theta = \frac{\pi}{3}, \sin\theta = \frac{1}{2}, \cos\theta = \frac{1}{2}$. Identify the earliest exact inconsistency and replace the row with the correct pair.

Required response

Angle	sin θ	cos θ
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{1}{2}$

AP Precalculus

Exact Trig Values at Special Angles

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A unit-circle table row for $\theta = 7\frac{\pi}{6}$ shows $\cos\theta$ missing and the other coordinate equal to $-\frac{1}{2}$. Complete the row exactly and use the angle's location to justify the square-root sign.

Required response

Angle	sin θ	cos θ
$7\frac{\pi}{6}$	$-\frac{1}{2}$	______

Your response: ______________________

Incoming: $\sin\theta = -\frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}$

A special-angle table row has $(\sin\theta, \cos\theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate.

Required response

Angle	sin θ	cos θ
______	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

Your response: ______________________

Incoming: $\theta = \frac{\pi}{6}$

On the finite one-turn window $(-\pi, \pi]$, locate every special-angle ray satisfying $\cos\theta = 1$. Use the unit-circle line intersection and check both endpoint flags.

Your response: ______________________

Incoming: $\theta = 0$

On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos\theta = -1$. Use the unit-circle line intersection and check both endpoint flags.

Your response: ______________________

AP Precalculus

Exact Trig Values at Special Angles

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A calculator in degree mode evaluates the radian expression $\cos(\frac{\pi}{3})$ and displays 0.9998. Diagnose the mode error, give the correct exact value, and explain why the display is not evidence of equality.

Exact Trig Values at Special Angles

Full Worked Solutions - excerpt

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{3}$ on the unit circle and evaluate the ordered pair $(\sin\theta, \cos\theta)$ exactly.

Answer. $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{3}$ has $x = \frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$. Therefore $(\sin\theta, \cos\theta) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$.

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{6}$ on the unit circle and evaluate the ordered pair $(\sin\theta, \cos\theta)$ exactly.

Answer. $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{6}$ has $x = \frac{\sqrt{3}}{2}$ and $y = \frac{1}{2}$. Therefore $(\sin\theta, \cos\theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Tangent Values and Domain

Tangent Values and Domain | APPC-U03-P05

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for classification and decisive evidence, exact slope and sign evidence, classification and earliest error, classification and evidence, exact ordering and earliest error and repair.

8 PDFs and a README; 146 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Tangent Values and Domain

Name: ______________ **Date:** ______________ **Class:** ______________

Give exact answers and all requested reasoning; record a bank letter. If bank entries give equivalent final responses for the same quantity and context, either label is valid and may be reused. A shared numerical value alone does not make entries interchangeable.

Order these exact scalar values from least to greatest, grouping ties and using no decimals: $\tan(2\frac{\pi}{3}), \sin(3\frac{\pi}{2}), \cos(5\frac{\pi}{6})$.

Solve the problem and show enough exact work for the requested result.

$\tan(2\frac{\pi}{3})$
$\sin(3\frac{\pi}{2})$
$\cos(5\frac{\pi}{6})$

Bank label: ______

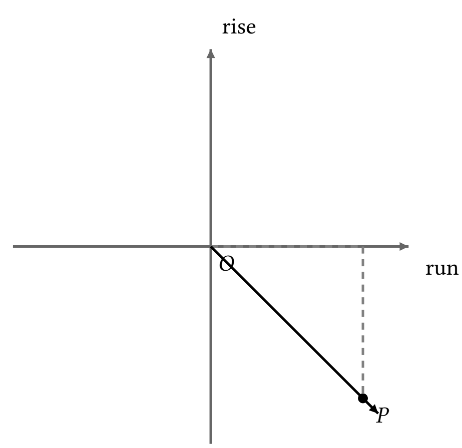
Given $\tan(\frac{\pi}{3}) = \sqrt{3}$, use “negate angle” to infer $\tan(-\frac{\pi}{3})$. Explain whether one or both coordinate signs change.

Bank label: ______

A terminal ray passes through $(-4, -4\sqrt{3})$. Evaluate its exact tangent from rise/run and explain why radius normalization is unnecessary.

Bank label: ______

A marked ray has signed run 12 and rise -12. Calculate its exact tangent and explain which geometric datum controls the sign.



Bank label: ______

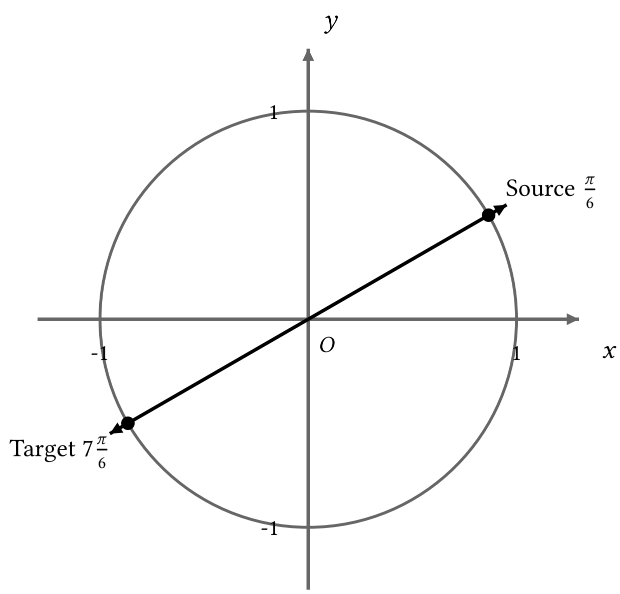
AP Precalculus

Tangent Values and Domain

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Given $\tan(\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$, use "add π" to infer $\tan(7\frac{\pi}{6})$. Explain whether one or both coordinate signs change.



AP Precalculus

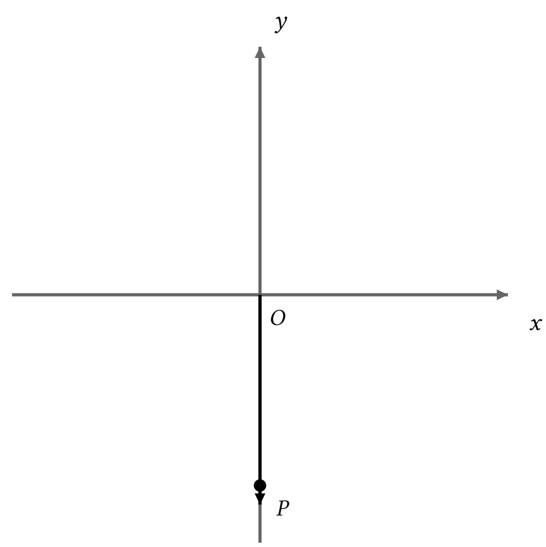
Tangent Values and Domain

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\tan(-5\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$

A ray is represented by the exact point $(0, -3)$. Classify its tangent as zero, nonzero, or undefined and cite numerator and denominator roles.



zero
nonzero
undefined

Your response: ________________

AP Precalculus

Tangent Values and Domain

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the claim "$\tan(\pi)$ is undefined because sine is zero." Classify tangent correctly and identify the first quotient or domain error.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

denominator
permission
sign

A student claims: "Because the x-coordinate contains a radical, only a decimal estimate is possible." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Evidence case: an exact point $(2\sqrt{3}, 2)$. Decide whether it determines an exact tangent, only an estimate, or no domain conclusion. State the decisive evidence and any exact datum still needed.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Value status	x	y
exact	$2\sqrt{3}$	2

Tangent Values and Domain

Full Worked Solutions - excerpt

Worked example

Problem. At $\theta = \pi, (\sin\theta, \cos\theta) = (0, -1)$. Evaluate $\tan\theta$ as a signed coordinate quotient and simplify exactly.

Answer. $\tan(\pi) = 0$

Explanation and check.

Cosine -1 is the denominator. The quotient $\frac{0}{-1}$ simplifies to 0.

Worked example

Problem. Given $\tan(3\frac{\pi}{4}) = -1$, use "add π" to infer $\tan(7\frac{\pi}{4})$. Explain whether one or both coordinate signs change.

Answer. $\tan(7\frac{\pi}{4}) = -1$

Explanation and check.

Adding π takes the opposite ray, so both x and y change sign. The quotient $\frac{y}{x}$ is unchanged, giving $\tan(7\frac{\pi}{4}) = -1$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Trig Signs and Reference Angles

Trig Signs and Reference Angles | APPC-U03-P06

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for three signs with coordinate evidence, all allowed angles, terminal location and reference angle, correct boundary classification, possible location or inconsistency and all consistent pairings or contradiction.

8 PDFs and a README; 107 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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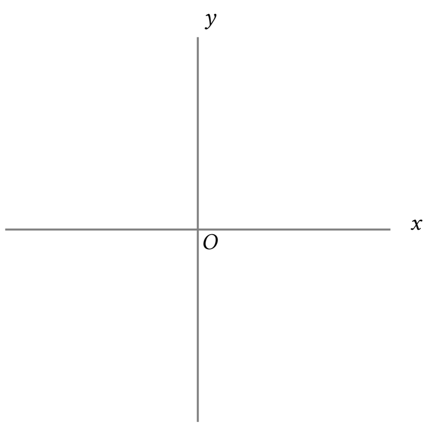
AP Precalculus

Trig Signs and Reference Angles

Name: ______________ **Date:** ______________ **Class:** ______________

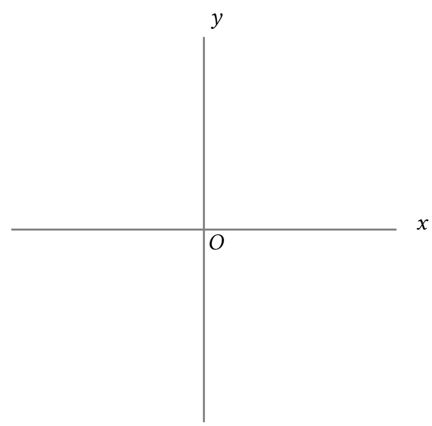
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Intersect the constraints $\sin\theta > 0, \cos\theta \neq 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.



Bank label: ______

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(2\pi - \alpha)$ using $\cos\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.



Bank label: ______

Intersect the constraints $\cos\theta < 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Bank label: ______

Construct every angle in $[0, 2\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant IV. Respect both endpoint flags and include all admitted turns.

Bank label: ______

AP Precalculus

Trig Signs and Reference Angles

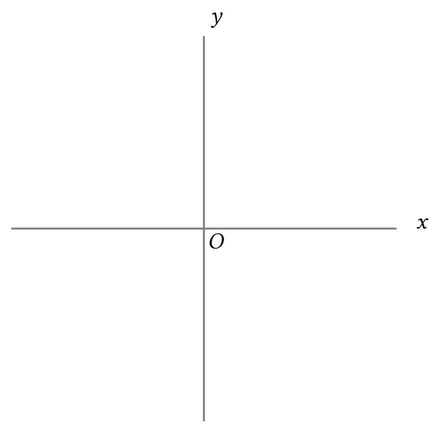
Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For quadrant IV coordinate box with $x > 0, y < 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.

Given evidence	**Sine**	**Cosine**	**Tangent**
$x > 0, y < 0$	______	______	______

For positive y-axis ray with $x = 0, y > 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.



Reduce the standard-position rotation $\theta = 7\frac{\pi}{6}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

Reduce the standard-position rotation $\theta = 11\frac{\pi}{4}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

AP Precalculus

Trig Signs and Reference Angles

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\theta = 2\frac{\pi}{3}$

For negative x-axis ray with $x < 0, y = 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required.

Given evidence	Sine	Cosine	Tangent
$x < 0, y = 0$	____	____	____

Your response: ____________

Incoming: $\theta = 7\frac{\pi}{6}$

Intersect the constraints $\sin\theta = 0, \cos\theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Your response: ____________

Incoming: $5\frac{\pi}{6}$

Intersect the constraints $\sin\theta > 0, \cos\theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Your response: ____________

Incoming: quadrant I

Construct every angle in $[0, 4\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant III. Respect both endpoint flags and include all admitted turns.

Your response: ____________

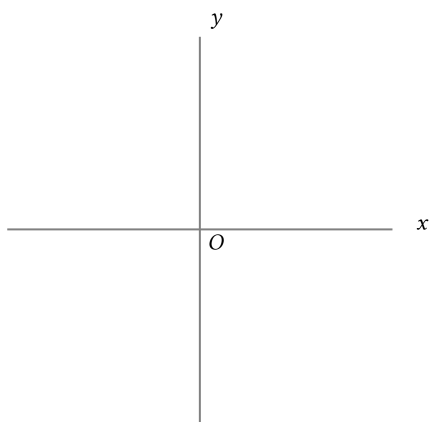
AP Precalculus

Trig Signs and Reference Angles

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(3\pi - \alpha)$ using $\cos\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α. First select and justify the valid method. Method A: Reduce $3\pi - \alpha$ to $\pi - \alpha$ and use the reflected x-coordinate. Method B: Cancel 3π as though it were an integer multiple of a full turn.



Trig Signs and Reference Angles

Full Worked Solutions - excerpt

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\sin(3\pi - \alpha)$ using $\sin\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.

Answer. $\sin(3\pi - \alpha) = \sin\alpha$

Explanation and check.

The reference magnitude is $\sin\alpha$; reduce to $\pi - \alpha$ in quadrant II. Therefore $\sin(3\pi - \alpha) = \sin\alpha$.

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\tan(2\pi - \alpha)$ using $\tan\alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α.

Answer. $\tan(2\pi - \alpha) = -\tan\alpha$

Explanation and check.

The reference magnitude is $\tan\alpha$; quadrant IV has opposite coordinate signs. Therefore $\tan(2\pi - \alpha) = -\tan\alpha$.

Sets & Steps

Math Practice & Worked Solutions

AP Precalculus

Sine and Cosine Parent Graphs

Sine and Cosine Parent Graphs | APPC-U03-P07

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for direction, sign, turning, and bending description, exact average rate and interpretation, missing projection and graph identity, parent graph and projection explanation, projection error and correction and missing anchors and verified curve.

8 PDFs and a README; 191 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

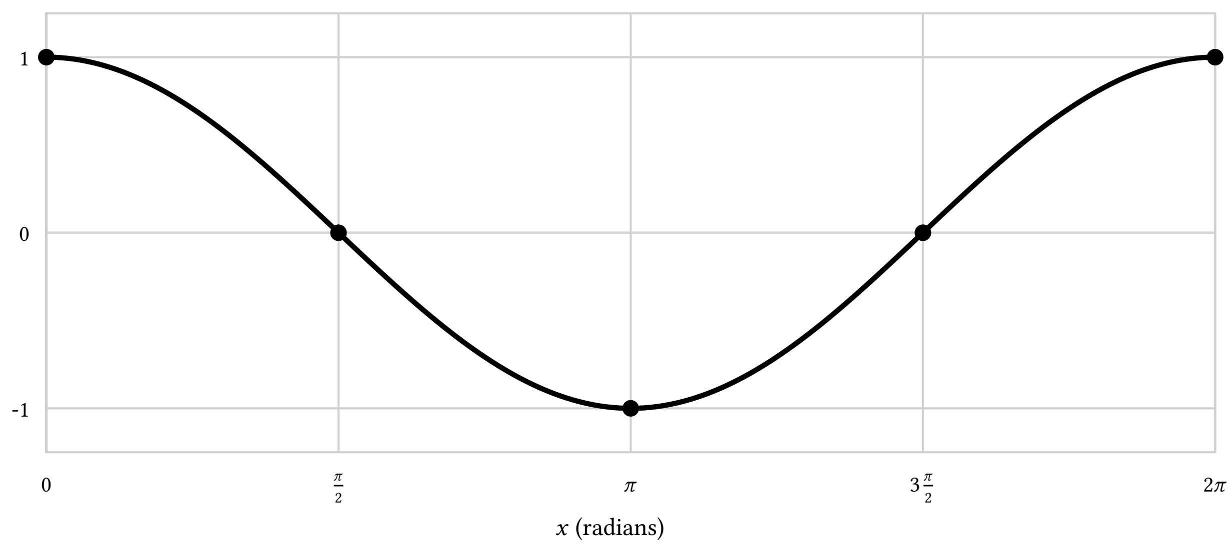
AP Precalculus

Sine and Cosine Parent Graphs

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

From the labeled parent graph $y = \cos x$ on the closed window $[0, 2\pi]$, identify its zeros. List event coordinates rather than confusing x-locations with output values, and include boundary events.



x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$3\frac{\pi}{2}$	0
2π	1

Bank label: ______

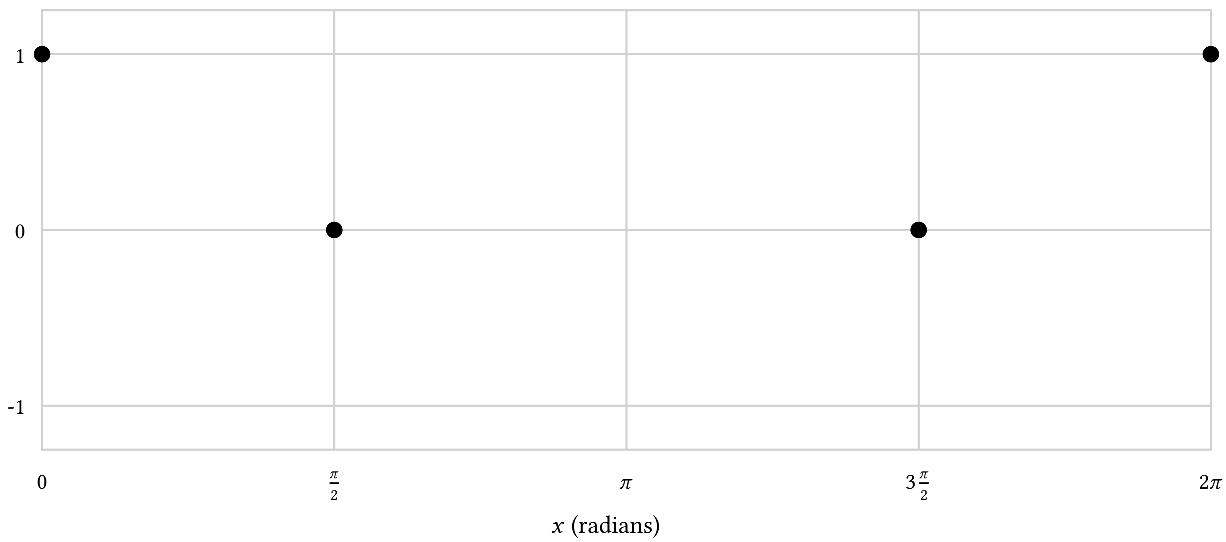
AP Precalculus

Sine and Cosine Parent Graphs

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For the stated parent $y = \cos x$ on $[0, 2\pi]$, restore every question-mark quarter-turn anchor from the partial data, then sketch one smooth continuous parent curve with exact endpoints.



x	y
0	1
$\frac{\pi}{2}$	0
π	____
$3\frac{\pi}{2}$	0
2π	1

AP Precalculus

Sine and Cosine Parent Graphs

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $y = \cos x$ on $[0, \frac{\pi}{2}]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Required response

x	**y**
0	1
$\frac{\pi}{2}$	0

Your response: ____

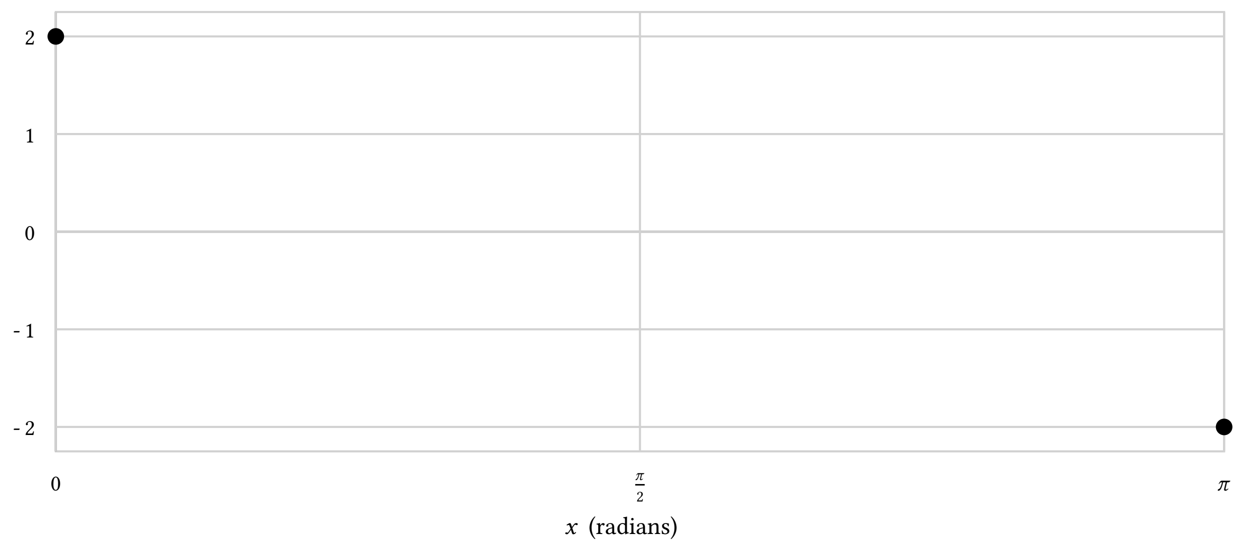
AP Precalculus

Sine and Cosine Parent Graphs

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the proposed $y = \cos x$ sketch using value at zero, quarter-turn anchors, range, cycle joins, and smoothness. Identify the first structural error and give the smallest sufficient repair.



x	**y**
0	2
π	-2

Sine and Cosine Parent Graphs

Full Worked Solutions - excerpt

Worked example

Problem. For $y = \cos x$ on $[\frac{\pi}{2}, 2\pi]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Answer. average rate = $\frac{2}{3\pi}$; output rises overall

Explanation and check.

Compute $\frac{(1)-(0)}{(2\pi)-(\frac{\pi}{2})} = \frac{2}{3\pi}$. Thus output rises overall.

Worked example

Problem. For $y = \cos x$ on $[3\frac{\pi}{2}, 2\pi]$, compute the finite average rate of change in output per radian from the exact endpoint values. Interpret the sign, and do not use derivative language.

Answer. average rate = $\frac{2}{\pi}$; output rises overall

Explanation and check.

Compute $\frac{(1)-(0)}{(2\pi)-(3\frac{\pi}{2})} = \frac{2}{\pi}$. Thus output rises overall.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sinusoidal Amplitude and Midline

Sinusoidal Amplitude and Midline | APPC-U03-P08

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for exact transformed local range, feature comparison and invariant, global pointwise relation and proof, vertical transformation equation, first error and minimal repair and invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Sinusoidal Amplitude and Midline

Name: ______ **Date:** ______ **Class:** ______

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

For $y = -4\cos x + 1$, complete the table by applying the vertical output map to each listed parent output. Use the sign of the scale rather than reading a decorative graph height.

Required response

parent output	transformed output
0	______
1	______

Bank label: ______

Under $y = 3\sin x + 2$, a transformed output $Y = \frac{23}{4}$ is requested. Reverse the output map to find the required parent output, then decide whether that output is available from the sin parent.

Required response

required parent output	transformed output
______	$\frac{23}{4}$

Bank label: ______

For $y = 2\cos x + 5$, complete the table by applying the vertical output map to each listed parent output. Use the sign of the scale rather than reading a decorative graph height.

Required response

parent output	transformed output
-1	______
0	______
1	______

Bank label: ______

A complete cycle has minimum 2 and maximum 6. For the supplied parent $p(x) = \sin x$, the labeled anchor has parent output -1 and transformed output 6. Recover amplitude, midline, and every coefficient sign supported by this evidence.

Required response

feature	output	model output	parent output
global minimum	2	______	______
global maximum	6	______	______
anchor $x = 3\frac{\pi}{2}$	______	6	-1

Bank label: ______

AP Precalculus

Sinusoidal Amplitude and Midline

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $y = (-\frac{1}{2}) \cos x + 2$, the horizontal rule is fixed as the cos parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	On the stated x-window $[0, \frac{\pi}{2}]$, the sin parent has certified local output range $[0, 1]$. Under $y = 2 \sin x + 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.
For $y = -3 \sin x - 2$, the horizontal rule is fixed as the sin parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	On the stated x-window $[-\frac{\pi}{2}, 0]$, the sin parent has certified local output range $[-1, 0]$. Under $y = 4 \sin x + 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.
For $y = 4 \sin x - 3$, the horizontal rule is fixed as the sin parent. State the amplitude, midline, full-cycle range, and whether the parent output is vertically reflected.	Compare $y = 3 \cos x - 1$ and $y = -3 \cos x - 1$. State amplitudes, midlines, full-cycle ranges, whether the ranges overlap, relative vertical orientation, and the horizontal invariant.

AP Precalculus

Sinusoidal Amplitude and Midline

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

On the stated x-window $[\frac{\pi}{3}, \pi]$, the cos parent has certified local output range $[-1, \frac{1}{2}]$. Under $y = -2\cos x + 2$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.

Your response: ______________

Incoming: $[1, 4]$; order reversed under the negative scale

Starting from $p(x) = \cos x$, perform this vertical-only task: Translate parent outputs by $\frac{3}{2}$, scale the result by 2, and finally translate by -5. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Your response: ______________

Incoming: $y = 2(\cos x + \frac{3}{2}) - 5$

On the stated x-window $[0, \frac{\pi}{2})$, the cos parent has certified local output range $(0, 1]$. Under $y = 3\cos x - 1$, determine the transformed range on this same window, preserving open or closed endpoints. Do not substitute the full-cycle range.

Your response: ______________

Incoming: $(-1, 2]$; order preserved under the positive scale

A complete cycle has minimum -9 and maximum -1. For the supplied parent $p(x) = \cos x$, the labeled anchor has parent output 1 and transformed output -1. Recover amplitude, midline, and every coefficient sign supported by this evidence.

Required response

feature	output	model output	parent output
global minimum	-9	______	______
global maximum	-1	______	______
anchor $x = 0$	______	-1	1

Your response: ______________

AP Precalculus

Sinusoidal Amplitude and Midline

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the student's vertical-feature inference using full-cycle extrema, the affine output map, operation order, and visible-window limits as relevant. Identify the first mathematical error and give the smallest sufficient repair. output map: $Y = -2p + 3$; restricted parent range: $[0, 1]$; student range: $[1, 5]$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

A student claims: "Using the first two rows gives $a = 1$ and $d = 3$, so $Y = \cos x + 3$ fits all three rows." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Fit $Y = ap + d$ to the first two distinct parent-output rows in the supplied cos table, then audit the third row. State whether one vertical-only sinusoid fits all observations.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

parent output	**target output**
-1	-2
0	1
1	4

Sinusoidal Amplitude and Midline

Full Worked Solutions - excerpt

Worked example

Problem. Starting from $p(x) = \cos x$, perform this vertical-only task: First multiply every cos output by -3, then translate outputs by -1. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Answer. y = $-3\cos x - 1$

Explanation and check.

Apply the actions to the output p in order. $Y = -3p - 1$; hence $y = -3\cos x - 1$.

Worked example

Problem. Starting from $p(x) = \cos x$, perform this vertical-only task: Reflect the graph $y = \cos x$ across the horizontal line $y = 4$. Write an equation with the same horizontal rule and show how the operation order controls the midline.

Answer. y = - $\cos x + 8$

Explanation and check.

Apply the actions to the output p in order. $Y = 2(4) - p = -p + 8$; hence $y = -\cos x + 8$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Period and Frequency of Sinusoids

Period and Frequency of Sinusoids | APPC-U03-P09

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for period and ordinary frequency, normalized frequencies and comparison, possible b values and verification, complete traversals, residual cycle, ending phase, first error and repair and supported candidates and needed evidence.

8 PDFs and a README; 75 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Period and Frequency of Sinusoids

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

For $y = \cos((-\frac{\pi}{4})x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: ______	Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 4π; input is measured in radian units. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units before using $\|b\| = \frac{2\pi}{T}$. Bank label: ______
A sinusoid has period $\frac{4}{3}$ seconds. An inclusive observation window lasts 5 seconds and begins $\frac{1}{2}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal. Bank label: ______	For $y = \cos((4)x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: ______
Construct $y = \sin(bx)$ or $y = \cos(bx)$ with frequency $\frac{1}{8}$ cycles per hour; input is measured in hours. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units if needed before using $\|b\| = \frac{2\pi}{T}$. Bank label: ______	A sinusoid has period 8 seconds. An inclusive observation window lasts 17 seconds and begins $\frac{7}{8}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal. Bank label: ______
For $y = \cos((3\frac{\pi}{2})x)$, the argument is in radians and x is measured in hours. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency. Bank label: ______	Construct $y = \sin(bx)$ or $y = \cos(bx)$ with frequency 2 cycles per second; input is measured in seconds. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units if needed before using $\|b\| = \frac{2\pi}{T}$. Bank label: ______

AP Precalculus

Period and Frequency of Sinusoids

Name: ____________ **Date:** ____________ **Class:** ____________

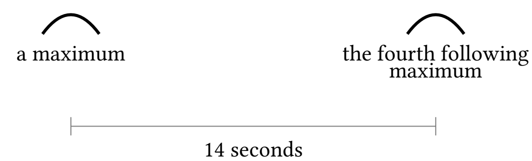
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Map the listed parent-angle anchors u to the graph of $y = \cos(-3x)$. Complete the transformed x-inputs using $u = bx$, retain each parent output, and state whether left-to-right anchor order is preserved or reversed.

Required response

parent angle	parent output	transformed input
0	1	______
$\frac{\pi}{2}$	0	______
π	-1	______
$3\frac{\pi}{2}$	0	______
2π	1	______

A sinusoidal graph labels a maximum and the fourth following maximum 14 seconds apart. The graph explicitly certifies the stated events as consecutive or gives the exact number skipped, and crossing directions are shown. Determine the smallest period and justify the cycle fraction used.



Map the listed parent-angle anchors u to the graph of $y = \cos(\frac{1}{3}x)$. Complete the transformed x-inputs using $u = bx$, retain each parent output, and state whether left-to-right anchor order is preserved or reversed.

Required response

parent angle	parent output	transformed input
0	1	______
$\frac{\pi}{2}$	0	______
π	-1	______
$3\frac{\pi}{2}$	0	______

Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 360; input is measured in degrees of physical input. The parent phase at the origin is fixed. Respect the stated sign information (either) and list every supported b, converting units before using $|b| = \frac{2\pi}{T}$.

AP Precalculus

Period and Frequency of Sinusoids

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A sinusoid has period 3 seconds. An inclusive observation window lasts 8 seconds and begins 0 of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal.

Your response: ____________

Incoming: 2 complete cycles; residual $\frac{2}{3}$ cycle; ending phase $\frac{2}{3}$ cycle

Construct $y = \sin(bx)$ or $y = \cos(bx)$ with period 5; input is measured in minutes. The parent phase at the origin is fixed. Respect the stated sign information (positive) and list every supported b, converting units before using $|b| = \frac{2\pi}{T}$.

Your response: ____________

Incoming: b = $2\frac{\pi}{5}$

For $y = \cos((\frac{1}{2})x)$, the argument is in radians and x is measured in radian input. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Your response: ____________

Incoming: period 4π; frequency $\frac{1}{4\pi}$ cycles per input unit

A sinusoid has period 2 seconds. An inclusive observation window lasts 6 seconds and begins $\frac{1}{4}$ of a cycle after the fixed cycle origin. Count complete cycle traversals within the elapsed window, the residual fraction of a traversal, and the ending phase modulo one cycle. Do not count an endpoint event as an extra traversal.

Your response: ____________

AP Precalculus

Period and Frequency of Sinusoids

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The supplied cos sample table is finite. Audit the listed candidate rules against every row and decide whether the period is identified. Explain any aliasing and name one minimal additional sample or bound that would resolve or narrow it. Measure the trigonometric arguments in radians. Compare the candidate rules $\cos(2\pi t), \cos(4\pi t)$. Work with sinusoidal recurrence with fixed origin and phase. No additional constraint is supplied. Only the displayed finite samples are given. The supplied samples are (output 1; $t0$), (output $-\frac{1}{2}$; $t\frac{1}{3}$), (output $-\frac{1}{2}$; $t\frac{2}{3}$), (output 1; $t1$).

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

output	**t**
1	0
$-\frac{1}{2}$	$\frac{1}{3}$
$-\frac{1}{2}$	$\frac{2}{3}$
1	1

Audit the proposed horizontal-scaling explanation. Check nonconstancy, full-cycle phase change, landmark direction, sampling limits, and input units as relevant. Identify the first error and give the smallest sufficient repair. Work with sinusoidal recurrence with fixed origin and phase. Use $y = \sin(\pi \frac{t}{30})$. A student claims: frequency is $\frac{\pi}{30}$ cycles/s Measure t in seconds.

Period and Frequency of Sinusoids

Full Worked Solutions - excerpt

Worked example

Problem. For $y = \sin((\pi)x)$, the argument is in radians and x is measured in seconds. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Answer. period 2; frequency $\frac{1}{2}$ cycles per input unit

Explanation and check.

For a nonconstant parent, $T = \frac{2\pi}{|b|} = 2$. Then ordinary frequency is $\frac{1}{T} = \frac{1}{2}$; b instead measures radians of phase per input unit.

Worked example

Problem. For $y = \sin((6)x)$, the argument is in radians and x is measured in days. Find the smallest positive period and the ordinary frequency in cycles per input unit. Explain why the inside multiplier itself is not generally the ordinary frequency.

Answer. period $\frac{\pi}{3}$; frequency $\frac{3}{\pi}$ cycles per input unit

Explanation and check.

For a nonconstant parent, $T = \frac{2\pi}{|b|} = \frac{\pi}{3}$. Then ordinary frequency is $\frac{1}{T} = \frac{3}{\pi}$; b instead measures radians of phase per input unit.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Phase Shifts of Sine and Cosine

Phase Shifts of Sine and Cosine | APPC-U03-P10

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for equivalence and canonical representatives, first error and minimal repair, new phase shift and event verification, shifted anchors and window qualification, phase classes or consistency conclusion and phase class and canonical representative.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Phase Shifts of Sine and Cosine

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \cos x$ horizontally by $h = 3\frac{\pi}{2}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** -1	______
x $-\frac{\pi}{2}$ **y** 0	______
x 0 **y** 1	______
x $\frac{\pi}{2}$ **y** 0	______

Bank label: ______

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: upward midline; x: 1); period: 4; signed amplitude: 1

Required response

event	x
upward midline	1

Bank label: ______

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (event: minimum; x: 7); period: 10; signed amplitude: 1

Required response

event	x
minimum	7

Bank label: ______

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: downward midline; x: 5); period: 8; signed amplitude: 1

Required response

event	x
downward midline	5

Bank label: ______

AP Precalculus

Phase Shifts of Sine and Cosine

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \sin x$ horizontally by $h = 13\frac{\pi}{4}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** 0	______
x 0 **y** 0	______
x π **y** 0	______
x 2π **y** 0	______

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (direction: not specified; level: midline; x: 3); period: 8; signed amplitude: 1

Required response

direction	level	x
not specified	midline	3

Translate every listed landmark of $y = \cos x$ horizontally by $h = -4\frac{\pi}{3}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ **y** -1	______
x $-\frac{\pi}{2}$ **y** 0	______
x 0 **y** 1	______
x $\frac{\pi}{2}$ **y** 0	______

For y=-cos(2π(x-h)/10), amplitude sign and period are fixed. A labeled maximum occurs at x=7. Find h modulo the period and give the representative in [0,10).

AP Precalculus

Phase Shifts of Sine and Cosine

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: First issue: a positive-sine peak occurs a quarter period after h, so $h = 0$; repair/defense: subtract $\frac{T}{4}$ from the peak location.

Rewrite the radian argument of y=cos((7/3)x-14π/9) in the form b(x-h). State h in x-units and distinguish it from the angular constant -14π/9.

Your response: ______________

Incoming: not equivalent; representatives $\frac{9}{2}$ and 2 differ modulo 5

Two sine rules have the same signed amplitude and period 7, with shifts h_1=−2 and h_2=12. Decide whether they define the same graph. Reduce each shift modulo the period; a half-period difference does not count as equivalent when the amplitude sign is fixed.

Your response: ______________

Incoming: $h \equiv \frac{3}{2}$ (mod 5); canonical $h = \frac{3}{2}$

Rewrite the radian argument of y=sin((-4/3)x+10π/9) in the form b(x-h). State h in x-units and distinguish it from the angular constant 10π/9.

Your response: ______________

Incoming: equivalent; difference 12 = 1 period(s); common representative 7

For y=sin(2π(x-h)/(5/2)), amplitude sign and period are fixed. A labeled minimum occurs at x=19/8. Find h modulo the period and give the representative in [0,5/2).

Your response: ______________

Incoming: argument $\frac{7}{3}(x - 2\frac{\pi}{3})$; $h = 2\frac{\pi}{3}$

Rewrite the radian argument of y=sin((-9/4)x-27π/8) in the form b(x-h). State h in x-units and distinguish it from the angular constant -27π/8.

Your response: ______________

Incoming: argument $10(x - 3\frac{\pi}{10})$; $h = 3\frac{\pi}{10}$

Rewrite $y = 2\cos(2\pi\frac{x-\frac{3}{2}}{9})$ using the sin reference while holding period and midline fixed. Track the signed coefficient and explain the quarter-cycle phase convention; for advanced cases give both same-sign and opposite-sign target representatives.

Your response: ______________

AP Precalculus

Phase Shifts of Sine and Cosine

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the proposed phase-shift claim. Normalize the argument, use the named event and signed family, and reduce representatives modulo the fixed period. Identify the first error or defend a valid noncanonical shift.

Given: domain: phase analysis with amplitude and period fixed; rule: $y = \sin(2x - \pi)$; student claim: shift right π

A student applies the horizontal shift to output values instead of translating the input landmarks with the stated sign. Diagnose the first mathematical error and state the correction. Then complete the original task: Translate every listed landmark of $y = \sin x$ horizontally by $h = \frac{\pi}{2}$. Complete the shifted coordinates, retain all y-values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	**shifted x**
x 0 **y** 0	_____
x $\frac{\pi}{2}$ **y** 1	_____
x π **y** 0	_____

Phase Shifts of Sine and Cosine

Full Worked Solutions - excerpt

Worked example

Problem. Rewrite the radian argument of y=cos((-7/2)x+21π/4) in the form b(x-h). State h in x-units and distinguish it from the angular constant 21π/4.

Answer. argument $-\frac{7}{2}(x-3\frac{\pi}{2})$; $h=3\frac{\pi}{2}$

Explanation and check.

Solve $-\frac{7}{2}(x\text{-}h)=(-\frac{7}{2})x+21\frac{\pi}{4}$. Since $h=-\frac{c}{b}, h=3\frac{\pi}{2}$; the inside constant itself is not the displacement unless $b=-1$.

Worked example

Problem. Rewrite the radian argument of y=cos(10x-3π) in the form b(x-h). State h in x-units and distinguish it from the angular constant -3π.

Answer. argument $10(x-3\frac{\pi}{10})$; $h=3\frac{\pi}{10}$

Explanation and check.

Solve $10(x\text{-}h)=10x-3\pi$. Since $h=-\frac{c}{b}, h=3\frac{\pi}{10}$; the inside constant itself is not the displacement unless $b=-1$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Writing Sinusoidal Equations from Features

Writing Sinusoidal Equations from Features | APPC-U03-P11

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for existence decision and first contradiction, equivalence decision and decisive normalized evidence, one compatible rule and sufficiency explanation, one complete rule and parameter verification, one example rule or impossibility proof and one sine rule with directional check.

8 PDFs and a README; 152 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Writing Sinusoidal Equations from Features

Name: ______ **Date:** ______ **Class:** ______

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$-\frac{5}{2}$
next global maximum	1	$\frac{5}{2}$

Bank label: ______

Determine which sinusoidal parameters are forced by the partial exact features and which remain free. Demonstrate any nonuniqueness by checking the two explicit candidate constructions, or replace either if it fails.

The first proposed rule is $y = 3\cos(2\pi \frac{x+\frac{5}{2}}{6})$.

The second proposed rule is $y = 3\cos(2\pi \frac{x+\frac{5}{2}}{3})$.

The global range is $[-3, 3]$.

The two stated maxima are not specified to be consecutive.

Maxima occur at $x = -\frac{5}{2}$ and $x = \frac{7}{2}$.

Bank label: ______

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$\frac{3}{2}$
next global maximum	3	$\frac{21}{2}$

Bank label: ______

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 10.

The claimed range is $[-4, 4]$.

The reference input is $x = -\frac{3}{2}$.

The first proposed rule is $y = 4\sin(2\pi \frac{x+\frac{3}{2}}{10})$.

The second proposed rule is $y = 4\cos(2\pi \frac{x-1}{10})$.

Bank label: ______

AP Precalculus

Writing Sinusoidal Equations from Features

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

An exact partial graph is supplied with its global range, labeled events and directions, and the stipulation that the curve continues sinusoidally. Construct one compatible cosine rule and explain why clipping does or does not hide needed information.

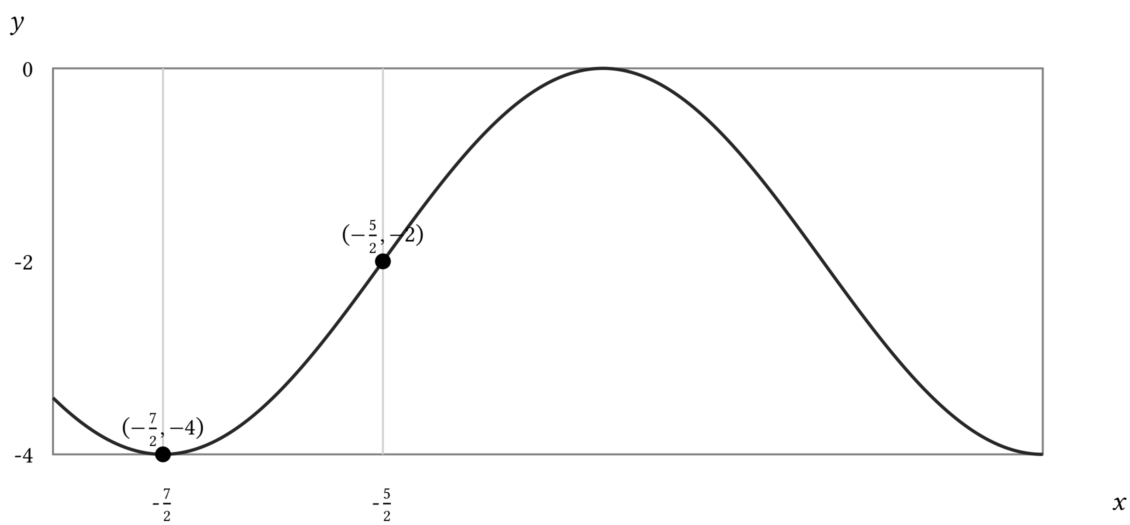
The curve continues sinusoidally beyond the displayed interval.

The global range is $[-4, 0]$.

The labeled minimum is $(-\frac{7}{2}, -4)$.

The labeled upward midline is $(-\frac{5}{2}, -2)$.

The displayed x-interval is $[-4, \frac{1}{2}]$.



event	x	y
minimum	$-\frac{7}{2}$	-4
upward midline	$-\frac{5}{2}$	-2

AP Precalculus

Writing Sinusoidal Equations from Features

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

event	value	x
global maximum	0	$\frac{21}{2}$
next global maximum	0	$\frac{57}{2}$
global minimum	-6	not specified

Your response: ____________

AP Precalculus

Writing Sinusoidal Equations from Features

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the student's multi-parameter reconstruction against every exact feature. Identify the earliest failed parameter relation, retain valid deductions, give a corrected complete rule, and verify the repaired model.

Preserve the valid deductions when repairing the model.

The supplied features are: range $[0, 6]$, period 4, upward midline at $x = 1$.

The student proposes $y = 3 + 3\sin(2\pi\frac{x-1}{8})$.

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 14.

The claimed range is $[-3, 1]$.

The reference input is $x = \frac{11}{2}$.

The first proposed rule is $y = -1 + 2\sin(2\pi\frac{x-\frac{11}{2}}{14})$.

The second proposed rule is $y = -1 + 2\sin(2\pi\frac{x-9}{14})$.

Writing Sinusoidal Equations from Features

Full Worked Solutions - excerpt

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = 1$.

The global range is $[-2, 2]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-2+2}{2} = 0$, not 1. Therefore the feature set is inconsistent.

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = -1$.

The global range is $[-4, 0]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-4+0}{2} = -2$, not -1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-4+0}{2} = -2$, not -1. Therefore the feature set is inconsistent.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Sinusoidal Modeling

Sinusoidal Modeling | APPC-U03-P12

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for family decision with evidentiary limit, model recommendation or insufficiency conclusion, invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result, invalid assumption and actionable repair, plausible fitted rule and uncertainty statement and signed-residual diagnosis and revision recommendation.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Sinusoidal Modeling

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 3\sin(2\pi\frac{t+\frac{5}{2}}{8})$. The event level is $y = 10$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[-\frac{31}{6}, \frac{27}{2}]$.

Bank label: ______

The contextual extrema are exact and the cycle is stipulated stationary only on the feasible time domain. Construct one sinusoidal model in the stated time unit, explain which events determine a full or half cycle, and retain the domain restriction. The feasible time domain is $[8, 80]$. Time is measured in days.

Required response

event	time	value
minimum	8	11
next minimum	26	11

Bank label: ______

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 20 + 3\cos(2\pi\frac{t+3}{8})$. The response is measured in units. The sample time window is $[-11, 13]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = -3$. Time is measured in hours.

Bank label: ______

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 3\sin(2\pi\frac{t-\frac{11}{2}}{14})$. The event level is $y = 12$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[\frac{5}{6}, \frac{67}{2}]$.

Bank label: ______

AP Precalculus

Sinusoidal Modeling

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compare the fitted sinusoids for the stated use case. Weigh observed-window error, physical range, prediction horizon, and divergence. If the evidence cannot distinguish them for the intended decision, recommend a specific validation measurement. The intended decision is at $t = 34$. The observed time window is $[0, 13]$. The response tolerance is 1 unit.

Required response

future prediction	model	observed window RMSE	period
21	A	0.43	11
26	B	0.45	14

The dataset and fully specified candidate regression outputs are supplied. Select every usable model in the stated angle mode. Verify at least one observed or held-out landmark and do not infer software behavior that is not shown. Use radians mode on the calculator.

Required response

formula	label
$y = 3 + 2\cos(2\pi \frac{t-10}{8})$	A
$y = 3 + 2\sin(2\pi \frac{t-8}{8})$	B

t	y
10	5
14	1
18	5

Use the signed residual definition shown. Assess both the pattern across phases and the held-out observation; a small total error does not excuse a systematic pattern. Recommend retention, revision, or investigation. The absolute tolerance is 0.75. At held-out time $t = 5$, the signed residual is 0.11.

Required response

observed minus predicted	t
0.11	0
−0.11	1
0.0	2
0.11	3
−0.11	4

Use several measurements rather than a single high or low value to estimate a stationary sinusoid. Give a plausible rule, identify the approximate baseline, amplitude, period, and phase, and state what the finite noisy sampling cannot establish exactly. The measured response is in units. Time is measured in hours.

Required response

measurement	time
24.2	$\frac{13}{2}$
19.8	$\frac{23}{2}$
20.2	$\frac{43}{2}$
23.8	$\frac{53}{2}$

AP Precalculus

Sinusoidal Modeling

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 23 + 4\cos(2\pi\frac{t-4}{12})$. The response is measured in units. The sample time window is $[-8, 28]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = 13$. Time is measured in hours.

Your response: ____________

Incoming: Prediction 8: 23 units; interpolation inside the sampled window.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 4\sin(2\pi\frac{t-\frac{5}{2}}{8})$. The event level is $y = 12$. Find the second occurrence of the midline level in the window, counting either direction. Time is measured in hours. Use the time window $[\frac{5}{2}, \frac{21}{2}]$.

Your response: ____________

Incoming: Event case 6: $t = \frac{13}{2}$ (downward).

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 9. Initially the point is initial vertical velocity zero, then downward. Use seconds as the time unit for t. The operating window is $\frac{1}{2} \le t \le \frac{85}{2}$. The radius is 6. The starting position is top. The starting time is $t = \frac{1}{2}$. One uniform rotation takes 14 seconds.

Your response: ____________

Incoming: $y = 9 + 6\cos(2\pi\frac{t-\frac{1}{2}}{14})$

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Treat negative depth as physically valid. A fitted water-depth sinusoid has range $[-2, 8]$ meters.

Your response: ____________

Incoming: Water depth in $[-2, 8]$ meters includes negative depth. Restrict use to nonnegative depths or refit with a nonnegative constrained model.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 6\sin(2\pi\frac{t-\frac{1}{2}}{14})$. The event level is $y = 10$. Find the all upward midline crossings in the window. Time is measured in hours. Use the time window $[\frac{1}{2}, \frac{57}{2}]$.

Your response: ____________

Incoming: Event case 4: $t = \frac{1}{2}, \frac{29}{2}, \frac{57}{2}$.

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 7. Initially the point is initial vertical velocity zero, then upward. Use seconds as the time unit for t. The operating window is $-\frac{5}{2} \le t \le \frac{43}{2}$. The radius is 3. The starting position is bottom. The starting time is $t = -\frac{5}{2}$. One uniform rotation takes 8 seconds.

Your response: ____________

AP Precalculus

Sinusoidal Modeling

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Combine times without a common clock origin. Tidal observations are in hours after midnight on different dates.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Repair only the phase of a stationary sinusoid. A sensor baseline rises steadily while residuals alternate.

Sinusoidal Modeling

Full Worked Solutions - excerpt

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Use $T = 3$ with t measured in seconds. The fitted period is 3 minutes but t is entered in seconds.

Answer. Assumption repair 3: convert the period to 180 seconds, or convert every input to minutes.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to convert the period to 180 seconds, or convert every input to minutes.

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Extrapolate the fitted cycle indefinitely. Seasonal data cover one year during an unusual shutdown.

Answer. Assumption repair 4: limit claims to the observed window and gather ordinary-year validation data.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to limit claims to the observed window and gather ordinary-year validation data.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Interpreting Sinusoidal Transformations

Interpreting Sinusoidal Transformations | APPC-U03-P13

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for changed and invariant physical features, relevant common-scale comparison, first misleading claim and corrected quantitative statement, phase event and initial-value qualification, contextual attained range and boundary status and parameter meanings and derived features.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Interpreting Sinusoidal Transformations

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes and $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 230 + 40\cos(2\pi\frac{t-90}{300})$; output unit before: centimeters. Bank label: ______	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 11 + 3\cos(2\pi\frac{t-3}{16})$; valid window: $0, 64$. Bank label: ______
Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 210 + 40\cos(2\pi\frac{t-30}{360})$; output unit before: centimeters. Bank label: ______	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 9 + 5\cos(2\pi\frac{t-4}{8})$; valid window: $0, 32$. Bank label: ______

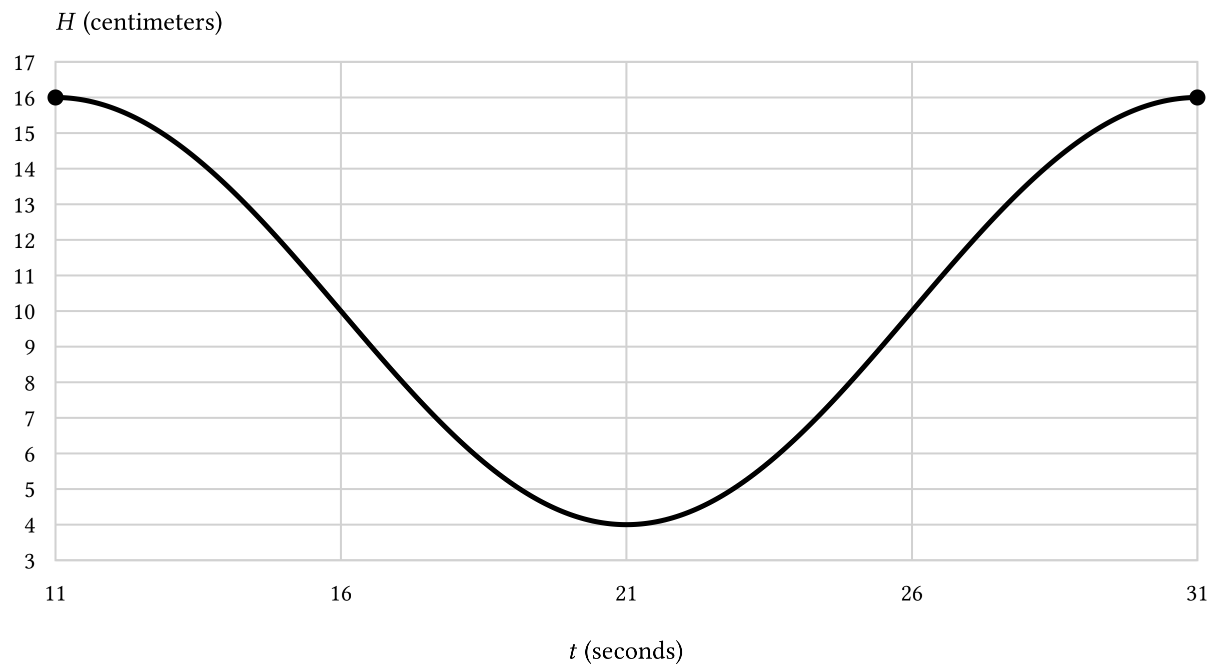
AP Precalculus

Interpreting Sinusoidal Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Interpret the supplied graph only on its physical operating window. State the attainable output interval, midline meaning, and whether displayed boundaries are attained; do not substitute the full mathematical range when the window clips an extremum.



AP Precalculus

Interpreting Sinusoidal Transformations

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{16})$; only changed category: period; original formula: $P(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{8})$; output unit: units; time unit: hours.

Your response: ______________________

Incoming: period doubles from 8 to 16 time units, so recurrence halves; range and the anchor at $t = \frac{7}{2}$ stay fixed.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: new datum is 52 cm above the old datum, so $R = Q - 52$; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 220 + 60\cos(2\pi\frac{t-60}{240})$; output unit before: centimeters.

Your response: ______________________

Incoming: $R(t) = 168 + 60\cos(2\pi\frac{t-60}{240})$; event times are unchanged while every height label decreases by 52 cm.

Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: meters; supplied formula: $Q(t) = 10 + 4\sin(2\pi\frac{t+\frac{3}{2}}{10})$; valid window: 0, 40.

Your response: ______________________

Incoming: amplitude 4 meters; midline 10 meters; period 10 hours; frequency $\frac{1}{10}$ cycle per hour; $h = -\frac{3}{2}$ hours labels an upward midline crossing.

Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 6 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12}), B(t) = 8 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12})$; practical measure: same-period.

Your response: ______________________

Incoming: same period 12 hours and same spread 10, but B's baseline and range are 2 units higher.

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(\tau) = 8 + 5\sin(2\pi\frac{\tau}{10})$; only changed category: clock; original formula: $P(t) = 8 + 5\sin(2\pi\frac{t-4}{10})$; output unit: units; time unit: hours.

Your response: ______________________

Incoming: with t = $\tau + 4$, the same physical anchor is labeled $\tau = 0$ rather than $t = 4$; range and physical period are unchanged.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 200 + 40\cos(2\pi\frac{t}{360})$; output unit before: centimeters.

Your response: ______________________

AP Precalculus

Interpreting Sinusoidal Transformations

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Test the student's contextual parameter statement against the supplied model. Identify the first misleading claim and replace it with a quantitatively correct statement including units or window limits where relevant. context units: stated in formula or prompt; student statement: The period is 4π .; supplied model: $Q(t) = 4 + 3\sin(4\pi(t-2))$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Interpreting Sinusoidal Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 2\sin(2\pi\frac{t+2}{8}), B(t) = 5 + 2\sin(2\pi\frac{t+2}{16})$; practical measure: same-range.

Answer. same range $[3, 7]$, but B recurs every 16 rather than 8 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[3, 7]$, but B recurs every 16 rather than 8 hours.

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 4\sin(2\pi\frac{t-1}{10}), B(t) = 5 + 4\sin(2\pi\frac{t-1}{20})$; practical measure: same-range.

Answer. same range $[1, 9]$, but B recurs every 20 rather than 10 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[1, 9]$, but B recurs every 20 rather than 10 hours.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Tangent Parent Function Graphs

Tangent Parent Function Graphs | APPC-U03-P14

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for signed unbounded behavior without an assigned value, method/conclusion choice, justification, and endpoint-aware output interval, tangent entries and branch conclusion, one disconnected increasing branch, domain, range, and monotonicity and endpoint-aware output interval.

8 PDFs and a README; 140 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Tangent Parent Function Graphs

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Select the candidate that can represent $y = \tan(x)$. Defend the choice with structural evidence and explain why local steepness or a cropped rising segment, by itself, cannot settle the match.

The axes have exact radian labels: yes.

Compare these candidates: A = decreasing branches between the same cosine-zero gaps; B = a continuous bounded wave with range $[-1, 1]$; C = U-shaped reciprocal branches with no x-intercepts; D = an odd graph through $(0, 0)$ whose central component approaches -∞ and +∞ at its excluded ends.

Apply this decision constraint: Distinguish increasing tangent branches from decreasing cotangent branches.

Some candidate cards are cropped: no.

Work on the domain structural identification of the tangent parent.

Bank label: _______

Complete the tangent column using $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Mark a row undefined rather than dividing by zero, then use the ordered rows to state one graph conclusion; do not connect points across an undefined input.

Required response

$\cos(x)$	$\sin(x)$	x
-1.000	0.025	$-\pi - 0.025$
-1.000	0.000	$-\pi$
-1.000	-0.025	$-\pi + 0.025$

Bank label: _______

AP Precalculus

Tangent Parent Function Graphs

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

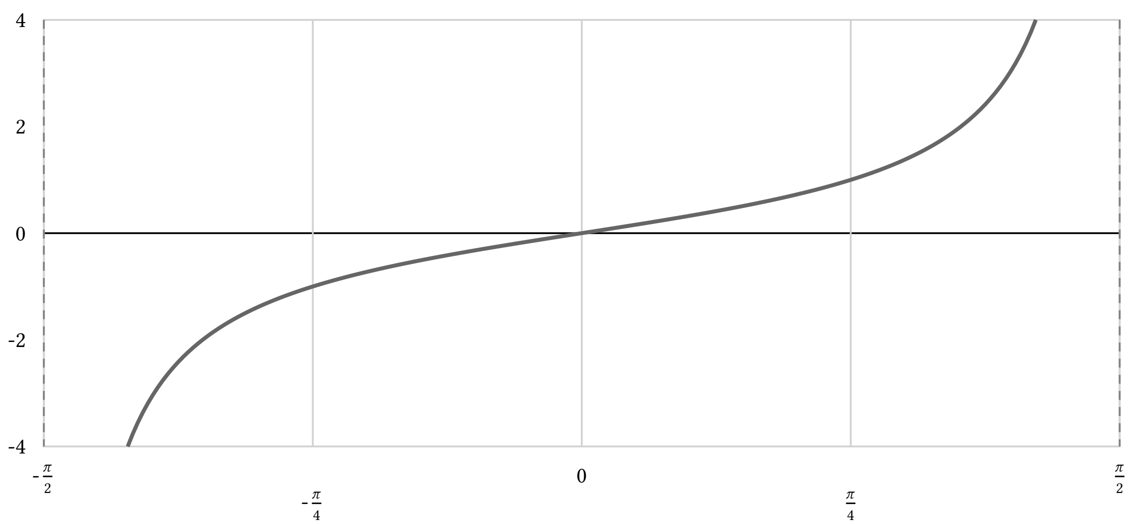
Reconstruct $y = \tan(x)$ on the displayed window using half-turn translations of the supplied central branch. List every visible zero and vertical asymptote, keep the branches disconnected, and state how many connected branch pieces meet the interior of the window.

The source branch is $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

The target window is display endpoints included is yes; left is $-9\frac{\pi}{4}$; right is $3\frac{\pi}{4}$.

The translation rule is (x, y) maps to $(x + k\pi, y)$.

Work on the domain half-turn repetition of the tangent parent.



AP Precalculus

Tangent Parent Function Graphs

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: the claim is false on $[5\frac{\pi}{4}, 7\frac{\pi}{4})$. The component ranges are $[1, \infty)$ and $(-\infty, -1)$; tangent increases on each component, but the gap at $3\frac{\pi}{2}$ prevents a single increasing interval claim.

Complete the tangent column using $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Mark a row undefined rather than dividing by zero, then use the ordered rows to state one graph conclusion; do not connect points across an undefined input.

Required response

$\cos(x)$	$\sin(x)$	x
0.999	−0.050	$-2\pi - 0.05$
1.000	0.000	-2π
0.999	0.050	$-2\pi + 0.05$

Your response: ______________

AP Precalculus

Tangent Parent Function Graphs

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Identify the defining contradiction in the student's tangent graph or statement. Give the smallest mathematical repair and support it with the quotient definition or branch structure; do not redraw unrelated valid features.

Evaluate claim 1.

Limit your correction to change only the contradicted feature.

Evaluate this student work: The graph labels $x = \frac{\pi}{2}$ as an x-intercept at $(\frac{\pi}{2}, 0)$.

Work on the domain diagnosis of a proposed tangent graph.

Identify the defining contradiction in the student's tangent graph or statement. Give the smallest mathematical repair and support it with the quotient definition or branch structure; do not redraw unrelated valid features.

Evaluate claim 2.

Limit your correction to change only the contradicted feature.

Evaluate this student work: The graph's y-axis states that the range of tangent is $[-1, 1]$.

Work on the domain diagnosis of a proposed tangent graph.

Tangent Parent Function Graphs

Full Worked Solutions - excerpt

Worked example

Problem. Use the supplied sine and cosine sign evidence to determine the one-sided behavior of $\tan(x)$ at the excluded input. Justify each quotient sign and explicitly state that the function has no value at the boundary.

The excluded input is $-5\frac{\pi}{2}$.

The requested order is both sides.

The supplied sign information is cos from left is -; cos from right is +; sin near boundary is $-$.

Work on the domain one-sided quotient behavior.

Answer. as $x \to -5\frac{\pi}{2^-}, \tan(x) \to +\infty$; as $x \to -5\frac{\pi}{2^+}, \tan(x) \to -\infty$; $\tan(-5\frac{\pi}{2})$ is undefined.

Explanation and check.

On the left, numerator and small denominator have the same sign, so the quotient is large positive. On the right their signs differ, so it is large negative. A one-sided trend does not define a value where cosine equals zero.

Worked example

Problem. Use the supplied sine and cosine sign evidence to determine the one-sided behavior of $\tan(x)$ at the excluded input. Justify each quotient sign and explicitly state that the function has no value at the boundary.

The excluded input is $-15\frac{\pi}{2}$.

The requested order is left side then right side.

The supplied sign information is cos changes sign is + to -; sin at boundary is $+1$.

Work on the domain one-sided quotient behavior.

Answer. as $x \to -15\frac{\pi}{2^-}, \tan(x) \to +\infty$; as $x \to -15\frac{\pi}{2^+}, \tan(x) \to -\infty$; $\tan(-15\frac{\pi}{2})$ is undefined.

Explanation and check.

On the left, numerator and small denominator have the same sign, so the quotient is large positive. On the right their signs differ, so it is large negative. A one-sided trend does not define a value where cosine equals zero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Tangent Function Transformations

Tangent Function Transformations | APPC-U03-P15

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for compatible pattern or precise contradiction, invariants, center outputs, and orientation, period, zero family, and asymptote family, failed relation and corrected feature statement, invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result and included zeros and asymptote locations.

8 PDFs and a README; 151 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Tangent Function Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

List every zero and every vertical-asymptote location inside the specified window. Solve integer bounds for the two families separately and honor each open or closed display endpoint exactly.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition	**Supplied value**
Given vertical-asymptote family	$x = \frac{\pi}{2} + k(\frac{\pi}{2})$
Required window	$(-3\frac{\pi}{4}, 5\frac{\pi}{4}]$
Given zero family	$x = \frac{\pi}{4} + k(\frac{\pi}{2})$

Bank label: ______

AP Precalculus

Tangent Function Transformations

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Compare the two supplied tangent models without solving a general tangent equation. State what happens to period, asymptotes, branch-center outputs, former zeros, and branch orientation after the vertical translation, with special attention to the required evidence focus. The supplied models are $f(x) = 3\tan(\frac{1}{2}(x))$ and $g(x) = 3\tan(\frac{1}{2}(x)) + 1$. The branch-center family is $x = 0 + k(2\pi)$. The required evidence focus is to state exactly which invariants come from the inner map.

Solve the problem and show enough exact work for the requested result.

Condition	Supplied value
Given branch-center input family	$x = 0 + k(2\pi)$
Required evidence focus	state exactly which invariants come from the inner map
Given output translation	1

AP Precalculus

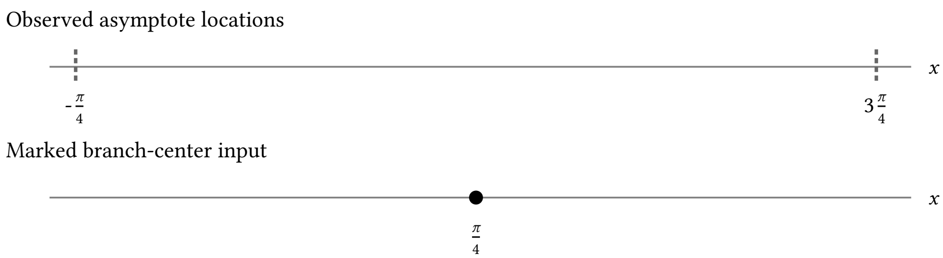
Tangent Function Transformations

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: zeros are $-15\frac{\pi}{4}, -7\frac{\pi}{4}, \frac{\pi}{4}, 9\frac{\pi}{4}, 17\frac{\pi}{4}$; asymptote locations are $-19\frac{\pi}{4}, -11\frac{\pi}{4}, -3\frac{\pi}{4}, 5\frac{\pi}{4}, 13\frac{\pi}{4}, 21\frac{\pi}{4}$.

Recover the horizontal period and a phase-anchor class from the graph evidence. Decide whether the asymptote spacing is one period or several, and state exactly what sign ambiguity remains; use orientation only when it is supplied.



Feature	Exact x-location(s)
Observed asymptote locations	$-\frac{\pi}{4}, 3\frac{\pi}{4}$
Marked branch-center input	$\frac{\pi}{4}$

Condition	Supplied value
Given adjacency information	certified consecutive
Given branch direction	not supplied
Given outer coefficient	not orientation-resolving

Your response: ____________________

AP Precalculus

Tangent Function Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “Omit $x = -7\frac{\pi}{3}$ and $x = 7\frac{\pi}{6}$ because boundary locations can never appear in requested zero or asymptote lists.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Use this route: audit both endpoint flags. Enumerate zeros from $x = -\frac{\pi}{3} + k(\pi)$ and poles from $x = \frac{\pi}{6} + k(\pi)$ inside $[-7\frac{\pi}{3}, 7\frac{\pi}{6}]$, honoring every endpoint.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition	Supplied value
Given vertical-asymptote family	$x = \frac{\pi}{6} + k(\pi)$
Required window	$[-7\frac{\pi}{3}, 7\frac{\pi}{6}]$
Given zero family	$x = -\frac{\pi}{3} + k(\pi)$

Tangent Function Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Find the first invalid relation in the worked tangent-feature calculation, repair only that relation, and verify the correction with one representative zero, asymptote, or spacing check.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition	Supplied value
Step to examine	calls every branch center an x-intercept after adding $d = 4$

Answer. centers have output 4 and need not be x-intercepts.

Explanation and check.

Check the tangent-specific period π, the separate inner-angle conditions for zeros and poles, and the role of any vertical offset. Preserve steps that remain valid after the single correction. At an old center, the inner angle is 0 and $\tan(0) = 0$. After the stated translation by 4, the output there is 4, not 0. This direct substitution verifies that the old center is not an x-intercept.

Worked example

Problem. Find the first invalid relation in the worked tangent-feature calculation, repair only that relation, and verify the correction with one representative zero, asymptote, or spacing check.

Compare the stated objects or approaches and justify the conclusion with exact evidence.

Condition	Supplied value
Step to examine	reports $T = -\frac{\pi}{3}$ for $b = -3$

Answer. period is positive $\frac{\pi}{3}$.

Explanation and check.

Check the tangent-specific period π, the separate inner-angle conditions for zeros and poles, and the role of any vertical offset. Preserve steps that remain valid after the single correction. With $b = -3$ and $T = \frac{\pi}{3}$, the inner-angle change is $-3(\frac{\pi}{3}) = -\pi$. Tangent has the same value after that change, while T is positive. The claimed negative value is not a period length.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Trigonometric Functions

Inverse Trigonometric Functions | APPC-U03-P16

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for valid inverse pair and reason for discarding a row, reason proposal fails and a valid restriction, validity and approximate corrected sign, defined or undefined, real-defined or undefined with failed restriction and validity and location of output.

8 PDFs and a README; 67 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Inverse Trigonometric Functions

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Evaluate $\arcsin(-\frac{\sqrt{3}}{2})$ exactly. Identify the compatible unit-circle angle that lies in the principal range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Bank label: ______	Let $\theta = \arcsin(\frac{5}{13})$. A right triangle for θ has opposite side 5 and hypotenuse 13. Find $\cos\theta$ and explain why its sign is forced by the arcsine branch. Bank label: ______
Evaluate $\arcsin(\sin(5\frac{\pi}{6}))$. Do not cancel the two function names; use a symmetry relation to place the result in the arcsine range. Bank label: ______	Find the exact principal value of $\arccos(-\frac{\sqrt{2}}{2})$. Use the sign of cosine and the required range $[0, \pi]$ to select the angle. Bank label: ______
Set $\theta = \arccos(-\frac{8}{17})$ and represent θ by a point (x, y) on a circle of radius 17 with $x = -8$. Determine $\tan\theta$, including its sign, from the principal arccos range. Bank label: ______	In radian mode, estimate $\arcsin(0.98)$ to the nearest thousandth. Verify that the rounded result lies in the correct principal interval. Bank label: ______
Reduce $\arccos(\cos(7\frac{\pi}{6}))$ to its principal value. Explain the reflection that produces an angle in $[0, \pi]$. Bank label: ______	Evaluate $\sin(\arctan(-\frac{7}{24}))$ exactly. Build a signed coordinate triangle compatible with the inverse-tangent range rather than treating the inner ratio as a sine value. Bank label: ______

AP Precalculus

Inverse Trigonometric Functions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Is $\arccos(\frac{6}{5})$ defined as a real number? Name the restriction that decides the question.

AP Precalculus

Inverse Trigonometric Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: -$\frac{\pi}{6}$ Place 13$\frac{\pi}{6}$ on the tangent graph and remove enough periods to reach the inverse-tangent branch. Evaluate the composition. Your response: ______________	**Incoming:** y = arccos x. Both -$\frac{\pi}{6}$ and 5$\frac{\pi}{6}$ have tangent -$\frac{\sqrt{3}}{3}$. Which one equals arctan($-\frac{\sqrt{3}}{3}$), and why is the other rejected? Your response: ______________
Incoming: $\sqrt{(}$1-v^2), for $-1 \le v \le 1$ Estimate arccos(-0.6) to the nearest tenth of a degree. Use the negative input to check the calculator's quadrant. Your response: ______________	**Incoming:** 126.9 degrees For θ = arccos($-\frac{3}{5}$), place a point of radius 5 with $x = -3$ in the correct half of the unit-circle model. Find sin θ. Your response: ______________
Incoming: 0.045 radians Without finding θ itself, evaluate cot(arctan($-\frac{9}{40}$)). State why this reciprocal is defined. Your response: ______________	**Incoming:** $\frac{\pi}{3}$ A calculation table records arcsin(0.73) = 0.818321 in radian mode. Report the value to the nearest hundredth and verify its range. Your response: ______________

AP Precalculus

Inverse Trigonometric Functions

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student writes, "$\arccos(\cos(4\frac{\pi}{3})) = 4\frac{\pi}{3}$ because inverse functions cancel." Identify the failed condition and replace the conclusion with the correct principal angle.

A solution interprets $\sin^{-1}(\frac{1}{2})$ as the reciprocal $\frac{1}{\sin(\frac{1}{2})}$. Diagnose the notation error, rewrite the inverse with unambiguous notation, and evaluate it exactly.

A symbolic solution states $\tan(\arcsin u) = \frac{u}{\sqrt{1-u^2}}$ for $-1 \leq u \leq 1$. Repair the domain and explain what happens at $u = \pm 1$ in the original composition, not just in the formula.

A calculator returns 45 for $\arctan(1)$, and a student reports 45 radians. Diagnose the display and give the correct radian answer.

Inverse Trigonometric Functions

Full Worked Solutions - excerpt

Worked example

Problem. Use radian mode to approximate $\arccos(0.2)$ to three decimal places.

Answer. 1.369 radians

Explanation and check.

The calculator value $1.369438\ldots$ rounds to 1.369 radians.

Worked example

Problem. Find the domain of $G(x) = \arcsin(3 - x)$. Preserve the inequality reversal when isolating x.

Answer. $[2, 4]$

Explanation and check.

Require $-1 \leq 3 - x \leq 1$. The two bounds give $x \leq 4$ and $x \geq 2$, hence $2 \leq x \leq 4$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Solving Trigonometric Equations

Solving Trigonometric Equations | APPC-U03-P17

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
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07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for count by h-region, count by level case, count for each L, number of distinct solutions for each case, solution count by case and all roots.

8 PDFs and a README; 47 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Solving Trigonometric Equations

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Solve $\sin x = \frac{1}{2}$ for $0 \le x \le 2\pi$. Use both unit-circle branches and check the endpoints. Bank label: ____	Solve $\sin(2x) = 0$ on $0 \le x \le \pi$. First map the x-interval to the complete $2x$-interval, then divide the enumerated angle values by 2. Bank label: ____
Solve $2\sin^2 x - 3\sin x + 1 = 0$ for $0 \le x \le 2\pi$. Treat $u = \sin x$, solve both feasible levels, and merge any overlapping angle sets. Bank label: ____	Find every x in $[0, 2\pi)$ satisfying $\sin x \cos x = 0$. Solve the two zero factors separately and combine without duplicating any root. Bank label: ____
Solve $\frac{\sin x}{1-\cos x}$=0 on $[0, 2\pi]$. Record denominator exclusions before using numerator-zero logic. Bank label: ____	Using $\sin^2 x$ = 1-$\cos^2 x$, solve $\sin^2 x$ = 1-$\cos x$ on $[0, 2\pi)$. Keep every root produced by the resulting factored equation. Bank label: ____
Solve $\cos x = -\frac{\sqrt{2}}{2}$ for -$\pi \le x < \pi$. Use the signed unit-circle interval rather than converting the final answers to $[0, 2\pi)$. Bank label: ____	Solve $\cos(x - \frac{\pi}{3}) = \frac{1}{2}$ on $[0, 2\pi]$. Enumerate every allowed angle $x - \frac{\pi}{3}$ in the mapped window, including boundary hits. Bank label: ____

AP Precalculus

Solving Trigonometric Equations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Solve $2\sin x + 1 = 2$ for $0 \le x \le 2\pi$. Normalize the output before selecting sine branches.	For $\sin x = c$ on $[0, 2\pi)$, determine the solution count when $c = 1$, when $0 < \lvert c\rvert < 1$, when $c = -1$, and when $\lvert c\rvert > 1$. Base the count on intersections with the extrema.
Solve $-3\cos x + 2 = 5$ on $[0, 2\pi]$. After isolating cosine, use the extreme level to determine multiplicity.	Solve $\sqrt{1-\cos x}=\sin x$ on $[0, 2\pi]$. After squaring, substitute every candidate into the original equation to enforce the nonnegative left side.
Solve $\sin x = 2$ on $[0, 2\pi]$. Give the range-based reason for your conclusion.	Solve $\sin^2 x = \sin x$ on $[0, 2\pi)$. Explain which solutions would disappear if both sides were divided by $\sin x$.

AP Precalculus

Solving Trigonometric Equations

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\{0\}$

Solve $\tan(2x - \frac{\pi}{4}) = 1$ for $0 \le x \le \pi$. Filter the angle family $\frac{\pi}{4} + k\pi$ through the mapped window.

Your response: ____________

Incoming: $[0, \frac{\pi}{2}]$ union $[3\frac{\pi}{2}, 2\pi]$

Solve $\sin^2 x - 2\sin x + 1 = 0$ on $[0, 2\pi]$. Explain why a repeated auxiliary root still produces a one-element angle set.

Your response: ____________

Incoming: $\{0, \frac{\pi}{2}, \pi, 2\pi\}$

A nonnegative graph of $(\sin x - 0.6)^2$ on $[0, \pi]$ touches zero at two certified minima. Report both roots to the nearest thousandth and explain why sign-change bracketing detects neither.

Required response

0.643501109	0
2.498091545	0

Your response: ____________

Incoming: $\{5\frac{\pi}{3}, 7\frac{\pi}{3}\}$

Solve $(2\sin x + 1)(2\sin x - 1) = 0$ on $[0, 2\pi)$. Enumerate the positive and negative half-levels separately.

Your response: ____________

Incoming: $\{\frac{\pi}{4}, 5\frac{\pi}{4}\}$

Solve $\cos(-x + \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ on $[-\pi, \pi]$. Reverse the mapped interval correctly before filtering the two cosine branches.

Your response: ____________

Incoming: $\{3.000\}$

Solve $\frac{\sin x}{\cos x} = 1$ on $[0, 2\pi)$. Preserve the cosine exclusions while using the equivalent tangent level where defined.

Your response: ____________

AP Precalculus

Solving Trigonometric Equations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student reports only $x = \frac{\pi}{3}$ for $\sin x = \frac{\sqrt{3}}{2}$ on $[0, 2\pi]$. Identify the missing branch and give the corrected complete set.

A student uses $a 2\pi$ repeat and lists $\{-2\pi, 0, 2\pi\}$ for $\tan x = 0$ on $[-2\pi, 2\pi]$. Repair the period and include every missing root.

A solution lists both 0 and 2π for $\cos x = 1$ on $[0, 2\pi)$. Repair the set by applying the interval notation.

A solution factors $\sin^2 x + \sin x - 2 = 0$ but tries to include an angle for $\sin x = -2$. Remove the impossible auxiliary level and finish the solution on $[0, 2\pi]$.

Solving Trigonometric Equations

Full Worked Solutions - excerpt

Worked example

Problem. Solve $\cos x = -1$ on $-2\pi \leq x \leq 2\pi$. Filter the extreme-value family $\pi + 2k\pi$ through the symmetric interval.

Answer. {-π, π}

Explanation and check.

The family $x = \pi + 2k\pi$ yields -π for $k = -1$ and π for $k = 0$. Adjacent values lie outside the interval.

Worked example

Problem. Solve $5 \tan x + 2 = 2$ on $[-\pi, \pi]$. Apply tangent's $\pi-$spaced zero family to the closed endpoints.

Answer. {-π, 0, π}

Explanation and check.

The equation reduces to $\tan x = 0$, whose zeros $x = k\pi$ in the interval are -$\pi, 0, \pi$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Trigonometric Inequalities

Trigonometric Inequalities | APPC-U03-P18

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for closed interval to tolerance, interval to tolerance, interval union to tolerance, correct intersection and counterexample, correct interval and boundary counterexample and correct set and boundary counterexample.

8 PDFs and a README; 111 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Trigonometric Inequalities

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Certified numerical evidence locates the crossings of $\sin x$ and $0.2x + 0.1$ at 0.125411 and 2.498473 on $[0, 4]$. Solve the nonstrict inequality to the nearest thousandth.

Certified numerical evidence locates the crossings of $\sin x$ and $0.2x + 0.1$ at 0.125411 and 2.498473 on $[0, 4]$. Solve the nonstrict inequality to the nearest thousandth.

Supplied evidence	Value
equality points	0.125410601 2.498473192

Condition or evidence	Supplied value
Original inequality	$\sin x \geq 0.2x + 0.1$
Requested precision	nearest thousandth
Given shape evidence	the difference is nonnegative exactly between the crossings
Given window	$[0, 4]$

Bank label: ______

Solve $-2\cos x + 1 \leq 0$ on $[0, 2\pi)$. Show the order reversal caused by division by -2.

Solve $-2\cos x + 1 \leq 0$ on $[0, 2\pi)$. Show the order reversal caused by division by -2.

Condition or evidence	Supplied value
Original inequality	$-2\cos x + 1 \leq 0$
Input interval	$[0, 2\pi)$

Bank label: ______

Solve -sin$(2x - \frac{\pi}{2}) \geq 0$ on $[0, \pi]$. Track the output reversal before mapping the inner-angle intervals.

Solve -sin$(2x - \frac{\pi}{2}) \geq 0$ on $[0, \pi]$. Track the output reversal before mapping the inner-angle intervals.

Condition or evidence	Supplied value
Original inequality	-sin(2x-pi/2)>=0
Input interval	$[0, \pi]$

Bank label: ______

Solve $\sin x(1 + \cos x) \geq 0$ on $[0, 2\pi]$. Use that $1 + \cos x$ never becomes negative but does vanish at π.

Solve $\sin x(1 + \cos x) \geq 0$ on $[0, 2\pi]$. Use that $1 + \cos x$ never becomes negative but does vanish at π.

Condition or evidence	Supplied value
Original inequality	$\sin x(1 + \cos x) \geq 0$
Input interval	$[0, 2\pi]$

Bank label: ______

AP Precalculus

Trigonometric Inequalities

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Solve $\sin x < 0$ OR $\cos x \leq 0$ on $[0, 2\pi)$. Decide which boundaries are contributed by only one condition.

Solve $\sin x < 0$ OR $\cos x \leq 0$ on $[0, 2\pi)$. Decide which boundaries are contributed by only one condition.

Condition or evidence	Supplied value
Given conditions	$\sin x < 0$ $\cos x \leq 0$
Logical connective	OR
Input interval	$[0, 2\pi)$

During $0 \leq t \leq 12$ hours, a temperature follows $T(t) = 50 + 10\sin(\pi \frac{t}{6})$. When is $T > 55$, and for how many total hours?

During $0 \leq t \leq 12$ hours, a temperature follows $T(t) = 50 + 10\sin(\pi \frac{t}{6})$. When is $T > 55$, and for how many total hours?

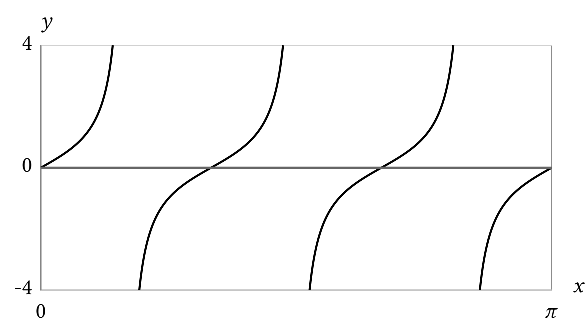
Condition or evidence	Supplied value
Required condition	$T > 55$
Given model	$T(t) = 50 + 10\sin(\pi \frac{t}{6})$
Time interval	$[0, 12]$ hours

For $0 \leq t \leq 10, V(t) = 12 + 4\cos(\pi \frac{t}{5})$. When is $V \leq 8$, and what is the total duration?

For $0 \leq t \leq 10, V(t) = 12 + 4\cos(\pi \frac{t}{5})$. When is $V \leq 8$, and what is the total duration?

Condition or evidence	Supplied value
Required condition	$V \leq 8$
Given model	$V(t) = 12 + 4\cos(\pi \frac{t}{5})$
Time interval	$[0, 10]$ hours

Solve $\tan(3x) < 0$ on $[0, \pi]$, marking the three scaled poles before reading signs.



Condition or evidence	Supplied value
Original inequality	$\tan(3x) < 0$
Input interval	$[0, \pi]$

AP Precalculus

Trigonometric Inequalities

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[0, 0.125] \cup [2.498, 4]$

Solve $\tan x \geq -\sqrt{3}$ on $[0, \pi)$, treating the two sides of $\frac{\pi}{2}$ as separate branches.

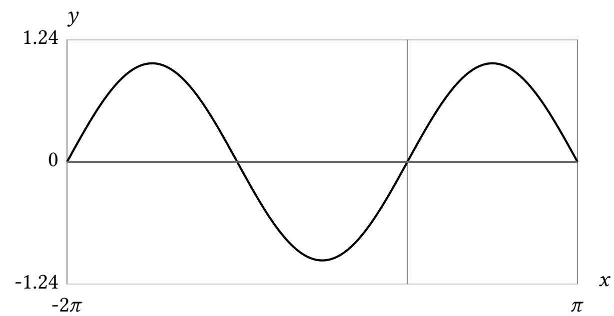
Solve $\tan x \geq -\sqrt{3}$ on $[0, \pi)$, treating the two sides of $\frac{\pi}{2}$ as separate branches.

Condition or evidence	Supplied value
Original inequality	$\tan x \geq -\sqrt{3}$
Input interval	$[0, \pi)$

Your response: ____________________

Incoming: $(\frac{\pi}{2}, \pi)$

Solve $\sin x \geq 0$ on $[-2\pi, \pi]$, using signed cycles and all equality zeros.



Condition or evidence	Supplied value
Original inequality	$\sin x \geq 0$
Input interval	$[-2\pi, \pi]$

Your response: ____________________

Incoming: $[-2\pi, \pi]$

Let $\alpha = \arccos(0.2)$. Solve $\cos x \leq 0.2$ on $[0, 2\pi)$ in terms of α.

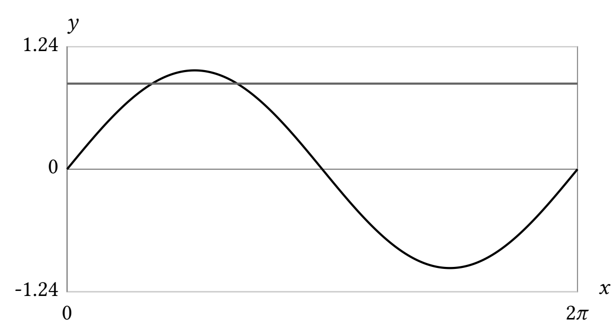
Let $\alpha = \arccos(0.2)$. Solve $\cos x \leq 0.2$ on $[0, 2\pi)$ in terms of α.

Condition or evidence	Supplied value
Original inequality	$\cos x \leq 0.2$
Input interval	$[0, 2\pi)$

Your response: ____________________

Incoming: $[-2, 2]$

Solve $\sin x > \frac{\sqrt{3}}{2}$ on $[0, 2\pi]$.



Condition or evidence	Supplied value
Original inequality	$\sin x > \frac{\sqrt{3}}{2}$
Input interval	$[0, 2\pi]$

Your response: ____________________

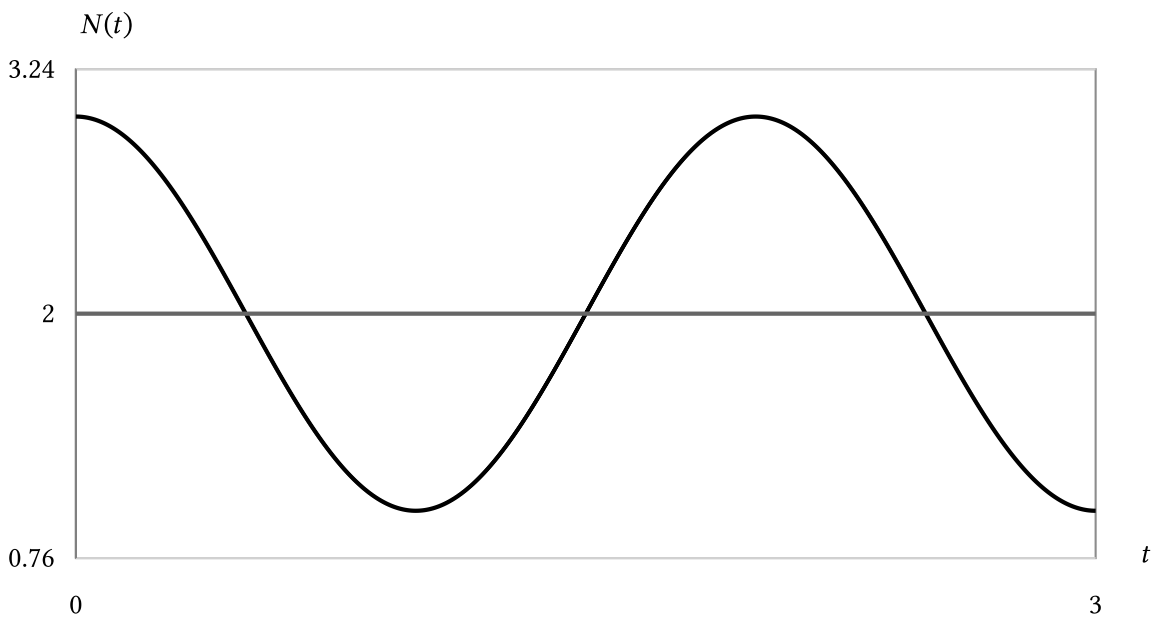
AP Precalculus

Trigonometric Inequalities

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $0 \le t \le 3, N(t) = 2 + \cos(\pi t)$. Find when $N > 2$ and add the unequal visible window lengths. First choose and justify a valid method. Method A: Add the actual lengths of the two admitted time windows. Method B: Multiply the number of windows by a common assumed length.



Condition or evidence	Supplied value
Required condition	$N > 2$
Given model	$N(t) = 2 + \cos(\pi t)$
Time interval	$[0, 3]$ hours

Trigonometric Inequalities

Full Worked Solutions - excerpt

Worked example

Problem. Determine the solution set of $\cos x > 1$ on $[0, 2\pi]$ without solving an equality first.

Determine the solution set of $\cos x > 1$ on $[0, 2\pi]$ without solving an equality first.

Condition or evidence	Supplied value
Original inequality	$\cos x > 1$
Input interval	$[0, 2\pi]$

Answer. $\varnothing$

Explanation and check.

Cosine's maximum value is 1, so it can never be strictly greater than 1. The solution set is empty.

Worked example

Problem. Solve $-3 \sin x + 2 < -1$ on $[0, 2\pi]$, reversing the inequality when the sine term is isolated.

Solve $-3 \sin x + 2 < -1$ on $[0, 2\pi]$, reversing the inequality when the sine term is isolated.

Condition or evidence	Supplied value
Original inequality	$-3 \sin x + 2 < -1$
Input interval	$[0, 2\pi]$

Answer. $\varnothing$

Explanation and check.

Isolation gives $\sin x > 1$. Since the upper endpoint of sine's range is 1, the strict inequality is impossible.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Secant and Cosecant Functions

Secant and Cosecant Functions | APPC-U03-P19

96 problems · Four activity formats

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08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for cosecant order and magnitude, cosecant value and order, cosecant value order and magnitude, exact order, exact signed and magnitude relation and exact values and order.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Secant and Cosecant Functions

Name: ________________ **Date:** ________________ **Class:** ________________

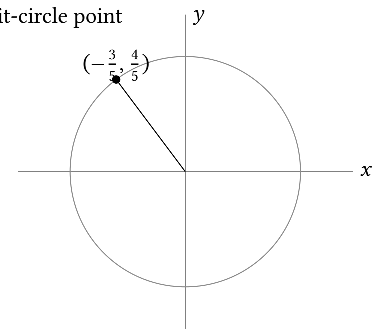
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A table gives $\cos\theta = \frac{1}{4}$ and $\sin\theta = \frac{\sqrt{15}}{4}$. Find $\sec\theta$ and $\csc\theta$ exactly.

Bank label: ______

The terminal point for θ is $(-\frac{3}{5}, \frac{4}{5})$ on the unit circle. Find $\sec\theta$ and $\csc\theta$.

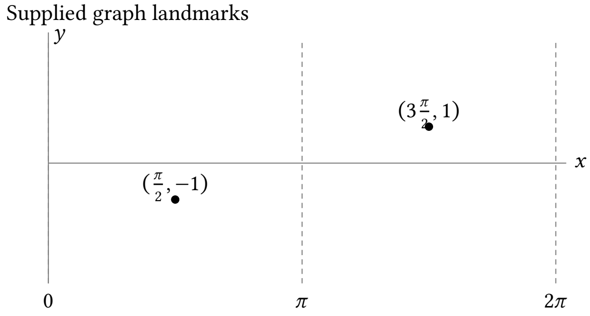
Supplied unit-circle point



Bank label: ______

A graph has poles at $0, \pi, 2\pi$ and vertices $(\frac{\pi}{2}, -1), (3\frac{\pi}{2}, 1)$. Which listed reciprocal formula matches it?

Supplied graph landmarks



$\frac{\pi}{2}$	-1
$3\frac{\pi}{2}$	1

Bank label: ______

If $\sin\theta = 0.8$, compute $\csc\theta$ and compare the two positive values.

Bank label: ______

AP Precalculus

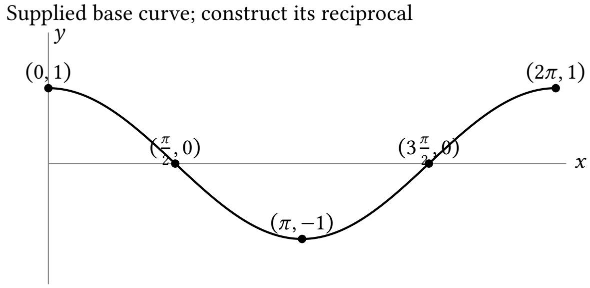
Secant and Cosecant Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Construct $y = \sec x$ on $[0, 2\pi]$ from the supplied cosine landmarks. State all poles, reciprocal vertices, and branch signs.

Supplied base curve; construct its reciprocal



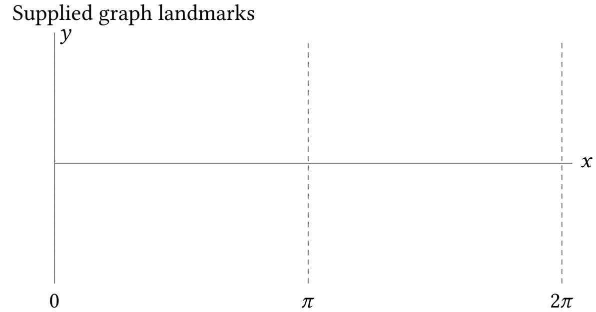
0	1
π	-1
2π	1

Derive the domain and range of $y = 2\sec x + 3$.

For $y = \csc(2x)$, give the period, all pole locations, and the reciprocal-vertex pattern.

A cropped graph has integer-π poles, a positive first branch, and a negative second branch. Match its reciprocal formula. Choose from $\csc x$, $-\csc x$, and $\sec x$.

Supplied graph landmarks



AP Precalculus

Secant and Cosecant Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

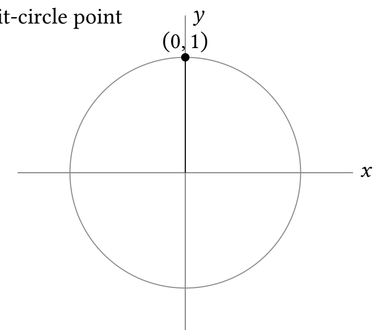
Give the repeating pole and vertex pattern of $y = \csc(3x)$.

Your response: ______________________

Incoming: period $2\frac{\pi}{3}$; poles $x = k\frac{\pi}{3}$; vertices $x = \frac{\pi}{6} + k\frac{\pi}{3}$ with alternating values $1, -1$

At the unit-circle point $(0, 1)$, evaluate $\sec\theta$ and $\csc\theta$.

Supplied unit-circle point



Your response: ______________________

Incoming: $\sec\theta$ is undefined; $\csc\theta = 1$

Restrict $y = \csc x$ to $[\frac{\pi}{6}, 5\frac{\pi}{6}]$. Determine its actual domain and range.

Your response: ______________________

Incoming: domain $[\frac{\pi}{6}, 5\frac{\pi}{6}]$; range $[1, 2]$

Recover the horizontal landmarks of $y = \sec(\frac{x}{3})$.

Your response: ______________________

AP Precalculus

Secant and Cosecant Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims $\sec(0) = 0$ because the angle is zero. Diagnose the reciprocal error.

A proposed $y = \csc x$ graph crosses the x-axis at $x = \frac{\pi}{2}$. Diagnose the error using the denominator value there and the reciprocal range.

A student marks $x = 0$ and $x = \pi$ as zeros of $\sec x$. Repair the graph landmarks on $[0, 2\pi]$.

A student copies $\cos\theta = 0.2$ into the secant column. Repair the table entry and state the operation required.

Secant and Cosecant Functions

Full Worked Solutions - excerpt

Worked example

Problem. A table gives $\cos\theta = \frac{1}{8}$. Find $\sec\theta$.

Answer. $\sec\theta = 8$

Explanation and check.

Invert the nonzero cosine value.

Worked example

Problem. A table lists $\sin\theta = 0$. Classify $\csc\theta$.

Answer. $\csc\theta$ is undefined

Explanation and check.

Cosecant would require division by zero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Cotangent Functions

Cotangent Functions | APPC-U03-P20

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for cotangent status and value, family of compatible rules and missing evidence, sign and domain on each interval, period consecutive pole and zero, range and existence of local extrema and values and domain explanation.

8 PDFs and a README; 142 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Cotangent Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$ to find $\cot\theta$.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given $\cos\theta$	$\frac{\sqrt{3}}{2}$
Given $\sin\theta$	$\frac{1}{2}$

Bank label: ______

AP Precalculus

Cotangent Functions

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$, find the period and give the first pole and first zero at or to the right of $x = \frac{\pi}{8}$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original rule	$y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$

AP Precalculus

Cotangent Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(a, b) = (2, 2)$ and $(-2, -2)$

For $y = 3\cot(-x + \frac{\pi}{5})$, solve for its pole and zero families and state the period and branch direction.

Solve the card and move only to the card whose incoming header matches the entire requested response.

Condition or evidence	Supplied value
Original rule	$y = 3\cot(-x + \frac{\pi}{5})$

Your response: ______________

AP Precalculus

Cotangent Functions

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Assess the statement "$\cot x = \frac{1}{\tan x}$ for every real x." Give a corrected domain-qualified statement and one counterexample to the original.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	**Supplied value**
Statement to examine	$\cot x = \frac{1}{\tan x}$ for every real x

Cotangent Functions

Full Worked Solutions - excerpt

Worked example

Problem. For $y = 2\cot(3x)$, state the period, all pole locations, all zeros, and the direction of each branch.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original rule	$y = 2\cot(3x)$

Answer. period $\frac{\pi}{3}$; poles $x = k\frac{\pi}{3}$; zeros $x = \frac{\pi}{6} + k\frac{\pi}{3}$; decreasing branches

Explanation and check.

Set $3x = k\pi$ for poles and $3x = \frac{\pi}{2} + k\pi$ for zeros. The positive output factor preserves cotangent's decreasing direction.

Worked example

Problem. For $y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$, find the period and give the first pole and first zero at or to the right of $x = \frac{\pi}{8}$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original rule	$y = (\frac{1}{2})\cot(4(x - \frac{\pi}{8}))$

Answer. period $\frac{\pi}{4}$; pole $x = \frac{\pi}{8}$; zero $x = \frac{\pi}{4}$

Explanation and check.

Input scale 4 gives period $\frac{\pi}{4}$. Starting at the phase pole, a zero lies half a period, $\frac{\pi}{8}$, later.

Sets & Steps
Math Practice & Worked Solutions

Precalculus
Reciprocal, Quotient, and Pythagorean Trig Identities

Trigonometric Functions | APPC-U03-P21

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Use reciprocal and quotient identities, derive Pythagorean forms, simplify expressions, and justify domain restrictions and sign choices.

8 PDFs and a README; 62 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values, method-choice labels, and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

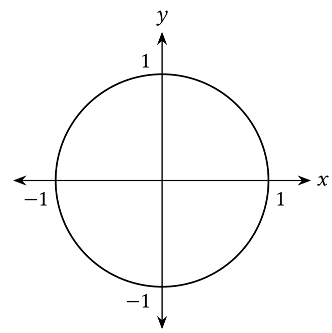
Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values and the requested justification. Angles are in radians; k is an integer.

Rewrite $\sec x$ using cosine and state the inputs allowed by the original expression.

Bank label: ______

A terminal ray lies in quadrant II and $\sin^2\theta = \frac{9}{25}$. Recover $\sin\theta$.



Bank label: ______

Use the Pythagorean identity to rewrite $1 - \sin^2\theta$ as one squared trigonometric function.

Bank label: ______

Simplify $\frac{1}{\csc x}$, but retain the domain of the written original expression.

Bank label: ______

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning. Use radians, exact values, and all stated domain restrictions. In excluded-input families, k is an integer.

Express $\tan x$ using sine and cosine, including its domain condition.	The expression $\frac{\sin x}{\tan x}$ simplifies to $\cos x$. On which inputs is the simplification equal to the original, and which inputs does the simplified formula newly admit?
Divide $\sin^2 x + \cos^2 x = 1$ by $\cos^2 x$ and name every quotient. State the required condition.	Refute the proposed identity $\sec x = \cos x$ using an input where both sides are defined.
Rewrite $\frac{\tan x}{\cot x}$ entirely in sine and cosine and simplify. Preserve the original restrictions.	After replacing $1 - \cos^2 x$, the expression $\frac{1-\cos^2 x}{\sin x}$ becomes $\sin x$. Compare the domains of the two formulas.

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\cot x$ for $\sin x \neq 0$

Rewrite $\sin^2 x + 2\cos^2 x$ as 1 plus one squared trig term.

Your response: ________________

Incoming: $\sin\theta = \frac{7}{8}$

Use the reciprocal definition to express $\sec x \sin x$ as a quotient function, retaining its domain.

Your response: ________________

Incoming: $\cos\theta = -2\frac{\sqrt{6}}{5}$

Rewrite $\tan x \cos^2 x$ using only sine and cosine and simplify.

Your response: ________________

Incoming: $\cot^2 t = \csc^2 t - 1$ for $\sin t \neq 0$

Rewrite $2(\cos^2 t - 1)$ in terms of $\sin^2 t$.

Your response: ________________

Precalculus

Reciprocal, Quotient, and Pythagorean Trig Identities

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statements or methods. Explain your decision and complete the requested task. Not every prompt contains an error.

A derivation divides $\sin^2 x + \cos^2 x = 1$ by $\sin^2 x$ "for every real x." Repair the step and state the resulting identity.

Test the claim $1 - \cos^2 x = \sin x$. Give a counterexample and replace it with the correct identity.

Repair the step $\sqrt{\cos^2 x} = \cos x$. Give a correct all-real formula and explain when the student' s version happens to agree.

Refute $\sqrt{\sin^2 x} = \sin x$ as an all-real identity, then write the correct version.

Reciprocal, Quotient, and Pythagorean Trig Identities

Full Worked Solutions - excerpt

Worked example

Problem. Rewrite $\sec x$ using cosine and state the inputs allowed by the original expression.

Answer. $\sec x = \frac{1}{\cos x}$ for $\cos x \neq 0$

Explanation and check.

$\sec x = \frac{1}{\cos x}$

Cosine is defined for every real x, but the denominator must be nonzero. Therefore $\cos x \neq 0$, or equivalently $x \neq \frac{\pi}{2} + k\pi$.

Worked example

Problem. A terminal ray lies in quadrant II and $\sin^2\theta = \frac{9}{25}$. Recover $\sin\theta$.

Answer. $\sin\theta = \frac{3}{5}$

Explanation and check.

$\sin^2\theta = \frac{9}{25}$

$\sin\theta = \pm(\frac{3}{5})$

Sin is positive on quadrant II. Select the matching sign.

$\sin\theta = \frac{3}{5}$

Check the given square: $\frac{9}{25}$; the selected sign also meets the location condition.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

Angle Sum, Difference, and Double-Angle Identities | APPC-U03-P22

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for $\cos(\alpha-\beta)$, $\cos 2x$, equivalent expression, rewrite and range, exact value and rewrite and maximum.

8 PDFs and a README; 75 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

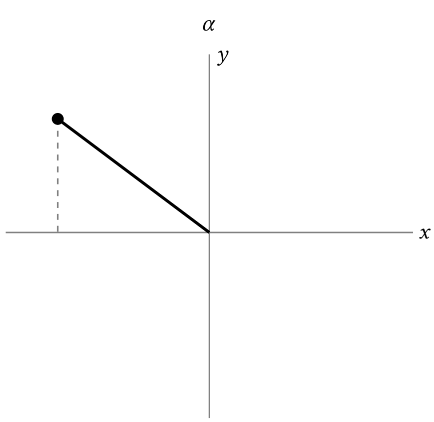
Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

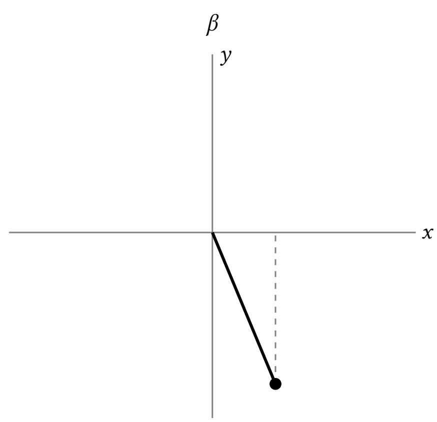
Use a difference identity to obtain the exact value of $\cos 15^{\circ}$.

Bank label: ______

Angle α lies in quadrant II with $\tan\alpha = -\frac{3}{4}$, and β lies in quadrant IV with $\sec\beta = \frac{13}{5}$. Find $\sin(\alpha - \beta)$.



$(-4, 3)$



$(5, -12)$

Bank label: ______

AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Prove $\frac{1-\cos 2x}{\sin 2x} = \tan x$ by transforming the left side, and state the common domain.	Use $120^\circ + 45^\circ$ to evaluate $\cos 165^\circ$. Write the result so its negative sign is visually clear.
Verify $\frac{1+\cos 2x}{\sin 2x} = \cot x$ from the left side and identify every excluded family in the original equality.	Find $\sin 105^\circ$ exactly and use its quadrant to check the sign of your result.
Starting from $\cos 2x = \frac{5}{13}$, select two forms of the cosine double-angle identity to determine $\cos^2 x$ and $\sin^2 x$.	A rewrite begins with $\sin(\alpha + \beta) = \sin\alpha + \sin\beta$. Identify the invalid assumption and replace it with a correct first line.

AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Evaluate $\sin 75^\circ$ exactly by decomposing the angle into familiar angles.

Your response: ______________

AP Precalculus

Angle Sum, Difference, and Double-Angle Identities

Name: ________________ **Date:** ________________ **Class:** ________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A short route cancels $\cos x$ and announces an identity for all real x. A second route checks domains first. Decide which conclusion is valid and repair the quantifier.

Evaluate the claim $\frac{\sin 2x}{2\cos x} = \sin x$ for all real x.

Work on the domain real x.

The route A is cancel $\cos x$.

The route B is compare original domains first.

The output is corrected claim.

A student writes $\cos 2x = 2\cos x - 1$. Repair the rule and give one input that reveals the missing square.

Angle Sum, Difference, and Double-Angle Identities

Full Worked Solutions - excerpt

Worked example

Problem. Transform $\sin 2x - 2\cos x$ into a product that displays its two zero-producing factors. Do not find the roots.

Answer. $2\cos x(\sin x - 1)$

Explanation and check.

Substitute $2\sin x\cos x$ for $\sin 2x$ and factor the common $2\cos x$.

Worked example

Problem. Rewrite $2\cos^2 x = 1 + \sin x$ as a polynomial equation in $\sin x$, without solving for x.

Answer. $2\sin^2 x + \sin x - 1 = 0$

Explanation and check.

Substitute $\cos^2 x = 1 - \sin^2 x$ and collect: $2 - 2\sin^2 x = 1 + \sin x$, so $2\sin^2 x + \sin x - 1 = 0$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Equivalent Polar Coordinates

Equivalent Polar Coordinates | APPC-U03-P23

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for diagnosis and corrected pair, same or different, distance and location, one pair, one polar pair and equivalence group.

8 PDFs and a README; 157 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

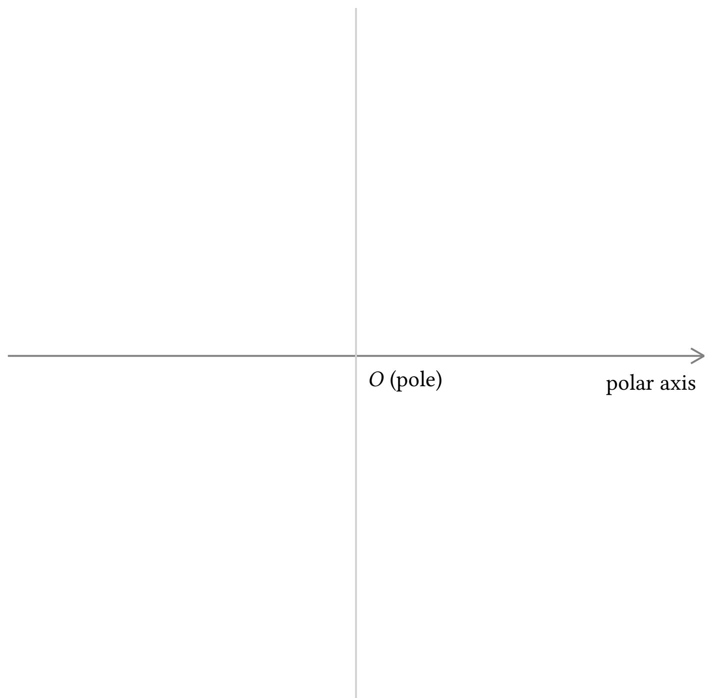
AP Precalculus

Equivalent Polar Coordinates

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the signed radius r that makes $(r, 5\frac{\pi}{6})$ equivalent to $(5, 11\frac{\pi}{6})$.



Condition or evidence	Supplied value
Incomplete polar record	$(r, 5\frac{\pi}{6})$
Known polar record	$(5, 11\frac{\pi}{6})$

Bank label: ______

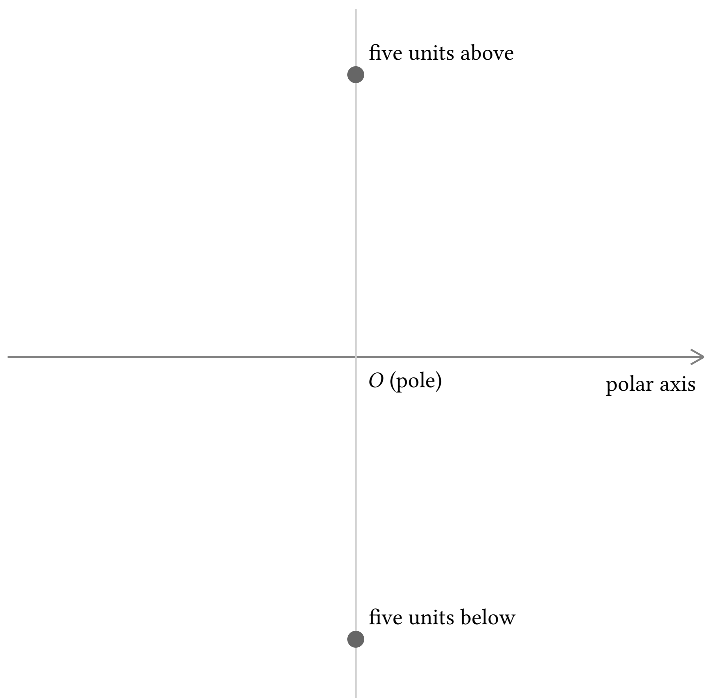
AP Precalculus

Equivalent Polar Coordinates

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A plot offers points five units above and five units below the pole. Which one represents $(-5, \frac{\pi}{2})$?



Condition or evidence	Supplied value
Candidate points	five units above five units below
Given polar record	$(-5, \frac{\pi}{2})$

AP Precalculus

Equivalent Polar Coordinates

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\theta = -3\frac{\pi}{10}$

An equivalent pair for $(7, 4\frac{\pi}{3})$ is required to have $\theta = \frac{\pi}{3} + 2k\pi$. What must r be for every integer k?

Solve the card and move only to the card whose incoming header matches the entire requested response.

Condition or evidence	Supplied value
Given angle form	$\theta = \frac{\pi}{3} + 2k\pi$
Incomplete polar record	(r, θ)
Known polar record	$(7, 4\frac{\pi}{3})$

Your response: ____

AP Precalculus

Equivalent Polar Coordinates

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $(-a, \frac{\pi}{6})$ with $a = -5$, evaluate the radius before interpreting the point. Correct the claim that the visible minus sign automatically reverses the ray. Justify the comparison by distinguishing signed radius, ordinary distance and actual location.

Compare the stated cases, methods, or representations and justify the conclusion using the supplied conditions.

Condition or evidence	**Supplied value**
Assumption	$a = -5$
Given polar record	$(-a, \frac{\pi}{6})$
Student claim to examine	The written minus sign always means a negative radius.

Equivalent Polar Coordinates

Full Worked Solutions - excerpt

Worked example

Problem. Fill in r so the table entry $(r, 13\frac{\pi}{6})$ names the same point as $(3, \frac{\pi}{6})$.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Candidate	Reference
$(r, 13\frac{\pi}{6})$	$(3, \frac{\pi}{6})$

Answer. r = 3

Explanation and check.

The angles differ by 2π, so the radius retains both its magnitude and sign.

Worked example

Problem. Complete the row $(r, 0)$ so it is equivalent to $(-8, \pi)$.

Solve the card and move only to the card whose incoming header matches the entire requested response.

Condition or evidence	Supplied value
Incomplete record	$(r, 0)$
Known record	$(-8, \pi)$

Answer. r = 8

Explanation and check.

The negative radius reverses the π ray to the 0 ray at distance 8, so the second pair uses $r = 8$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Polar and Rectangular Coordinate Conversion

Polar and Rectangular Coordinate Conversion | APPC-U03-P24

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for polar pair plus check, polar coordinates, polar pair with quadrant check, rounded polar coordinates, rounded coordinates and positive-radius polar pair.

8 PDFs and a README; 154 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Polar and Rectangular Coordinate Conversion

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A point has rectangular form $(3, y)$, polar radius 5, and lies above the x-axis. Complete the missing coordinate.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Condition	$y > 0$
Given polar radius	5
Incomplete rectangular record	$(3, y)$

Bank label: ______

Convert the measured point $(0.4, -6)$ to positive-radius polar form using the stated rounding. Preserve the quadrant near the negative y-axis.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

x	y
0.4	-6

Condition or evidence	Supplied value
Required angle interval	$0^\circ \le \theta < 360^\circ$
Angle precision	nearest tenth
Radius precision	nearest hundredth

Bank label: ______

For $(-5, 2)$, find positive-radius polar coordinates to the nearest tenth of a degree. Explain the quadrant correction and check the signs of the two projections.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

x	y
-5	2

Condition or evidence	Supplied value
Required angle interval	$0^\circ \le \theta < 360^\circ$
Precision	nearest tenth of a degree

Bank label: ______

A calculator returns a reference angle near 20.6° for $(-8, -3)$. Produce the correct polar angle, radius, and a sign check by reprojection.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Required angle interval	$0^\circ \le \theta < 360^\circ$
Given rectangular point	-8 -3
Precision	**Given angle θ** nearest tenth degree **Radius** nearest hundredth
Given calculator ratio angle	about 20.6°

Bank label: ______

AP Precalculus

Polar and Rectangular Coordinate Conversion

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

A positive-radius point has $\theta = 3\frac{\pi}{2}$ and $y = -4$. Determine r and the missing x-coordinate.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	**Supplied value**
Given angle θ	$3\frac{\pi}{2}$
Radius convention	positive
Given x-coordinate	missing
Given y-coordinate	-4

AP Precalculus

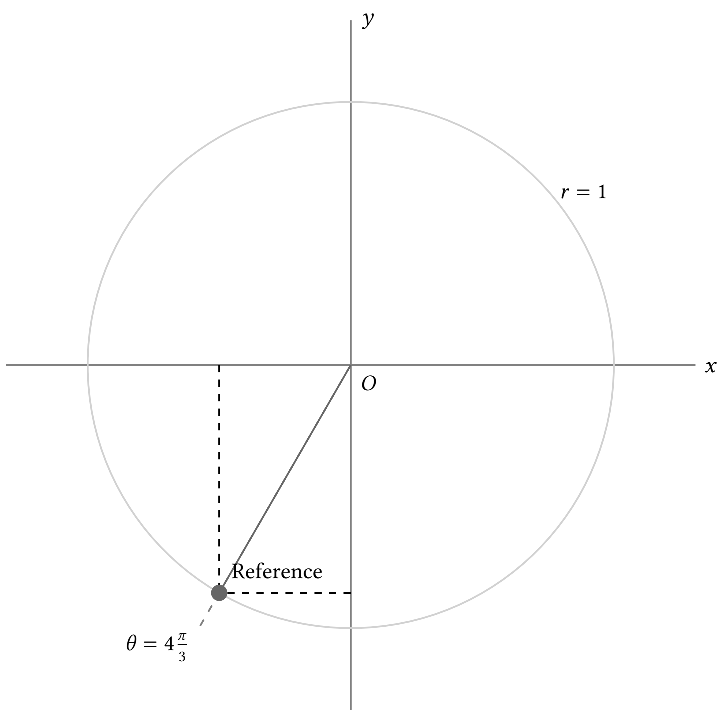
Polar and Rectangular Coordinate Conversion

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(11, 3\frac{\pi}{2})$

Convert $(-9, 4\frac{\pi}{3})$ and explain why both coordinates become positive.



cos θ	sin θ
$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$

Condition or evidence	Supplied value
Given quadrant to verify	I
Given polar record	$(-9, 4\frac{\pi}{3})$

Your response: ______________

AP Precalculus

Polar and Rectangular Coordinate Conversion

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student converts $(4, \frac{\pi}{6})$ to $(2, 2\sqrt{3})$. Identify the swapped roles and repair the ordered pair.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Given polar record	$(4, \frac{\pi}{6})$
Student rectangular result to examine	$(2, 2\sqrt{3})$

Polar and Rectangular Coordinate Conversion

Full Worked Solutions - excerpt

Worked example

Problem. A point has rectangular form $(3, y)$, polar radius 5, and lies above the x-axis. Complete the missing coordinate.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Condition	$y > 0$
Given polar radius	5
Incomplete rectangular record	$(3, y)$

Answer. y = 4

Explanation and check.

From $3^2 + y^2 = 5^2, y^2 = 16$. The upper-half-plane condition selects $y = 4$ rather than -4.

Worked example

Problem. A point has radius $10, y = -6$, and lies left of the y-axis. Reconstruct x.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Condition	$x < 0$
Given polar radius	10
Incomplete rectangular record	$(x, -6)$

Answer. x = −8

Explanation and check.

The norm equation gives $x^2 + 36 = 100$, so $x = \pm 8$. The left-half-plane condition selects $x = -8$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polar Graphs from Tables

Polar Graphs from Tables | APPC-U03-P25

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for two new rows and adequacy explanation, two radius values, ordered rows, directed partial trace, polar graph and directed trace.

8 PDFs and a README; 93 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polar Graphs from Tables

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Complete the radius column for $r = 2$ at $\theta = 0, \frac{\pi}{2}$, and π, preserving the given angle order. Bank label: ______	For $r = 1 + \sin\theta$, evaluate the outputs at $0, \frac{\pi}{2}$, and π and place them in the same order. Bank label: ______

AP Precalculus

Polar Graphs from Tables

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Plot the ordered table for $r = 1$ and add an arrow showing the direction as θ increases from 0 to 2π.

Required response

0	1
$\frac{\pi}{2}$	1
π	1
$3\frac{\pi}{2}$	1
2π	1

Sketch only the portion traced by $r = 2$ for $0 \leq \theta \leq \frac{\pi}{2}$, marking both endpoint statuses.

The $\theta - r$ graph is the horizontal line $r = 3$ for one full turn. Which plane trace matches, and why is the line $y = 3$ not the same graph?

The listed samples for $r = \sin(2\theta)$ are all zero. Explain why they do not show the trace, then evaluate the proposed extra input $\frac{\pi}{4}$.

Required response

0	0
$\frac{\pi}{2}$	0
π	0

Use the ordered rows for $r = \theta$ to construct the trace on $[0, \frac{\pi}{2}]$ and describe its direction.

Required response

0	0
$\frac{\pi}{4}$	$\frac{\pi}{4}$
$\frac{\pi}{2}$	$\frac{\pi}{2}$

Evaluate $r = 2\cos\theta - 1$ at the four listed angles. Keep zero and negative outputs unchanged for later plotting. The supplied angles, in order, are $\theta = 0, \frac{\pi}{3}, \pi, 3\frac{\pi}{2}$.

AP Precalculus

Polar Graphs from Tables

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(0,1), (\frac{\pi}{4}, 0), (\frac{\pi}{2}, -1), (\pi, -1)$

Determine the plane arc traced by $r = -1$ for $0 \le \theta \le \frac{\pi}{2}$.

Required response

0	-1
$\frac{\pi}{2}$	-1

Your response: ______________

Incoming: The upper arc from filled $(2, 0)$ through $(1, 1)$, approaching but not including the pole.

Complete the table for $r = 3\cos\theta$ at $\frac{\pi}{6}$, $\frac{\pi}{2}$, and $5\frac{\pi}{6}$.

Your response: ______________

Incoming: An outward quadrant-III curve from $(-1, 0)$ to $(0, -\frac{3}{2})$, with both endpoints filled.

Determine the portion traced by $r = -1$ for -$\pi \le \theta \le \frac{\pi}{2}$, including its direction.

Required response

-π	-1
-$\frac{\pi}{2}$	-1
0	-1
$\frac{\pi}{2}$	-1

Your response: ______________

Incoming: A three-quarter unit-circle arc from $(1, 0)$ through $(0, 1), (-1, 0)$, to $(0, -1)$.

Sketch the windowed trace of $r = \theta + \pi$ from $\theta = -\pi$ to $\theta = -\frac{\pi}{2}$.

Required response

-π	0
$-3\frac{\pi}{4}$	$\frac{\pi}{4}$
-$\frac{\pi}{2}$	$\frac{\pi}{2}$

Your response: ______________

AP Precalculus

Polar Graphs from Tables

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student replaces the negative output of $r = \cos\theta$ by its magnitude. Repair the $\theta = \pi$ row and state where that signed-radius point plots.

Required response

0	1
$\frac{\pi}{2}$	0
π	1

A student submits the horizontal $\theta - r$ graph as the plane trace of $r = 2$. Explain the coordinate-system confusion and replace it with the correct polar graph.

Polar Graphs from Tables

Full Worked Solutions - excerpt

Worked example

Problem. Evaluate $r = 2 + \sin\theta - \cos\theta$ at the four quadrantal angles.

Answer. $(0, 1), (\frac{\pi}{2}, 3), (\pi, 3), (3\frac{\pi}{2}, 1)$

Explanation and check.

Insert the four sine-cosine pairs $(0, 1), (1, 0), (0, -1)$, and $(-1, 0)$ into the rule.

Worked example

Problem. Compute $r = 2\sin\theta + \cos(2\theta)$ at the four quadrantal angles.

Answer. $(0, 1), (\frac{\pi}{2}, 1), (\pi, 1), (3\frac{\pi}{2}, -3)$

Explanation and check.

Substitution gives $0 + 1, 2 - 1, 0 + 1$, and $-2 - 1$, respectively.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polar Symmetry and Curve Tracing

Polar Symmetry and Curve Tracing | APPC-U03-P26

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for sufficient interval and retrace explanation, symmetry conclusion, once-through subinterval and retrace explanation, pole and nonpole multiplicities, pole multiplicity and nonpole verdict and partial/full/retraced judgment.

8 PDFs and a README; 156 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polar Symmetry and Curve Tracing

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

On $r = 2\sin(3\theta)$, show that $\theta = \frac{\pi}{6}$ and $\theta = 7\frac{\pi}{6}$ represent the same petal tip and give its Cartesian coordinates.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Given θ	Given r
$\frac{\pi}{6}$	2
$7\frac{\pi}{6}$	-2

Condition or evidence	Supplied value
Original polar rule	$r = 2\sin(3\theta)$
Given target point	distance 2 on ray $\frac{\pi}{6}$

Bank label: ______

How many times does $r = 2 + 2\cos\theta$ visit the pole on $0 \le \theta < 2\pi$? Distinguish that count from the closed-curve endpoint convention.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original polar rule	$r = 2 + 2\cos\theta$
Given equation for zero radius	$\cos\theta = -1$

Bank label: ______

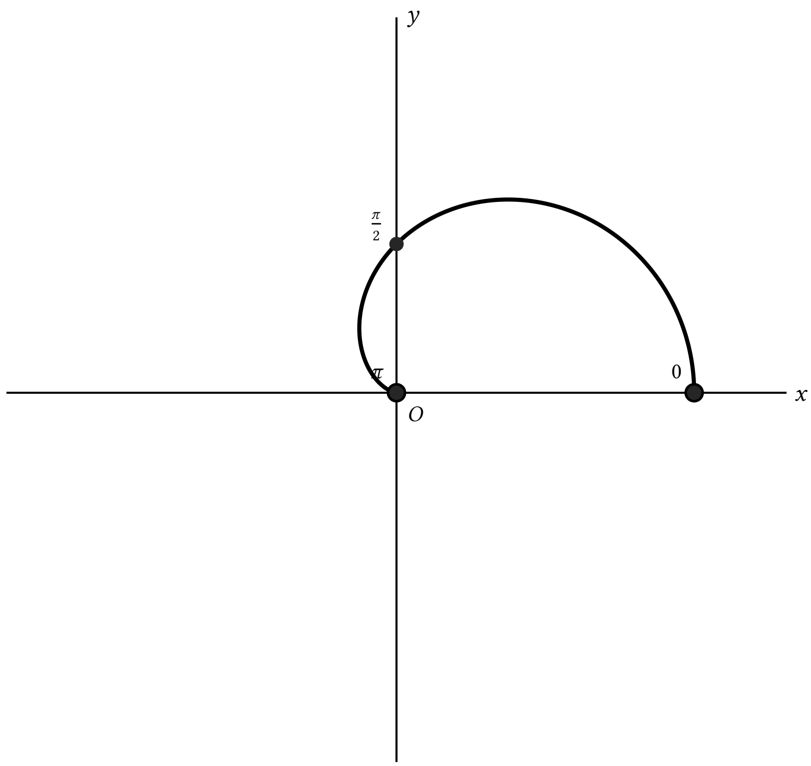
AP Precalculus

Polar Symmetry and Curve Tracing

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $r = 1 + \cos\theta$ on $0 \leq \theta \leq \pi$, describe the direction of travel through the three labeled anchors as θ increases.



Given parameter	Given point or location
0	$(2, 0)$
$\frac{\pi}{2}$	$(0, 1)$
π	pole

Condition or evidence	Supplied value
Original polar rule	$r = 1 + \cos\theta$

AP Precalculus

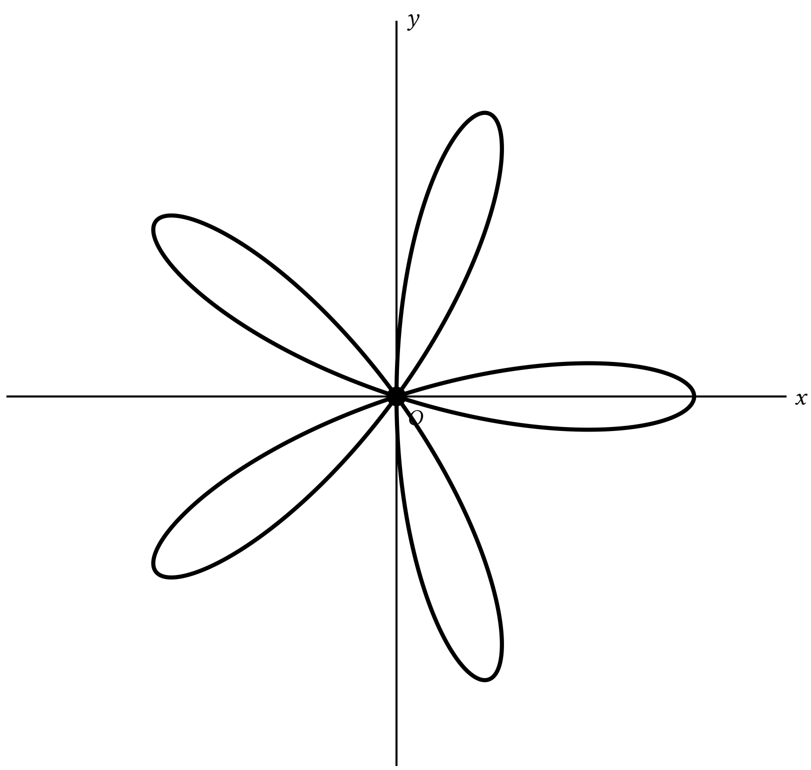
Polar Symmetry and Curve Tracing

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Determine whether $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ is sufficient to trace every petal of $r = \cos(5\theta)$ once. Justify with a half-turn correspondence rather than a memorized rule.



Condition or evidence	Supplied value
Half turn relation	$r(\theta + \pi) = -r(\theta)$
Interval length	π
Original polar rule	$r = \cos(5\theta)$

Your response: ____________

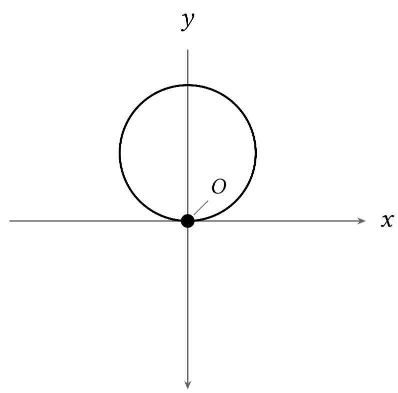
AP Precalculus

Polar Symmetry and Curve Tracing

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student says $r = \sin\theta$ needs all of $[0, 2\pi]$ for one trace because $\sin\theta$ has period 2π. Repair the claim using signed-radius equivalence.



Condition or evidence	Supplied value
Half turn relation	$r(\theta + \pi) = -r(\theta)$
Original polar rule	$r = \sin\theta$
Student claim to examine	A full 2π parameter interval is required because the radius has period 2π.

Polar Symmetry and Curve Tracing

Full Worked Solutions - excerpt

Worked example

Problem. Which parameters repeat the closing point of $r = 4$ on the closed interval $0 \leq \theta \leq 2\pi$?

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given common-point reference	$(4, 0)$
Original polar rule	$r = 4$

Answer. $\theta = 0$ and $\theta = 2\pi$ both give $(4, 0)$.

Explanation and check.

The radius is 4 at both coterminal endpoint angles, so the two polar pairs locate the same point.

Worked example

Problem. A polar table contains $(\theta, r) = (\frac{\pi}{3}, 2)$ and $(4\frac{\pi}{3}, -2)$. Do the rows locate the same point? Give the coordinate.

Solve the card and move only to the card whose incoming header matches the entire requested response.

θ	Given r
$\frac{\pi}{3}$	2
$4\frac{\pi}{3}$	-2

Answer. Yes; both locate $(1, \sqrt{3})$.

Explanation and check.

The angles differ by π and the radii are opposites, so the polar pairs are equivalent. Converting the first gives $(2\cos(\frac{\pi}{3}), 2\sin(\frac{\pi}{3})) = (1, \sqrt{3})$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Features of Polar Equations

Features of Polar Equations | APPC-U03-P27

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for minimum distance and locations, distance and parameter, ordered exact parameter list, closest point and distance, distance and location and distinct coordinate set.

8 PDFs and a README; 83 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Features of Polar Equations

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

In the family $r = a + \cos\theta$, choose a so that the curve reaches the pole at $\theta = \pi$. Bank label: ______	For $r = a + 2\sin\theta$ with $a \geq 2$, determine a if the greatest distance from the pole is 5. Bank label: ______
Determine a in $r = a + \sin\theta$ if every radius is negative and the closest distance to the pole is 2. Bank label: ______	At what parameter does $r = 5\theta$ reach the pole on $-1 \leq \theta \leq 1$? Bank label: ______

AP Precalculus

Features of Polar Equations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Find every θ in $0 \le \theta \le \pi$ for which $r = 2\sin\theta$ is zero, and name the plane point reached.

AP Precalculus

Features of Polar Equations

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $0 \leq \varphi < 2\pi$, the curve $r = 3 + 3\cos(\theta - \varphi)$ has its cusp at the pole when $\theta = 5\frac{\pi}{6}$. Find φ.

Your response: ______________________

Incoming: $\varphi = 11\frac{\pi}{6}$.

Choose a in $r = 2 + a\cos\theta$ so that $\theta = 0$ is a pole parameter. Verify the resulting zero.

Your response: ______________________

Incoming: -2

Find a if $r = a + 4\cos\theta$ must pass through the pole at $\theta = 2\frac{\pi}{3}$.

Your response: ______________________

Incoming: a = 2.

For k>0, the curve r=k(1-cos θ) has farthest distance 10. Determine k.

Your response: ______________________

AP Precalculus

Features of Polar Equations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A table row gives $(\theta, r) = (\frac{\pi}{4}, -7)$, and a student reports a distance of -7. Correct both the distance and the plotted direction.

A student finds seven zero parameters for $r = \sin(3\theta)$ on $0 \le \theta \le 2\pi$ and reports seven different poles. Correct the feature count.

Features of Polar Equations

Full Worked Solutions - excerpt

Worked example

Problem. Choose a in $r = 2 + a\cos\theta$ so that $\theta = 0$ is a pole parameter. Verify the resulting zero.

Answer. -2

Explanation and check.

At $\theta = 0$ the radius is $2 + a$. Setting it to zero gives $a = -2$, and substitution yields $r(0) = 2 - 2 = 0$.

Worked example

Problem. For k>0, the curve r=k(1-cos θ) has farthest distance 10. Determine k.

Answer. k = 5.

Explanation and check.

The factor $1 \text{-} \cos\theta$ is largest at $\theta = \pi$, where it equals 2. Thus the farthest distance is $2k$, and $2k = 10$ gives $k = 5$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polar Rates of Change and Radial Motion

Polar Rates of Change and Radial Motion | APPC-U03-P28

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for supported or undetermined with alternative, estimated radius, exact signed rate and interpretation, local closest or farthest, local feature and inward and outward intervals.

8 PDFs and a README; 108 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polar Rates of Change and Radial Motion

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

The average signed-radius rate from $\theta = 0$ to $\theta = 2$ is 3 units per radian, and $r(0) = -1$. Recover $r(2)$. Bank label: ______	The average radial rate over $2 \le \theta \le 4$ is -3, and $r(4) = 1$. Use the secant model to estimate $r(3)$. Bank label: ______
The average radial rate from $\theta = \frac{1}{2}$ to $\theta = \theta_2$ is -4. If $r(\frac{1}{2}) = 6$ and $r(\theta_2) = 2$, find θ_2. Bank label: ______	The average signed-radius rate from 30° to 90° is 2 units per radian, and $r(30^\circ) = 1$. Find $r(90^\circ)$. Bank label: ______

AP Precalculus

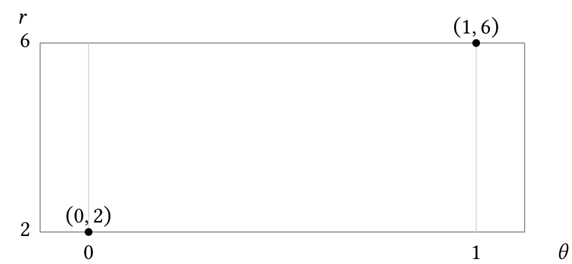
Polar Rates of Change and Radial Motion

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Throughout an angle interval, r is negative and increases from -7 toward -2. Is the point moving inward or outward relative to the pole?

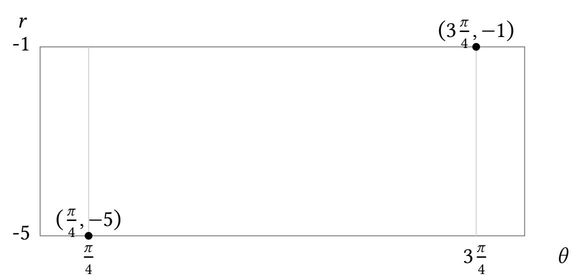
Using the average rate between the two table rows, estimate r at $\theta = \frac{1}{2}$. Is the result necessarily exact?



0	2
1	6

Near $\theta = a$, a radius graph stays positive and has a local minimum $r = 2$. What distance feature does the polar point have at a?

Compute the average signed-radius rate from $\theta = \frac{\pi}{4}$, where $r = -5$, to $\theta = 3\frac{\pi}{4}$, where $r = -1$.



$\frac{\pi}{4}$	-5
$3\frac{\pi}{4}$	-1

AP Precalculus

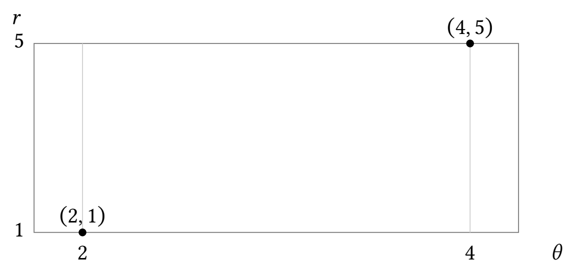
Polar Rates of Change and Radial Motion

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $r(90^\circ) \approx 4$.

A table gives $r(2) = 1$ and $r(4) = 5$. Use the average rate to estimate $r(3)$, and say whether the estimate must be exact.



2	1
4	5

Your response: ____________

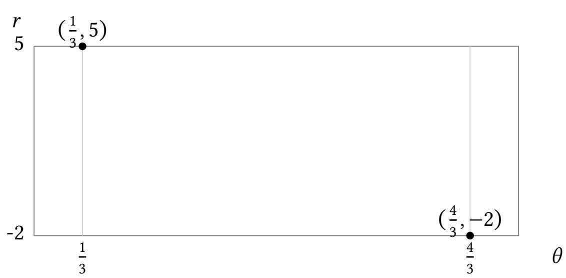
Incoming: Signed-radius rate 2; distance rate $\frac{4}{3}$ radial units per radian.

For $r = 2^{\theta}$, find the average radial rate from $\theta = 0$ to $\theta = 1$.

Your response: ____________

Incoming: Both rates are -3 radial units per radian.

Find the average radial rate from $r(\frac{1}{3}) = 5$ to $r(\frac{4}{3}) = -2$.

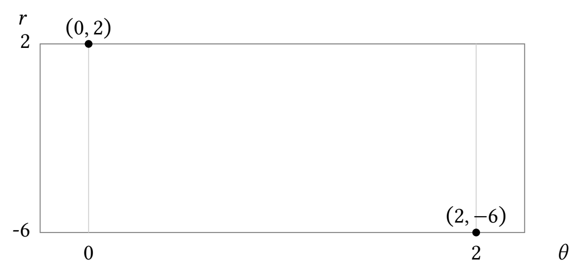


$\frac{1}{3}$	5
$\frac{4}{3}$	-2

Your response: ____________

Incoming: The estimate is 10 and the actual radius lies in $[9.5, 10.5]$.

A diagram labels $r = 2$ at $\theta = 0$ and $r = -6$ at $\theta = 2$. Find the average radial and distance rates.



0	2
2	-6

Your response: ____________

AP Precalculus

Polar Rates of Change and Radial Motion

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student computes $\frac{5-2}{90-30} = \frac{1}{20}$ and labels it radial units per radian. Repair the interpretation.

Required response

30°	2
90°	5

Polar Rates of Change and Radial Motion

Full Worked Solutions - excerpt

Worked example

Problem. A signed radius rises from 2 to 12 at average rate 5. What is the angular interval width?

Answer. $\theta_2 - \theta_1 = 2$ radians.

Explanation and check.

The radius change is 10. Dividing change by rate gives a width of 2 radians.

Worked example

Problem. For $r = 2^\theta$, find the average radial rate from $\theta = 0$ to $\theta = 1$.

Answer. 1 radial unit per radian.

Explanation and check.

The radius changes from 1 to 2 over one radian, so the secant quotient is 1.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Radius Graphs vs Polar Traces

Radius Graphs vs Polar Traces | APPC-U03-P29

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for graph sets, qualification and least point repeat, change descriptions, opposite trends, qualification about least period and two graph descriptions.

8 PDFs and a README; 167 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Radius Graphs vs Polar Traces

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

For $r = \sin(3\theta)$, determine the scalar period and the least positive shift that repeats the same parameterized polar point for every generic θ. Explain why they differ.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given parameter range or context	all real θ
Given half-turn relation	$r(\theta + \pi) = -r(\theta)$
Original rule	$r = \sin(3\theta)$

Bank label: ______

A table gives $r = 1, 2, 1$ at $\theta = 0, \frac{\pi}{2}, \pi$. State the plotted samples in the scalar and polar views.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

θ	Given r
0	1
$\frac{\pi}{2}$	2
π	1

Condition or evidence	Supplied value
Given parameter range or context	three sampled angles

Bank label: ______

AP Precalculus

Radius Graphs vs Polar Traces

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A complete scalar radius graph has highest value 5 and lowest value -7. Repair the claim that its top point must correspond to the farthest polar point.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	**Supplied value**
Given parameter range or context	complete scalar graph
Claim to examine	The top of the scalar graph is always the farthest polar point.
Marked values	-7 5

AP Precalculus

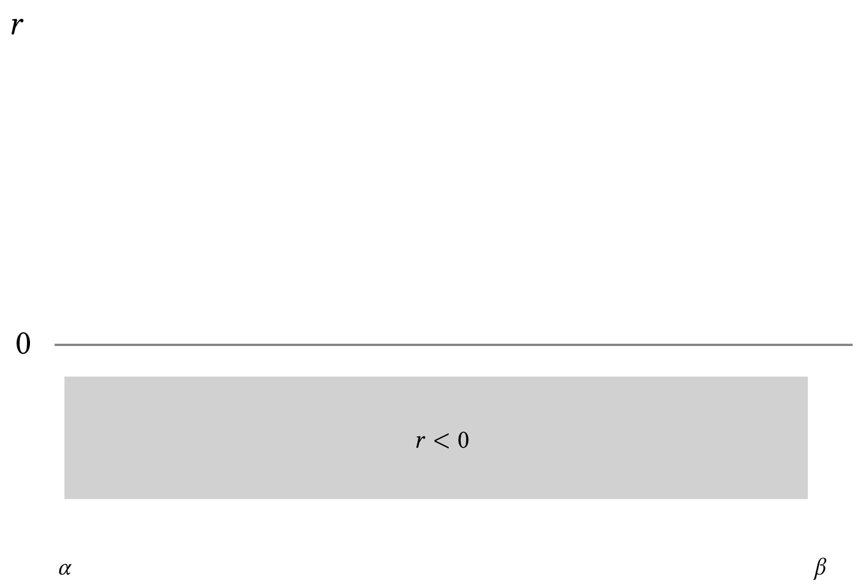
Radius Graphs vs Polar Traces

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The scalar value decreases, but the distance increases from 1 to 3.

A scalar $\theta - r$ graph is negative only on $\alpha < \theta < \beta$ and is zero at both endpoints. Translate this entire feature into a statement about the polar trace.



Condition or evidence	Supplied value
Boundary values	$r(\alpha) = r(\beta) = 0$
Condition	$r(\theta) < 0$ for $\alpha < \theta < \beta$
Given parameter range or context	interval (α, β)

Your response: ____________

AP Precalculus

Radius Graphs vs Polar Traces

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student says every negative value on $a\theta - r$ graph becomes a polar point below the x-axis. Use $r = -2$ at $\theta = 3\frac{\pi}{2}$ to repair the claim.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	**Supplied value**
Given parameter range or context	$\theta = 3\frac{\pi}{2}$
Polar pair	-2 $3\frac{\pi}{2}$
Student claim to examine	negative r means the plotted point has $y < 0$

A student claims: "Because the scalar radius repeats after π, the same polar point also repeats after π." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: The scalar rule $\cos(2\theta)$ has period π. Does shifting θ by π repeat the same polar point?

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	**Supplied value**
Given parameter range or context	all real θ
Original rule	$r = \cos(2\theta)$
Given scalar period	π
Student claim to examine	Because the scalar radius repeats after π, the same polar point also repeats after π.

Radius Graphs vs Polar Traces

Full Worked Solutions - excerpt

Worked example

Problem. On an interval, r rises from -6 to -1. Compare the scalar graph's change with the polar point's distance change.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given parameter range or context	an interval with $r < 0$
Scalar behavior	r increases from -6 to -1

Answer. The scalar graph increases, while the polar point moves inward because its distance decreases from 6 to 1.

Explanation and check.

Scalar order uses signed outputs; polar distance uses absolute radius. Those measures move in opposite directions in this negative regime.

Worked example

Problem. For constant radius $r = -4$ while θ varies, compare scalar output change, polar distance change, and polar position change.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given parameter range or context	θ varies over an interval
Radius	constant -4

Answer. Scalar output and distance are both constant, but the polar position changes around the circle on opposite rays.

Explanation and check.

Neither r nor $|r|$ changes. The angle still changes the direction, so the point is not stationary in the plane.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polar Models in Context

Polar Models in Context | APPC-U03-P30

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for feasibility decision, two model choices and noninterchangeability, supported model, admissibility decision, feasibility and repair and decision.

8 PDFs and a README; 56 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Polar Models in Context

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

A rotating sensor obeys $r = 2 + \cos\theta$ and $\theta = \pi t$, with t in seconds. Write its radius as a function of time.

Bank label: ______

Held-out radii are 4.1 and 2.2 at $\theta = \frac{\pi}{3}$ and $2\frac{\pi}{3}$. Model A predicts 4.0 and 2.0; Model B predicts 3.6 and 2.7. With tolerance 0.25 at each angle, which model is supported?

Required response

$\frac{\pi}{3}$	4.1
$2\frac{\pi}{3}$	2.2

Bank label: ______

Engineers measure the maximum audible distance from a siren separately for each compass bearing. Choose a model that directly supports drawing the warning boundary.

Bank label: ______

A radius graph has maximum 9 at $\theta = \pi$ and minimum 3 at $\theta = 0$. Select the compatible model from $r = 6 - 3\cos\theta, r = 6 + 3\cos\theta$, and $r = 3 + 6\sin\theta$.

Bank label: ______

AP Precalculus

Polar Models in Context

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A boundary circle centered at $(3, 0)$ with radius 3 satisfies $r^2 = 6r\cos\theta$. Find its nonzero radial model and the angles where it gives a forward boundary point.	An accepted model $r = 5 - \sin\theta$ is explicitly valid only for $0 \le \theta \le \pi$. A report requests the modeled location at $\theta = 7\frac{\pi}{6}$. What should be reported?
Models A and B have equal held-out RMSE 0.18. In two safety-critical directions A's residuals are 0.08 and 0.11, while B's are 0.32 and 0.29; tolerance there is 0.15. Which model is validated for the safety use?	A signed radial signal is $r = 5\cos\theta$, and $\theta = \pi\frac{t-2}{4}$ so that the polar axis is reached at $t = 2s$. Express r as a function of t.
A radius graph has maxima at 0 and π and minima at $\frac{\pi}{2}$ and $3\frac{\pi}{2}$. Which candidate fits: $r = 5 + 2\cos\theta, r = 5 + 2\cos(2\theta)$, or $r = 5 + 2\sin(2\theta)$?	Radar gives the storm edge's distance from a station for every bearing at one instant. Choose a representation for drawing the current edge, then decide whether the same data can predict when the edge reaches a town.

AP Precalculus

Polar Models in Context

Name: __________ **Date:** __________ **Class:** __________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The matching model is r = 4 − 2 $\sin\theta$.

A scanner starts at angle $\frac{\pi}{6}$ and turns counterclockwise at 3 revolutions per minute. For $r = 4 + \cos\theta$ and t in minutes, write $\theta(t)$ and $r(t)$.

Your response: __________

Incoming: $\theta(t) = \frac{\pi}{6} + 6\pi t$ and $r(t) = 4 + \cos(\frac{\pi}{6} + 6\pi t)$.

A scanner follows $r = 3 + 2\sin\theta$ and turns according to $\theta = \pi\frac{t}{6}$. Write $r(t)$ and find the radius at $t = 3$ seconds.

Your response: __________

Incoming: $r(\theta)$ =min$(\frac{4}{|\cos\theta|}, \frac{3}{|\sin\theta|})$, with the usual convention that division by zero is ignored. A corner is hit when $\frac{4}{|\cos\theta|} = \frac{3}{|\sin\theta|}$, or $|\tan\theta| = \frac{3}{4}$.

Radii at $0, \frac{\pi}{2}, \pi, 3\frac{\pi}{2}$ are $3, 5, 7, 5$. Which candidate fits:
$r = 5 - 2\cos\theta, r = 5 + 2\cos\theta$, or $r = 5 - 2\sin\theta$?

Required response

0	3
$\frac{\pi}{2}$	5
π	7
$3\frac{\pi}{2}$	5

Your response: __________

Incoming: $r(t) = 7 + \sin(\pi\frac{t-1}{4})$, and the scan covers $0 \le \theta \le \pi$.

For the signed rule $r = 1 + 3\cos\theta$, evaluate at $\theta = 4\frac{\pi}{3}$ and report the signed radius, distance from the origin, and Cartesian point.

Your response: __________

AP Precalculus

Polar Models in Context

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A device claims that $d(\theta) = 2\cos\theta$ is its nonnegative distance to an obstacle for every $0 \le \theta \le 2\pi$. Is the model physically admissible as stated?

An unlabeled polar loop is used to claim that a vehicle moved more slowly where the curve bends sharply. Diagnose the claim.

A student claims: “Model B has the smaller mean absolute error because it exactly matches the last observation more closely.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Held-out radii are $5.0, 4.2, 3.1, 4.1$. Models A and B predict $(4.9, 4.0, 3.2, 4.3)$ and $(5.4, 4.1, 2.7, 4.0)$. Which has the smaller mean absolute radial error?

A scanner can measure at most $10\ \text{ m}$, but a proposed all-bearing model is $r = 7 + 4\cos\theta$. Is the model within the hardware range?

Polar Models in Context

Full Worked Solutions - excerpt

Worked example

Problem. A rotating sensor obeys $r = 2 + \cos\theta$ and $\theta = \pi t$, with t in seconds. Write its radius as a function of time.

Answer. $r(t) = 2 + \cos(\pi t)$

Explanation and check.

Substitute the supplied angle law $\theta = \pi t$ into the radial rule.

Worked example

Problem. A circular safety boundary has model $r = 6$. What boundary point corresponds to $\theta = \pi$?

Answer. $(-6, 0)$

Explanation and check.

At angle π, cosine is -1 and sine is 0, so $(r\cos\theta, r\sin\theta) = (-6, 0)$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Parametric Tables and Curve Tracing

Parametric Tables and Curve Tracing | APPC-U04-P01

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for equal x and equal y input pairs, filled table, repeated point, equal-y pair, (t,x,y) table, ordered triples, anchor order and repeated-point times and vertex order and closure.

8 PDFs and a README; 157 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Parametric Tables and Curve Tracing

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

For $x = t + 3$ and $y = t^2 - 1$, complete the (t, x, y) rows for $t = -1, 0, 2$.

Bank label: _______

Find the common parameter domain of $x = \sqrt{t+2}$ and $y = \frac{1}{t-1}$.

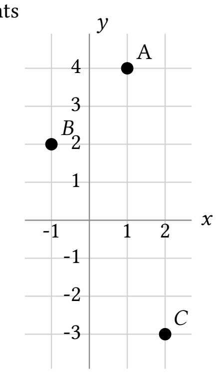
Bank label: _______

For $x = \cos t$, $y = \cos(2t)$ on $0 \le t \le 2\pi$, find every visit to $(0, -1)$.

Bank label: _______

A unit-scale plane trace labels $t = -2$ at $(1, 4)$, $t = 0$ at $(-1, 2)$, and $t = 3$ at $(2, -3)$. Write the corresponding component table.

Supplied labeled points



Given points only; connections between labels are unspecified here.
A: $t = -2(1, 4)$
B: $t = 0(-1, 2)$
C: $t = 3(2, -3)$

Bank label: _______

AP Precalculus

Parametric Tables and Curve Tracing

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Connect consecutive table points by straight segments for increasing t. List the vertex order and decide whether the path closes.

Required response

t	**x**	**y**
0	1	0
1	0	2
2	-1	0
3	0	-2
4	1	0

For $x = t^2 - 1, y = t^2$ on $-2 \leq t \leq 2$, find every t that visits $(0, 1)$. What causes the repeat?

AP Precalculus

Parametric Tables and Curve Tracing

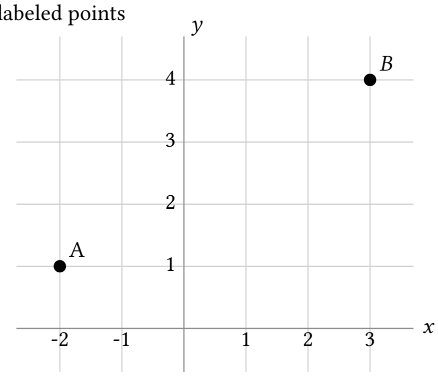
Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A trace labels $t = 0$ at $(-2, 1)$, $t = 2$ at $(3, 4)$, and $t = 5$ again at $(-2, 1)$. Write the component rows and identify the repeated visit.

Supplied labeled points



Given points only; connections between labels are unspecified here.
A: $t = 0(-2, 1)$
B: $t = 2(3, 4)$
A: $t = 5(-2, 1)$

Your response: ____________

Incoming: Rows are $(0, -2, 1), (2, 3, 4), (5, -2, 1)$. The point $(-2, 1)$ is visited at both $t = 0$ and $t = 5$.

Connect consecutive samples in increasing t. Give the point sequence and identify any repeated location.

Required response

t	x	y
0	0	0
1	2	1
2	0	0
3	-1	2

Your response: ____________

Incoming: The sequence is $(0, 0) \to (2, 1) \to (0, 0) \to (-1, 2)$. The origin is visited at $t = 0$ and $t = 2$.

Build the paired table for $x = t + 1$ and $y = 2t$ at $t = -2, 0, 3$.

Your response: ____________

Incoming: $[[-2, -1, -4], [0, 1, 0], [3, 4, 6]]$

Determine the joint domain, identify the two denominator holes, and evaluate each attained outer boundary point. Work with -5≤t≤5. Restrict the parameter to [-5,5]. Use x=√((t+1)(t-4))/(t-5). Use y=ln(6-t)/(t+3).

Your response: ____________

AP Precalculus

Parametric Tables and Curve Tracing

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $x = t^2$ and $y = 2t - 1$, a student writes rows $(0, 0, 1), (1, 1, 3), (2, 4, -1)$. Identify the pairing error and give the correct table.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

t	**x**	**y**
0	0	1
1	1	3
2	4	-1

A student claims: "At $t = 0$ record $y = 0$, so all four requested rows are valid." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Build the paired table for $x = t^2 - 3t, y = \frac{4}{t}$ at $t = -2, 0, 2, 4$, marking invalid inputs.

Parametric Tables and Curve Tracing

Full Worked Solutions - excerpt

Worked example

Problem. Build the (t, x, y) table for $x = |t| - 1, y = t^2 - 4$ at $t = -2, -1, 1, 3$. Identify any repeated plane point.

Answer. Rows are $(-2, 1, 0), (-1, 0, -3), (1, 0, -3), (3, 2, 5)$. The point $(0, -3)$ repeats at $t = -1$ and $t = 1$.

Explanation and check.

Both components are even in t on the paired ± 1 inputs, so those two rows have the same x and y outputs.

Worked example

Problem. For $x = (t - 1)(t + 1), y = (t - 2)(t + 1)$, find all visits to $(0, 0)$ on $-2 \leq t \leq 3$.

Answer. Only t = -1 visits $(0, 0)$.

Explanation and check.

$x = 0$ at $t = -1$ or 1, while $y = 0$ at $t = -1$ or 2. The only common root is -1.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Eliminating Parameters and Reparameterizing Curves | APPC-U04-P02

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for covered old t-values and missing branch, parametric pair and parameter domain, pair and mixed t-domain, pair and parameter domain, relation and exact missing point and two exact set differences.

8 PDFs and a README; 63 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Both pairs eliminate to the same familiar relation. Are their actual images equal?

Work on the domain $t \in \mathbb{R}; s \in \mathbb{R} \setminus \{0\}$.

The first parametrization is $x = t; y = t^2$.

The second parametrization is $x = \frac{1}{s}; y = \frac{1}{s^2}$.

Bank label: ______

Eliminate t from the two affine component rules.

Work on the domain all real t.

The x-coordinate satisfies $x = t - 4$.

The y-coordinate satisfies $y = 2t + 3$.

Bank label: ______

Eliminate t and keep the branch restriction.

Work on the domain $t \geq 0$.

The x-coordinate satisfies $x = t^2$.

The y-coordinate satisfies $y = t - 3$.

Bank label: ______

Eliminate t and describe the traced half of the locus with a Cartesian inequality.

Work on the domain $\frac{\pi}{3} \leq t \leq 4\frac{\pi}{3}$.

The endpoints is $((1, \frac{\sqrt{3}}{2}), (-1, -\frac{\sqrt{3}}{2}))$.

The x-coordinate satisfies $x = 2\cos t$.

The y-coordinate satisfies $y = \sin t$.

Required response

1	$\frac{\sqrt{3}}{2}$
-1	-$\frac{\sqrt{3}}{2}$

Bank label: ______

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: __________ **Date:** __________ **Class:** __________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Use $t = -s$ to reparameterize the curve without changing its point set. Give the required s-domain and compare direction. Work on the domain $-2 \leq t \leq 1$. Use the parameter substitution $t = -s$. The original parametrization is $x = t$; $y = t^2$.	Compare the point set, orientation, and number of visits for the two descriptions. Work on the domain $0 \leq t \leq 2\pi$ and $0 \leq s \leq 2\pi$. The first parametrization is $x = \cos t$; $y = \sin t$. The second parametrization is $x = \cos(2s)$; $y = \sin(2s)$.
Use $t = y$, not $t = x$, to parametrize the restricted graph. Derive $x(t)$ and verify the supplied y-range. Work on the domain $-1 \leq x \leq 3$. The observed y range is $[-2, \frac{2}{5}]$. The relation is $y = \frac{x-1}{x+2}$.	Eliminate t and identify the algebraic-locus point that is excluded by the strict bound. Work on the domain $0 < t \leq 3$. The x-coordinate satisfies $x = t^2$. The y-coordinate satisfies $y = t + 1$.
Eliminate t and state how much of the locus is covered. Work on the domain $0 \leq t \leq 2\pi$. The x-coordinate satisfies $x = 4\sin t$. The y-coordinate satisfies $y = 4\cos t$.	Eliminate t and distinguish the two horizontal-axis endpoint statuses. Work on the domain $\pi < t \leq 2\pi$. The x-coordinate satisfies $x = 2\cos t$. The y-coordinate satisfies $y = 2\sin t$.

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: They have the same image y = x^2 for $-1 \le x \le 1$. The first crosses it once from left to right; the second goes left-to-right and then right-to-left.

Eliminate t from the affine pair and state the x-domain.

Work on the domain all real t.

The x-coordinate satisfies $x = 2t - 1$.

The y-coordinate satisfies $y = t + 4$.

Your response: ______________

Incoming: $x^2 + y^2 = 1$ with $x + y > 0$.

Eliminate t and retain the coordinate interval created by the asymmetric parameter bounds.

Work on the domain $-2 \le t \le 4$.

The x-coordinate satisfies $x = |t - 1|$.

The y-coordinate satisfies $y = t + 2$.

Your response: ______________

Incoming: $x^2 + y^2 = 1$ with $x \ge 0$ and $y \ge 0$, including $(0, 1)$ and $(1, 0)$.

Do the two descriptions have the same image, and do they trace it in the same way?

Work on the domain $-1 \le t \le 1$ and -$\frac{\pi}{2} \le s \le 3\frac{\pi}{2}$.

The first parametrization is $x = t$; $y = t^2$.

The second parametrization is $x = \sin s$; $y = \sin^2 s$.

Your response: ______________

Incoming: x = -2 with $-1 < y \le 8$.

Repair the student's ± choice and test the supplied extra point against the original rules.

Work on the domain $t \ge 0$.

Evaluate this student result: $y = \pm\sqrt{x} - 1$.

Check the point $(4, -3)$.

The x-coordinate satisfies $x = t^2$.

The y-coordinate satisfies $y = t - 1$.

Your response: ______________

Incoming: x = $t + 2$, $y = \frac{1}{t}$ on $1 \le t \le 4$.

Eliminate t and identify the branch restriction inherited from $t \le 1$.

Work on the domain $t \le 1$.

The x-coordinate satisfies $x = (t - 1)^2$.

The y-coordinate satisfies $y = t + 2$.

Your response: ______________

Incoming: The new pair is x = $s^3 - s$, $y = s^3 - s + 1$ and has the same point set. Its t-image is $[-\frac{2}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}]$; $t = 0$ occurs at $s = -1, 0, 1$, and each nonzero interior t-value is attained twice within the interval.

Eliminate t without dropping either sign of the original parameter.

Work on the domain $-2 \le t \le 2$.

The x-coordinate satisfies $x = t^2$.

The y-coordinate satisfies $y = 2t + 1$.

Your response: ______________

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Repair the student's elimination and use the supplied point to expose the extra branch.

Work on the domain $t < 0$.

The original parametrization is $x = t^2$; $y = t$.

Evaluate this student result: $x = y^2$ with both signs of y.

Check the point $(1, 1)$.

Repair the canceled expression result and use the supplied point to show why the domain matters.

Work on the domain $t \in \mathbb{R}, t \neq 1$.

Evaluate this student result: $y = x + 1$ for every real x.

Check the point $(1, 2)$.

The x-coordinate satisfies $x = t$.

The y-coordinate satisfies $y = \frac{t^2-1}{t-1}$.

Eliminating Parameters and Reparameterizing Curves

Full Worked Solutions - excerpt

Worked example

Problem. Eliminate t from $x = t + 2, y = 3t - 1$ and state the x-domain.

Answer. y = $3x - 7$ for all real x.

Explanation and check.

From $x = t + 2, t = x - 2$. Substitution into y gives $3(x - 2) - 1 = 3x - 7$.

Worked example

Problem. Introduce a parameter for $y = x^2 - 3$ restricted to $-2 \le x \le 1$, and verify coverage.

Answer. x = $t, y = t^2 - 3$ with $-2 \le t \le 1$.

Explanation and check.

Choosing the free x-coordinate as t transfers the x restriction directly to t and reaches each permitted x once.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Parametric Motion and Intersections

Parametric Motion and Intersections | APPC-U04-P03

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for all extrema with times and points, rightmost and highest positions, feasible interval, paired rules, paired model and all times and positions.

8 PDFs and a README; 149 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Parametric Motion and Intersections

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Do the robots collide during the observation window?

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 3$ seconds
Robot A	**X** t **Y** $2t$
Robot B	**X** $6 - 2t$ **Y** $8 - 2t$

Bank label: ______

AP Precalculus

Parametric Motion and Intersections

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Interpret the vehicle's location at $t = 4$ under the stated coordinate convention.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Coordinate direction convention	positive x west, positive y north
Original time range or clock definition	$0 \le t \le 5$ minutes
Time to evaluate	4
Given $x(t)$	$x = 6 - 2t$ kilometers
Given $y(t)$	$y =$ 1-t kilometers

Where is the object at the final modeled time?

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Coordinate direction convention	positive x east, positive y upward
Original time range or clock definition	$0 \le t \le 5$ seconds
Time to evaluate	5
Given $x(t)$	$x = 1 - 2t$ meters
Given $y(t)$	$y = (t - 3)^2$ meters

Write the position model for the four-second interval.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 4$ seconds
Horizontal change	2 meters west each second
Initial horizontal coordinate	7
Vertical rule	$y = 3 + (t - 1)^2$ meters

List every ground event in the time window and the position at each one.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 5$ seconds
Given $x(t)$	$x = 2t - 3$ meters
Given $y(t)$	$y = (t - 1)(t - 4)$ meters

AP Precalculus

Parametric Motion and Intersections

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: At t = 4 seconds, at $(4, 3)$.

When is the object inside the corridor and at nonnegative height?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Conditions	$-2 \le x \le 4$ $y \ge 0$
Original time range or clock definition	$0 \le t \le 8$ seconds
Given $x(t)$	$x = t - 3$ meters
Given $y(t)$	$y = 12 - 2t$ meters

Your response: ______________

Incoming: Leftmost and highest occur at t = 0, 4, both at $(0, 3)$. Rightmost and lowest occur at $t = 2$ at $(4, -1)$.

When does the object cross the x-axis?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 4$ seconds
Given $x(t)$	$x = \frac{1}{t+2}$ meters
Given $y(t)$	$y = t - 1$ meters

Your response: ______________

Incoming: $t \in [0, 2] \cup [4, 5]$.

Find the position at $t = 3$ seconds.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 6$ seconds
Time to evaluate	3
Given $x(t)$	$x = t - 4$ meters
Given $y(t)$	$y = 2t + 1$ meters

Your response: ______________

Incoming: At t = 1 the position is $(0, 0)$, and at the included endpoint $t = 3$ it is $(0, 2)$.

When is the object both on or right of the y-axis and at nonnegative height?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Conditions	$x \ge 0$ $y \ge 0$
Original time range or clock definition	$0 \le t \le 6$ seconds
Given $x(t)$	$x = t^2 - 1$ meters
Given $y(t)$	$y = 5\text{-}t$ meters

Your response: ______________

AP Precalculus

Parametric Motion and Intersections

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Repair the student's claim using the two supplied times.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Comparison times	2 3
Original time range or clock definition	$0 \le t \le 6$ seconds
Student claim to examine	At $t = 2$ the cart is moving left because $x(2) < 0$.
Given $x(t)$	$x = -5 + t$ meters
Given $y(t)$	$y = 2$ meters

A student claims the motion never reaches the y-axis during $0 \le t \le 3$ because the event would occur only after the interior of the interval. Identify the error, then give the correct event time and position.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 3$ seconds
Event	y-axis means $x = 0$
Given $x(t)$	$x = 3\text{-}t$ meters
Given $y(t)$	$y = t^2 + 1$ meters

Parametric Motion and Intersections

Full Worked Solutions - excerpt

Worked example

Problem. When does the object cross the y-axis?

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 4$ seconds
Given $x(t)$	$x = 3t - 6$ meters
Given $y(t)$	$y = t^2$ meters

Answer. At t = 2 seconds, at $(0, 4)$ meters.

Explanation and check.

Set $x = 0$ to get $t = 2$, then evaluate y at that same time.

Worked example

Problem. List every ground event in the time window and the position at each one.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 5$ seconds
Given $x(t)$	$x = 2t - 3$ meters
Given $y(t)$	$y = (t - 1)(t - 4)$ meters

Answer. At t = 1 the position is $(-1, 0)$, and at $t = 4$ it is $(5, 0)$.

Explanation and check.

The height factors vanish at $t = 1$ and $t = 4$; the horizontal coordinate must be evaluated separately at each event time.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Average Rates and Speed in Parametric Motion

Average Rates and Speed in Parametric Motion | APPC-U04-P04

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
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Topic focus

Across 96 tasks this topic asks for classification and slope, classification, slope, and orientation, slope status, slope status and direction, two ordered pairs and two rate pairs and scale factor.

8 PDFs and a README; 141 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Average Rates and Speed in Parametric Motion

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the average horizontal and vertical rates over the interval.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

t	x	y
1	2	-1
3	8	5

Condition or evidence	Supplied value
Original interval or clock definition	$1 \leq t \leq 3$ seconds
Units	meters

Bank label: ______

AP Precalculus

Average Rates and Speed in Parametric Motion

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Find average speed and average-velocity magnitude.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

t	x	y
0	0	0
2	8	0
5	3	0

Condition or evidence	Supplied value
Original interval or clock definition	$0 \le t \le 5$ seconds
Segments	straight between rows
Units	meters

Find the endpoint after the stated interval.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given component averages	-4 5
Original interval or clock definition	three-parameter-unit interval
Start	8 -1

Compute both exact average coordinate rates.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original interval or clock definition	$3 \le t \le 8$ seconds
X rule	$x = \sqrt{t+1}$ meters
Y rule	$y = \frac{1}{t}$ meters

Determine the endpoint-secant slope from the exact rates.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given average x-rate	$\frac{1}{3}$
Given average y-rate	0
Original interval or clock definition	common nonzero interval

AP Precalculus

Average Rates and Speed in Parametric Motion

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Average speed 2 meters per second; average-velocity magnitude $\frac{1}{2}$ meter per second.

Reconstruct the initial position that is compatible with the endpoint and both average rates.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	**Supplied value**
Given component averages	-2 3
Original interval or clock definition	four-second observation window
End	5 8

Your response: ________________________________

AP Precalculus

Average Rates and Speed in Parametric Motion

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Diagnose and repair the student's calculation.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Original interval or clock definition	$2 \le t \le 5$ seconds
End point	7 11
Start point	1 2
Student claim to examine	The average vertical rate is $\frac{\Delta y}{\Delta x} = \frac{9}{6} = \frac{3}{2}$ meters per second.

A student claims: "The endpoint displacement is right and down, so the object moves right and down throughout the sampled interval." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Use all rows, not just the endpoints, to describe net direction and any observed reversals.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

t	x	y
0	-2	6
1	-5	4
3	-1	4
4	2	1

Condition or evidence	Supplied value
Original interval or clock definition	$0 \le t \le 4$ seconds
Student claim to examine	The endpoint displacement is right and down, so the object moves right and down throughout the sampled interval.

Average Rates and Speed in Parametric Motion

Full Worked Solutions - excerpt

Worked example

Problem. Determine the endpoint-secant slope from the exact rates.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given average x-rate	$\frac{1}{3}$
Given average y-rate	0
Original interval or clock definition	common nonzero interval

Answer. 0

Explanation and check.

The nonzero horizontal rate permits division, and 0 divided by $\frac{1}{3}$ is 0.

Worked example

Problem. Determine the two finite coordinate rates across the entire parameter interval.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original interval or clock definition	$1 \leq t \leq 4$
X	$3t - 2$
Y	$t^2 - 5$

Answer. $(3, 5)$

Explanation and check.

The endpoints change from $(1, -4)$ to $(10, 11)$, giving changes 9 and 15 over an interval of length 3.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Parametric Equations of Lines

Parametric Equations of Lines | APPC-U04-P05

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
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Topic focus

Across 96 tasks this topic asks for domain, interval notation, closed interval, interval, inequality and sufficiency statement and family.

8 PDFs and a README; 125 buyer pages.

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Published by Tusils LLC.

AP Precalculus

Parametric Equations of Lines

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use a normal direction of the reference line to parametrize the perpendicular line through the point. point: $[3, -2]$; reference line: $x - 2y = 7$; required relation: perpendicular.

Bank label: ______

AP Precalculus

Parametric Equations of Lines

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Write parametric equations for the line through the point with the given direction. direction: $[3, 4]$; point: $[2, -1]$.	State the parameter interval that produces the full supporting line.
Determine whether changing both the anchor and direction scale changed the geometric line. domains: t and s real; first: $(x, y) = (-2 + t, 4 + 3t)$; second: $(x, y) = (1 + 2s, 13 + 6s)$.	Use the given direction angle and scale to parametrize the full line from the marked point. coordinate speed: 2; direction angle degrees: 150; point: $[1, 1]$.

AP Precalculus

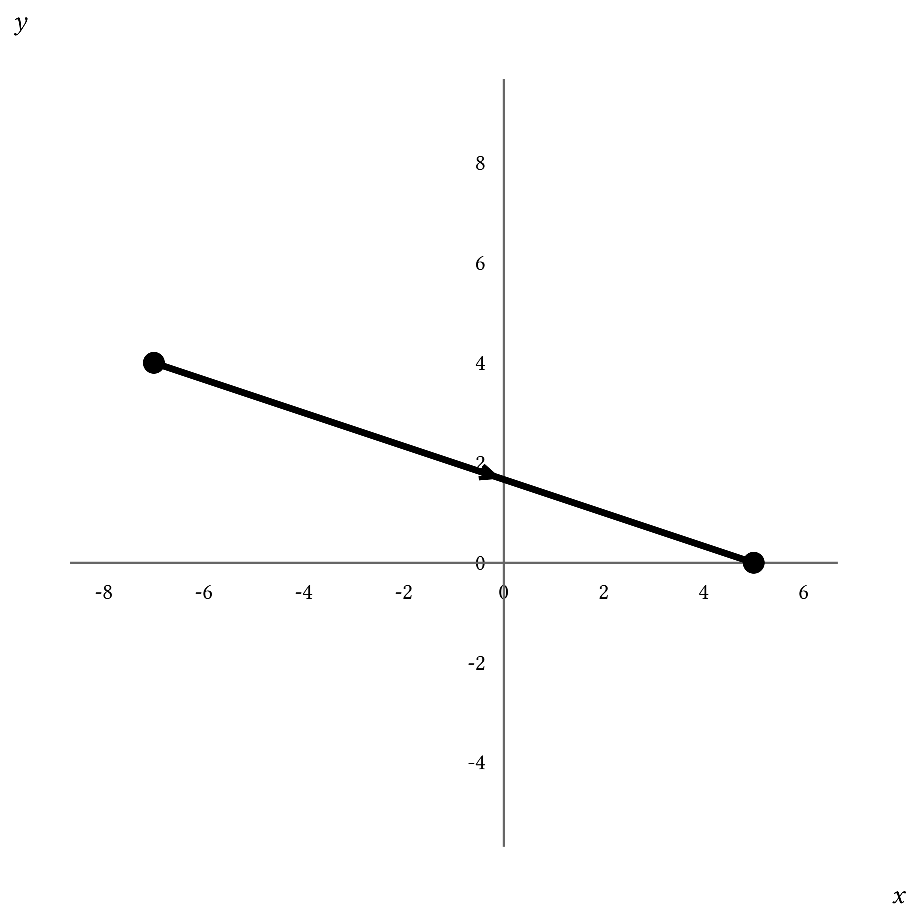
Parametric Equations of Lines

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: a = 2 and b = 4

Choose the t-values that trace exactly the highlighted closed segment. endpoint membership: both included; endpoint parameters: $[-2, 2]$; map: $x = 3t - 1$, y = 2-t; domain: to be selected.



Your response: ____________

AP Precalculus

Parametric Equations of Lines

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The student interchanged point coordinates and direction components. Repair the parametrization. required direction: $[3, 1]$; required point: $[4, -2]$; student map: $x = 3 + 4t, y = 1 - 2t$.

The rates are right but the clock origin is not. Repair the student's timed segment. required: $(1, 4)$ at $t = 2$ and $(7, 1)$ at $t = 5$; student map: $x = 1 + 2t, y = 4 - t, 0 \leq t \leq 3$.

Parametric Equations of Lines

Full Worked Solutions - excerpt

Worked example

Problem. Find a parameter-free equation for the line image. map: $x = -3 + 2t, y = 5 + 5t$; domain: t real.

Answer. $5x - 2y = -25$

Explanation and check.

The combination $5(-3 + 2t) - 2(5 + 5t)$ is constantly -25.

Worked example

Problem. Give an affine parametrization of this horizontal line. equation: $y = -7$; domain: full line.

Answer. x = $t, y = -7$, with t real.

Explanation and check.

Let x vary freely while y remains equal to -7.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Parametric Circles

Parametric Circles | APPC-U04-P06

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
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Topic focus

Across 96 tasks this topic asks for center, radius, start, orientation, exact set-builder list, interval, motion comparison and times, set, direction, parameter-rate comparison and locus, start, period, orientation.

8 PDFs and a README; 139 buyer pages.

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AP Precalculus

Parametric Circles

Name: ____________ **Date:** ____________ **Class:** ____________

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Read the geometric and traversal features from the paired rule.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$3 + 4\cos(t)$
Given $y(t)$	$-2 + 4\sin(t)$

Bank label: _______

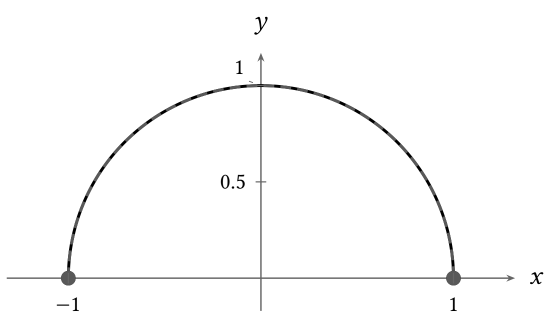
Model the ninety-second clockwise revolution from the delayed leftmost start.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given center	1 2
Original parameter range or clock definition	time t in seconds
Given direction	clockwise
Period seconds	90
Given radius	3
Start point	−2 2
Start time	30

Bank label: _______

Compare the two restricted maps as arc sets and as directed traversals.



Condition or evidence	Supplied value
Original parameter range or clock definition	$0 \leq t \leq \pi$ and -$\pi \leq s \leq 0$
First motion rule	$(\cos t, \sin t)$
Second motion rule	$(\cos(-s), \sin(-s))$

Bank label: _______

Account for the doubled angle when choosing the lower half-circle parameter window.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given arc	lower semicircle
Original parameter range or clock definition	to be selected
Given ending position	1 0
Given endpoint membership	both included
Given direction	counterclockwise
Given coordinate rule	$x = \cos(2t)$, $y = \sin(2t)$
Given starting position	−1 0

Bank label: _______

AP Precalculus

Parametric Circles

Name: ____________ **Date:** ____________ **Class:** ____________

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Read the circle geometry and zero-time position from the rule.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$2 + 3\cos(t)$
Given $y(t)$	$4 + 3\sin(t)$

Construct the clockwise motion from the $3 - 4 - 5$ radius position without using a decimal phase.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given center	2 -3
Original parameter range or clock definition	t real
Given direction	clockwise
Given radius	5
Given starting position	5 1

AP Precalculus

Parametric Circles

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Incoming: $x = 3\sin(t), y = 2 - 3\cos(t)$.

Construct a rule that begins at the top and initially moves right.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Given center	-3 2
Original parameter range or clock definition	t real
Given direction	clockwise
Given radius	4
Given starting position	-3 6

Your response: ______________________

Incoming: $x = -4\text{-}\cos(t), y = \sin(t)$.

Can any increasing parameter interval satisfy the endpoint and sweep requirements simultaneously?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original parameter range or clock definition	to be selected
Given ending position	1 0
Required sweep	exactly one-half revolution
Given coordinate rule	$x = \cos(t), y = \sin(t)$
Given starting position	1 0

Your response: ______________________

AP Precalculus

Parametric Circles

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The radius, phase, and orientation are correct, but one center coordinate is not. Repair it.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	**Supplied value**
Original parameter range or clock definition	t real
Student rule to examine	$x = 1 + 3\cos(t), y = 2 + 3\sin(t)$
Given target requirements	**Given center** 1, −2 **Given direction** counterclockwise **Given radius** 3 **Given starting position** 4, −2

Parametric Circles

Full Worked Solutions - excerpt

Worked example

Problem. Read the basic circle geometry from the constants and amplitudes.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$-2 + 6\cos(t)$
Given $y(t)$	$4 + 6\sin(t)$

Answer. Center $(-2, 4)$, radius 6.

Explanation and check.

The coordinate midlines are -2 and 4, and both components have amplitude 6.

Worked example

Problem. Identify the circle's center and radius from the paired rule.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$5 + 2\cos(t)$
Given $y(t)$	$1 + 2\sin(t)$

Answer. Center $(5, 1)$ and radius 2.

Explanation and check.

The additive constants locate the center, and the equal sinusoidal amplitudes give the radius.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Implicit Relations and Curves

Implicit Relations and Curves | APPC-U04-P07

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Topic focus

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AP Precalculus

Implicit Relations and Curves

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Exactly one row misses the relation. Which one?

Given: domain: x, y real; relation: $(x-2)(y+1)=6$; rows: $(5,1)$; $(-1,-3)$; $(4,2)$; $(3,4)$

Required response

x	y
5	1
-1	-3
4	2
3	4

Bank label: ______

Identify every coordinate pair that satisfies the scaled relation.

Given: domain: x, y real; relation: $\frac{x^2}{4}+\frac{y^2}{9}=1$; rows: $(0,3)$; $(2,0)$; $(1,2)$; $(\sqrt{2}, 3\frac{\sqrt{2}}{2})$

Required response

x	y
0	3
2	0
1	2
$\sqrt{2}$	$3\frac{\sqrt{2}}{2}$

Bank label: ______

How many rows lie on the relation, and which are they?

Given: domain: x, y real; relation: $2x$-$y=5$; rows: $(3,1)$; $(\frac{7}{2},2)$; $(\frac{1}{2},-4)$; $(-1,-7)$

Required response

x	y
3	1
$\frac{7}{2}$	2
$\frac{1}{2}$	-4
-1	-7

Bank label: ______

Mark the valid rows, then explain what they show about y-values at $x=10$.

Given: domain: x, y real; relation: $y^2=x-1$; rows: (x: 10; y: 3); (x: 10; y: -3); (x: 10; y: 0)

Required response

x	y
10	3
10	-3
10	0

Bank label: ______

AP Precalculus

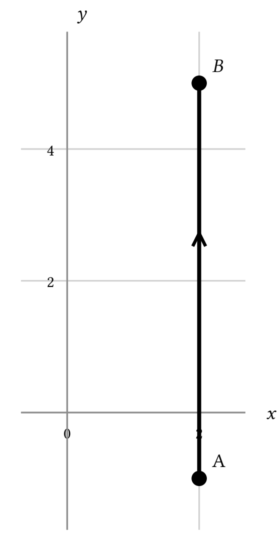
Implicit Relations and Curves

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Describe both coordinate changes along the arrow.

Given graph



A: $(2, -1)$

B: $(2, 5)$

AP Precalculus

Implicit Relations and Curves

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Write every real y-branch and state the x-values for which it exists.

Given: domain: x, y real; isolated variable: y; relation: y^2 = 4-x

Your response: ______________

Incoming: y = $\sqrt{4-x}$ or $y = -\sqrt{4-x}$, with $x \leq 4$.

Express the relation as a rule with x as input.

Given: domain: x, y real; isolated variable: y; relation: $2x + 3y = 12$

Your response: ______________

Incoming: y = $4 - (\frac{2}{3})x$ for every real x.

List every relation point with the prescribed first coordinate.

Given: domain: x, y real; input x: 3; relation: y^2 = 12-x

Your response: ______________

Incoming: $(3, 3)$ and $(3, -3)$.

Determine whether the relation has any real point above this x-value.

Given: domain: x, y real; input x: 3; relation: $(y-1)^2 = x - 4$

Your response: ______________

Incoming: It has none.

Solve for x, keeping every real branch and the output-variable domain.

Given: domain: x, y real; isolated variable: x; relation: $(x-2)^2 = y+1$

Your response: ______________

Incoming: x = 2+$\sqrt{y+1}$ or x = 2−$\sqrt{y+1}$, for $y \geq -1$.

Make y explicit and state the excluded vertical line.

Given: domain: x, y real; isolated variable: y; relation: $(x+3)(y-2) = 4$

Your response: ______________

AP Precalculus

Implicit Relations and Curves

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Which proposed points belong to the relation?

Given: candidates: $(5,3)$; $(5,-3)$; $(5,1)$; domain: x, y real; relation: $y^2 = x + 4$

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

x	y
5	3
5	−3
5	1

A student claims: "All four listed rows satisfy $x^2 - y = 3$." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Three rows satisfy the relation. Identify the one that does not.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

x	y
−1	−2
0	−3
2	1
3	5

Implicit Relations and Curves

Full Worked Solutions - excerpt

Worked example

Problem. Determine whether the relation has any real point above this x-value.

Given: domain: x, y real; input x: 3; relation: $(y-1)^2 = x - 4$

Answer. It has none.

Explanation and check.

Substitution would make the square $(y-1)^2$ equal -1, impossible over the reals.

Worked example

Problem. Give a complete explicit description of the relation.

Given: domain: x, y real; relation: $y^2 = (x-1)^2$

Answer. y = $x - 1$ or $y = 1$ - x, for every real x.

Explanation and check.

Equality of the squares means $y = \pm(x-1)$; the two lines meet at $(1, 0)$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Conic Sections from Equations and Data

Conic Sections from Equations and Data | APPC-U04-P08

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for axis, direction, x-intercepts, vertex, axis, direction, sample point, exact points, normalized form, equation or contradiction and equation or impossibility.

8 PDFs and a README; 158 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Conic Sections from Equations and Data

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the two labeled rows to determine the vertex-form coefficient.

Required response

role	x	y
vertex	1	-2
curve point	3	6

Bank label: ______

Normalize the equation and determine whether any real point remains. domain: x, y real; equation: $x^2 + y^2 - 4x + 2y + 9 = 0$.

Bank label: ______

AP Precalculus

Conic Sections from Equations and Data

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

List the graph-defining features of the hyperbola. domain: x, y real; equation: $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

AP Precalculus

Conic Sections from Equations and Data

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Complete both squares and normalize the constant to 1. domain: x, y real; equation: $4x^2 + 9y^2 - 16x + 18y - 11 = 0$.

Your response: ______________

Incoming: $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$.

Use the labeled extreme-point table to construct the ellipse.

Required response

label	**x**	**y**
right	6	0
left	-6	0
top	0	2
bottom	0	-2

Your response: ______________

AP Precalculus

Conic Sections from Equations and Data

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Find the constant error in the work and repair the conic description. domain: x, y real; equation: $4x^2 - 16x + y^2 + 6y - 11 = 0$; student work: $4(x-2)^2 + (y+3)^2 = 20$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

A student claims: “The focus $(0, 2)$ gives $p = 1$, so the equation is $x^2 = 4y$ and $(4, 2)$ is not on the parabola.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Build the focal equation and test the proposed point. candidate point: $4, 2$; domain: vertical parabola; focus: $0, 2$; rule: $x^2 = 4py$; student claim: The focus $(0, 2)$ gives $p = 1$, so the equation is $x^2 = 4y$ and $(4, 2)$ is not on the parabola.; vertex: $0, 0$. Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

Conic Sections from Equations and Data

Full Worked Solutions - excerpt

Worked example

Problem. State the two defining geometric features. domain: x, y real; equation: $x^2 + y^2 = 81$.

Answer. Center $(0, 0)$, radius 9.

Explanation and check.

The equation is a sum of squared displacements equal to 9^2.

Worked example

Problem. Write the exact equation without converting the radius to a decimal. center: $-3, 1$; domain: circle; radius: $\sqrt{11}$.

Answer. $(x + 3)^2 + (y - 1)^2 = 11$.

Explanation and check.

Coordinate differences measure displacement from $(-3, 1)$, and the equation uses radius squared.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Parametric Conic Sections

Parametric Conic Sections | APPC-U04-P09

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for coverage and orientation, image, number of traversals, start visits, coverage, start, direction, branches and parameter pieces, parametric pair and parametric pair and domain.

8 PDFs and a README; 49 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Parametric Conic Sections

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Parametrize one counterclockwise traversal from the stated start. direction: counterclockwise; domain: $0 \leq t \leq 2\pi$; equation: $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1$; start: rightmost point.

Bank label: _______

Parametrize only the component above the center. desired: upper branch only; domain: $-\frac{\pi}{2} < t < \frac{\pi}{2}$; equation: $\frac{(y+3)^2}{25} - \frac{(x-2)^2}{4} = 1$.

Bank label: _______

Separate curve membership from traversal multiplicity. domain: $0 \leq t \leq 2\pi$; rules: x: $5\cos(2t)$; y: $2\sin(2t)$; target: $\frac{x^2}{25} + \frac{y^2}{4} = 1$.

Bank label: _______

Let the vertical coordinate be the free parameter. curve: $x = 3y - 5$; domain: y real.

Bank label: _______

The formulas are correct but reach only one branch on the student's domain. Repair only the domain so both branches are covered. desired: both complete branches; domain: student used $-\frac{\pi}{2} < t < \frac{\pi}{2}$; implicit curve: $\frac{x^2}{9} - \frac{y^2}{4} = 1$; rules: x: $3\sec t$; y: $2\tan t$.

Bank label: _______

Use a free horizontal parameter while retaining the branch endpoints. curve: $y = \sqrt{(9\text{-}x^2)}$; domain: $y \geq 0$; x interval: $-3 \leq x \leq 3$.

Bank label: _______

Use the supplied identity to parametrize the hyperbola. domain: $\cos t \neq 0$; equation: $x^2 - y^2 = 1$; identity: $\sec^2 t - \tan^2 t = 1$.

Bank label: _______

Trace the specified lower half exactly once. desired arc: left endpoint to right endpoint through bottom; domain: $\pi \leq t \leq 2\pi$; equation: $\frac{x^2}{36} + \frac{(y-1)^2}{4} = 1$.

Bank label: _______

AP Precalculus

Parametric Conic Sections

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Decide whether the pairs trace the same ellipse and relate their parameters. domain: $0 \le$parameter$\le 2\pi$; pair A: x: $3\cos t$; y: $2\sin t$; pair B: x: $3\cos u$; y: $-2\sin u$.	Trace only the component below the center. desired: lower branch; domain: $\frac{\pi}{2} < t < 3\frac{\pi}{2}$; equation: $\frac{(y+1)^2}{16} - \frac{(x-3)^2}{9} = 1$.
Determine whether the enlarged parameter interval changes the branch image. domain: -$\frac{\pi}{2}$ <parameter< $\frac{\pi}{2}$; pair A: x: $3\sec t$; y: $2\tan t$; pair B: domain: -$\pi < u < \pi$; x: $3\sec(\frac{u}{2})$; y: $2\tan(\frac{u}{2})$.	Parametrize only the specified quarter arc with both endpoints included. desired arc: from $(7,-2)$ to $(3,1)$ in quadrant relative to center; domain: $0 \le t \le \frac{\pi}{2}$; equation: $\frac{(x-3)^2}{16} + \frac{(y+2)^2}{9} = 1$.
Use y as the free coordinate and preserve its forbidden value. curve: $x = \frac{y+2}{y-1}$; domain: $y \neq 1$.	Describe the traversal produced by this nonstandard one-period interval. domain: $\frac{\pi}{2} \le t \le 5\frac{\pi}{2}$; rules: x: $2 + 6\cos t$; y: $1 + 3\sin t$; target: $\frac{(x-2)^2}{36} + \frac{(y-1)^2}{9} = 1$.

AP Precalculus

Parametric Conic Sections

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The first covers the whole parabola; the second covers only the upper half $y \geq 0$, with each nonvertex point reached by u and -u.

Test the proposed assignment against the signed equation. candidate: x: $3\tan t$; y: $4\sec t$; domain: $\cos t \neq 0$; implicit curve: $\frac{x^2}{9}$-$\frac{y^2}{16} = 1$.

Your response: ______________________

Incoming: The open arc with angles $\frac{\pi}{3} < t < 2\frac{\pi}{3}$ is omitted; both boundary points at angles $\frac{\pi}{3}$ and $2\frac{\pi}{3}$ are included.

Identify the relative quadrant and endpoint membership. domain: $\frac{\pi}{2} < t < \pi$; rules: x: $2 + 4\cos t$; y: $-1 + 3\sin t$; target: $\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1$.

Your response: ______________________

Incoming: x = $3 + 7\cos t, y = -2 - 4\sin t, 0 \leq t \leq \pi$.

Parametrize both branches across one trigonometric period. domain: $0 \leq t \leq 2\pi$ excluding poles; equation: $\frac{(x-1)^2}{9} - \frac{(y+4)^2}{4} = 1$.

Your response: ______________________

Incoming: x = $-1 + 2\cos t, y = 2 + 5\sin t, 0 \leq t \leq \frac{\pi}{2}$.

Give one rule that follows the specified right-bottom-left route without tracing the upper half. direction: clockwise; domain: half arc; end: $[-4, -2]$; implicit curve: $\frac{(x-3)^2}{49} + \frac{(y+2)^2}{16} = 1$; start: $[10, -2]$; via: $[3, -6]$.

Your response: ______________________

Incoming: x = $1 + 3\sec t, y = -4 + 2\tan t$; $0 \leq t \leq 2\pi$ with $t \neq \frac{\pi}{2}, 3\frac{\pi}{2}$.

Place secant on the coordinate with the positive normalized square. domain: $\cos t \neq 0$; equation: $\frac{y^2}{16}$-$\frac{x^2}{9} = 1$; identity: $\sec^2 t$-$\tan^2 t = 1$.

Your response: ______________________

Incoming: Invalid; it gives $\tan^2 t$-$\sec^2 t = -1$ instead of 1.

Use the horizontal coordinate while preserving the real-power branch. curve: $y = (x-1)^{\frac{3}{2}}$; domain: $x \geq 1, y \geq 0$.

Your response: ______________________

AP Precalculus

Parametric Conic Sections

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Change the minimum necessary component to fit the ellipse. domain: t real; student pair: $x:\ 5\cos t$; $y:\ 5\sin t$; target: $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Fix both the semiaxis error and the excessive trace domain. domain: $0\le t\le 2\pi$; student domain: $0\le t\le 2\pi$; student pair: $x:\ 1+4\cos t$; $y:\ -2+2\sin t$; target: $\frac{(x-1)^2}{25}+\frac{(y+2)^2}{4}=1$, upper half only.

Decide whether the proposed pair remains on the ellipse for every parameter. candidate: $x:\ 3\cos t$; $y:\ 3\sin t$; domain: t real; implicit curve: $\frac{x^2}{9}+\frac{y^2}{4}=1$.

The formulas lie on the ellipse, but the student's interval selects the wrong half. Change only the interval. desired: lower arc from left vertex to right vertex; domain: student used $0\le t\le \pi$; implicit curve: $\frac{(x-1)^2}{25}+\frac{(y-2)^2}{4}=1$; rules: $x:$ $1+5\cos t$; $y:\ 2+2\sin t$.

Parametric Conic Sections

Full Worked Solutions - excerpt

Worked example

Problem. Use a direct horizontal free parameter. curve: $y = -3x + 7$; domain: x real.

Answer. $\mathrm{x} = t, y = -3t + 7$, for all real t.

Explanation and check.

Each real input is represented once and its output follows the explicit rule.

Worked example

Problem. Factor the relation and use the coordinate that keeps the formula simplest. curve: $xy + y = 4$; domain: $y \neq 0$.

Answer. $\mathrm{x} = \frac{4}{t} - 1, y = t$, with $t \neq 0$.

Explanation and check.

The relation is $y(x + 1) = 4$, so choosing $y = t$ gives the stated reciprocal and excludes zero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Vector Components and Direction

Vector Components and Direction | APPC-U04-P10

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for classification with decisive evidence, vector, components, relations with components, equal, opposite, and non-equivalent relations and components and relation.

8 PDFs and a README; 146 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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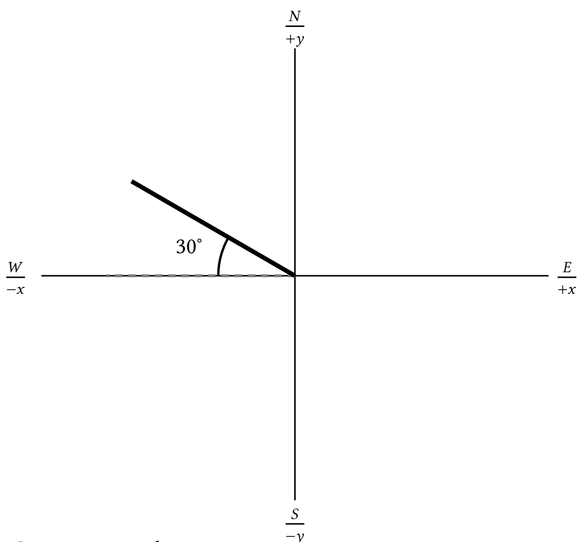
AP Precalculus

Vector Components and Direction

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Resolve the west-based heading into east and north components.  Given magnitude: 12 Bank label: ______	Find the directed component changes. Domain: Cartesian plane; Head: $(2, 1)$; Tail: $(5, 5)$ Bank label: ______

AP Precalculus

Vector Components and Direction

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Do the two directed segments represent the same free vector even though they start at different points?

Required response

Arrow	Tail (x, y)	Head (x, y)
A	$1, 2$	$5, 5$
B	$-3, 0$	$1, 3$

State the magnitude and standard-position direction of the vector.

Components: $(0, 7)$; Domain: Cartesian plane; angles in degrees when stated

AP Precalculus

Vector Components and Direction

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Find the vector directed from the stated tail to the stated head.

Domain: Cartesian plane; angles in degrees when stated; Head: $(7, 1)$; Tail: $(2, 3)$

Your response: ______________

AP Precalculus

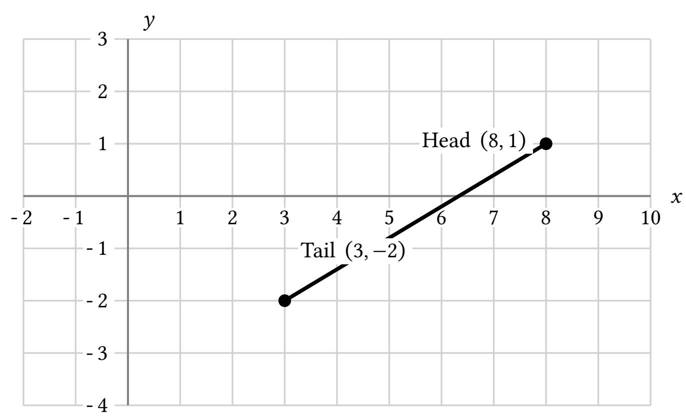
Vector Components and Direction

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Explain why the student's ordered pair is not the vector and give the correct components.

Domain: Cartesian plane; angles in degrees when stated; Head: $(8, 1)$; Student claim: The vector is $\langle 8, 1 \rangle$.; Tail: $(3, -2)$



Given arrow

Vector Components and Direction

Full Worked Solutions - excerpt

Worked example

Problem. Find the vector directed from the stated tail to the stated head.

Domain: Cartesian plane; angles in degrees when stated; Head: $(7, 1)$; Tail: $(2, 3)$

Answer. $\langle 5, -2 \rangle$

Explanation and check.

Subtracting tail coordinates from head coordinates gives horizontal change 5 and vertical change -2.

Worked example

Problem. Find the terminal point of the directed segment.

Domain: Cartesian plane; angles in degrees when stated; Tail: $(-2, 5)$; Vector: $(4, -3)$

Answer. The head is $(2, 2)$.

Explanation and check.

The head equals the tail plus the displacement, giving $(-2 + 4, 5 - 3) = (2, 2)$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Vector Operations and Magnitude

Vector Operations and Magnitude | APPC-U04-P11

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for components, components of u+v, resultant and endpoint, resultant and final head, squared norms and longer expression and longer derived vector and exact squared norms.

8 PDFs and a README; 97 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Vector Operations and Magnitude

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Combine the three displacements u, v, and w. Use the expression $u + v + w$. $u = \langle 1,\ 4 \rangle$ $v = \langle -3,\ 2 \rangle$ $w = \langle 5,\ -1 \rangle$ Bank label: ______	Multiply each component by the scalar. Use the expression $3u$. $u = \langle -2,\ 3 \rangle$ Bank label: ______
Compute the vector obtained by reversing u and multiplying its length by four. Use the expression $-4u$. $u = \langle -1,\ 2 \rangle$ Bank label: ______	Construct the unit vector preserving the direction. The given vector is $\langle -7,\ 24 \rangle$. Bank label: ______
Scale the direction to length 5 without rounding. The required magnitude is 5. Use the same direction as the given vector. The given vector is $\langle 12,\ -5 \rangle$. Bank label: ______	Construct the requested length without changing the downward direction. The required magnitude is 4. Use the same direction as the given vector. The given vector is $\langle 0,\ -9 \rangle$. Bank label: ______
Undo the subtraction to recover u. Use the equation $u - \langle 2, 7 \rangle = \langle -5, 1 \rangle$. Bank label: ______	Find the missing second summand. Use the equation $u + v =$ resultant. The resultant is $\langle 7,\ 2 \rangle$. $u = \langle -4,\ 9 \rangle$ Bank label: ______

AP Precalculus

Vector Operations and Magnitude

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Use the right triangle formed by the two vectors to find the resultant magnitude.

The included angle, in degrees, is 90.

The vector magnitudes are $|u| = 3$ and $|v| = 4$.

One permitted placement

Reflected permitted placement

The two panels show alternative placements of the same data.

No absolute coordinate orientation is specified.

AP Precalculus

Vector Operations and Magnitude

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Vector v points in the same direction as u and has magnitude $2\sqrt{13}$. Find v.

The magnitude of v is $2\sqrt{13}$.

The relation is v has the same direction as u.

$u = \langle 2, -3\rangle$

Your response: ______________

Incoming: $v = \langle 4, -6\rangle$.

Isolate and determine v from the given scaled difference.

The given linear combination equals $\langle 1, -14\rangle$.

Use the equation $3u - v = \langle 1, -14\rangle$.

$u = \langle 2, -3\rangle$

Your response: ______________

Incoming: $v = \langle 5, 5\rangle$.

Given $|u| = 5, |v| = 10$, and $u \cdot v = 20$, determine the smaller angle between the vectors.

The dot product $u \cdot v$ is 20.

The magnitude of u is 5.

The magnitude of v is 10.

Your response: ______________

Incoming: $\cos\theta = \frac{2}{5}$, so $\theta \approx 66.4^\circ$.

Scale the direction to length 9.

The required magnitude is 9.

Use the same direction as the given vector.

The given vector is $\langle -8, -15\rangle$.

Your response: ______________

Incoming: $\langle -\frac{72}{17}, -\frac{135}{17}\rangle$

Undo the vector subtraction and verify the recovered minuend.

Use the equation $u - v = \langle 4, -7\rangle$.

$v = \langle 2, 5\rangle$

Your response: ______________

Incoming: $u = \langle 6, -2\rangle$; subtracting v returns $\langle 4, -7\rangle$.

Isolate the unknown vector and check the original equation.

Use the equation $2u - v = w$.

$v = \langle 1, 5\rangle$

$w = \langle 7, -3\rangle$

Your response: ______________

AP Precalculus

Vector Operations and Magnitude

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Determine whether the scalar relationship gives angle 0° or 180°.

$u = \langle -2,\ 5\rangle$

$v = \langle -6,\ 15\rangle$

Vector Operations and Magnitude

Full Worked Solutions - excerpt

Worked example

Problem. Solve the two vector equations simultaneously for u and v.

Use these conditions: $u + v = \langle 1, 9 \rangle,\ 2u - v = \langle 8, 3 \rangle$.

Answer. u = $\langle 3, 4 \rangle$ and $v = \langle -2, 5 \rangle$.

Explanation and check.

Adding the equations eliminates v and gives $3u = \langle 9, 12 \rangle$; substitution into the sum then determines v.

Worked example

Problem. Construct the unit vector preserving the direction.

The given vector is $\langle -7,\ 24 \rangle$.

Answer. $\langle -\frac{7}{25}, \frac{24}{25} \rangle$

Explanation and check.

The component pair has norm 25, so divide both entries by 25.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Velocity Vector Applications

Velocity Vector Applications | APPC-U04-P12

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for elapsed time and endpoint, required component and impossibility proof, final coordinates, exact coordinates, displacement and exact path length and net displacement and exact distance.

8 PDFs and a README; 123 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Velocity Vector Applications

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An aircraft flies due east at $120\frac{km}{h}$ relative to the air while $a50\frac{km}{h}$ wind blows due north. Find its ground velocity, speed, and course.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Air velocity ($\frac{km}{h}$)	$\langle 120, 0\rangle$
Coordinate convention	east-north
Given context	aircraft motion
Wind velocity ($\frac{km}{h}$)	$\langle 0, 50\rangle$

Bank label: ______

A boat covers displacement $\langle 24, 8\rangle$ km in 2 hours while its water-relative velocity is $\langle 10, 1\rangle$ $\frac{km}{h}$. Assuming a constant current, find it.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Coordinate convention	east-north
Boat velocity relative water ($\frac{km}{h}$)	$\langle 10, 1\rangle$
Given context	boat and current
Duration (hours)	2
Observed displacement (km)	$\langle 24, 8\rangle$

Bank label: ______

AP Precalculus

Velocity Vector Applications

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A plane flies at airspeed $200\frac{km}{h}$ on heading $N30^\circ E$ while the wind is $\langle -40, 20\rangle\ \frac{km}{h}$. Find the ground velocity and approximate ground course.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Airspeed ($\frac{km}{h}$)	200
Coordinate convention	east-north
Given context	aircraft navigation
Heading	$N30^\circ E$
Wind ($\frac{km}{h}$)	$\langle -40, 20\rangle$

A person walks at $\langle 1, 2\rangle\ \frac{m}{s}$ relative to a walkway moving at $\langle 3, 0\rangle\ \frac{m}{s}$ relative to the floor. Describe the person's ground motion.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Coordinate convention	forward-left
Given context	person on moving walkway
Person velocity relative walkway ($\frac{m}{s}$)	$\langle 1, 2\rangle$
Walkway velocity relative ground ($\frac{m}{s}$)	$\langle 3, 0\rangle$

Infer the current independently from each observation row and decide whether the constant-current model is supported.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

interval	duration h	displacement km
1	2	$\langle 12, 6\rangle$
2	0.5	$\langle 3, 1.5\rangle$

Condition or evidence	Supplied value
Coordinate convention	east-north
Boat velocity relative water ($\frac{km}{h}$)	$\langle 8, 0\rangle$
Given context	repeated boat observations

Convert every duration to seconds before using the velocity table. Then find displacement and distance.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

stage	duration	velocity
1	$30s$	$\langle 2, 0\rangle$
2	1 min	$\langle 0, 1\rangle$
3	0.5 min	$\langle -1, 0\rangle$

Condition or evidence	Supplied value
Given context	stages with mixed time units
Velocity unit	$\frac{m}{s}$

AP Precalculus

Velocity Vector Applications

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The stage displacements sum to $\langle 0, 4\rangle\ km$, whose magnitude is 4 km; the total distance is 20 km.

Find the current that changes water-relative velocity $\langle 10, 0\rangle\ \frac{m}{s}$ into ground velocity $\langle 6, 8\rangle\ \frac{m}{s}$.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Coordinate convention	east-north
Boat velocity relative water ($\frac{m}{s}$)	$\langle 10, 0\rangle$
Given context	boat velocity frames
Ground velocity ($\frac{m}{s}$)	$\langle 6, 8\rangle$

Your response: ______________

AP Precalculus

Velocity Vector Applications

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student adds the two perpendicular speeds as scalars. Explain the error and correct the ground-speed calculation.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	**Supplied value**
Coordinate convention	east-north
Given context	drone in wind
Drone air velocity ($\frac{m}{s}$)	$\langle 40, 0\rangle$
Student claim to examine	The ground speed is $40 + 30 = 70\frac{m}{s}$.
Wind ($\frac{m}{s}$)	$\langle 0, 30\rangle$

Infer the current from the two velocities and decide whether it can be due east as claimed.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	**Supplied value**
Coordinate convention	east-north
Boat velocity relative water ($\frac{m}{s}$)	$\langle 3, 4\rangle$
Claimed current direction to test	due east
Given context	boat observation with directional assumption
Ground velocity ($\frac{m}{s}$)	$\langle 8, 5\rangle$

Velocity Vector Applications

Full Worked Solutions - excerpt

Worked example

Problem. The velocity does not change between the two table rows. Find displacement and distance after all 5 minutes.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Duration (min)	Given velocity (m/min)
2	$\langle 3, 0\rangle$
3	$\langle 3, 0\rangle$

Condition or evidence	Supplied value
Given context	collinear stages
Units	**Time** min **Given velocity** m/min

Answer. The displacement is $\langle 15, 0\rangle$ m and the distance is 15 m.

Explanation and check.

The durations combine because the velocity is unchanged, so $5\langle 3, 0\rangle$ gives the entire trip.

Worked example

Problem. Find the one-vector displacement and the scalar path length for the three stages.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Duration (s)	Given velocity (m/s)
2	$\langle 4, 0\rangle$
1	$\langle 0, 3\rangle$
1	$\langle -4, 0\rangle$

Condition or evidence	Supplied value
Given context	three-sided path
Units	**Time** s **Given velocity** $\frac{m}{s}$

Answer. The net displacement is $\langle 4, 3\rangle$ m, and the distance is 15 m.

Explanation and check.

The displacements $\langle 8, 0\rangle$, $\langle 0, 3\rangle$, and $\langle -4, 0\rangle$ add to the net; their lengths add to 15.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Vector-Valued Functions: Operations and Motion

Vector-Valued Functions: Operations and Motion | APPC-U04-P13

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for direction intervals, axis event, and stop, basis form and vector value, i-j notation, ordered-pair rule, vector table and insufficiency statement and formal component sum and interpretation verdict.

8 PDFs and a README; 65 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the supplied velocity value to describe the motion at $t = 3s$. Use $v(3) = \langle -2, 0\rangle$. Velocity components are measured in meters per second.

Bank label: ______

Determine the discrete domain of $3u - 2w$ and evaluate the function everywhere it is defined. The table with inputs $0, 1, 2$ gives $u(t)$. The table with inputs $1, 2, 3$ gives $w(t)$.

Required response

0	$1, 0$
1	$2, 3$
2	$-1, 4$

1	$5, -2$
2	$0, 6$
3	$7, 1$

Bank label: ______

AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Describe the velocity direction on the interval and identify every stationary time. Use $v(t) = \langle t-2, (t-2)(t+1)\rangle \frac{m}{s}$. Use the time restriction $0 \le t \le 4$.	Combine the radical interval with the denominator exclusion, then evaluate at the left endpoint. Use $r(t) = \langle \sqrt{(9\text{-}t^2)}, \frac{1}{t-1}\rangle$.
Find when the object is at the origin and compare its distances from the origin at $t = 0$ and $t = 2$. Use $p(t) = \langle t-1, 2t-2\rangle$. Position components are measured in meters. Use the time restriction $0 \le t \le 2$.	Derive the speed function, then find the speeds at $t = -1$ and $t = 1$. Use $v(t) = \langle t+1, -(t+1)\rangle$. Use the times $t = -1, 1$.
Form the resultant velocity and describe its direction for every real t. Use $v_1(t) = \langle t, t^2\rangle$. Use $v_2(t) = \langle -t, 1\rangle$. The stated time domain is all real t. Velocity components are measured in meters per second.	Write $d(t)$ in angle-bracket notation and find $d(-1)$. Use $d(t) = -(t-1)i + (2-3t)j$. The specified input is $t = -1$.

AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

<table>
<tr>
<td>

START

Describe the motion implied by this supplied velocity function. Use $v(t) = \langle 0, 5\rangle$. The stated time domain is all t. Velocity components are measured in meters per second.

Your response: ____________

</td>
<td>

Incoming: The object always moves straight upward at $5\frac{m}{s}$, with no horizontal motion.

Use both the time restriction and radical condition to find the domain, then evaluate the boundary input. Use $r(t) = \langle\sqrt{(5\text{-}t)}, t + 2\rangle$. Use the time restriction $t \geq 0$.

Your response: ____________

</td>
</tr>
<tr>
<td>

Incoming: The domain is $[0, 5]$, and $r(5) = \langle 0, 7\rangle$.

Build $2u$-v componentwise and state every allowed real input. Use $u(t) = \langle\sqrt{t}, t\rangle, t \geq 0$. Use $v(t) = \langle t, \frac{1}{t-1}\rangle, t \neq 1$.

Your response: ____________

</td>
<td>

Incoming: $(2u\text{-}v)(t) = \langle 2\sqrt{t}\text{-}t, 2t - \frac{1}{t-1}\rangle$ on $[0, 1) \cup (1, \infty)$.

Express p in i-j notation and evaluate the same vector function at $t = \frac{\pi}{2}$. Use $p(t) = \langle\cos t, \sin t\rangle$. The specified input is $t = \frac{\pi}{2}$.

Your response: ____________

</td>
</tr>
<tr>
<td>

Incoming: $p(t) = (\cos t)i + (\sin t)j$, and $p(\frac{\pi}{2}) = j = \langle 0, 1\rangle$.

Find the speed function and determine its least value. Use $v(t) = \langle t + 1, t - 1\rangle$. The stated time domain is all real t. Velocity components are measured in meters per second.

Your response: ____________

</td>
<td>

Incoming: $s(t) = \sqrt{(t + 1)^2 + (t - 1)^2} = \sqrt{2t^2 + 2}$; its minimum is $\sqrt{2}\frac{m}{s}$ at $t = 0$.

Combine the supplied component functions into a single basis-vector rule. Use $x(t) = \sin t$. Use $y(t) = \cos t$.

Your response: ____________

</td>
</tr>
</table>

AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Explain what the position value actually tells and why the student's motion claim is unsupported. Use $p(4) = \langle -3, 4 \rangle$ meters. The student claims: The object is moving left and up at $t = 4$.

Vector-Valued Functions: Operations and Motion

Full Worked Solutions - excerpt

Worked example

Problem. Combine the supplied component functions into a single basis-vector rule. Use $x(t) = \sin t$. Use $y(t) = \cos t$.

Answer. $r(t) = (\sin t)i + (\cos t)j$.

Explanation and check.

Both scalar functions use the same input and become the two basis coefficients.

Worked example

Problem. Rewrite $q(t)$ in ordered-pair notation. Use $q(t) = (2 - t)i + (3t + 1)j$.

Answer. $q(t) = \langle 2 - t, 3t + 1 \rangle$.

Explanation and check.

The coefficient of i is the first component and the coefficient of j is the second.

Sets & Steps

Math Practice & Worked Solutions

AP Precalculus

Matrix Operations and Multiplication

Matrix Operations and Multiplication | APPC-U04-P14

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for constraint comparison and verdict, numeric and label judgments, constraint and uniqueness conclusion, compatibility verdict and repair, valid and invalid labels with reason and intermediate and final sizes.

8 PDFs and a README; 66 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Matrix Operations and Multiplication

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

State the dimensions of A and give a_{23}.

matrix =

4	0	7
−2	5	−1

Bank label: ______

Compute Ax and confirm the result's dimensions. A: see the labelled matrix A; domain: matrix-vector multiplication; x: $[4, 5]$.

A =

2	−1
0	3

Bank label: ______

Recover x and y from both product entries and explain why neither condition is redundant. B: see the labelled matrix B; domain: row vector times invertible-looking coefficient matrix; product: $[5, 1]$; row: $[x, y]$.

B =

1	2
1	−1

Bank label: ______

Use a row-column product to find the meal's total calories. calories per serving: $[80, 120, 50]$; domain: meal calorie total; foods: [fruit, grain, milk]; servings: $[2, 1, 3]$.

Bank label: ______

AP Precalculus

Matrix Operations and Multiplication

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compute AB and BA and identify the entry that disproves commutativity. A: see the labelled matrix A; B: see the labelled matrix B; domain: diagonal matrix and shear.

A =

2	0
0	1

B =

1	3
0	1

Determine all possible x and y. Explain why the two output entries do not give two independent constraints. B: see the labelled matrix B; domain: row vector product with repeated columns; product: $[5, 5]$; row: $[x, y]$.

B =

1	1
1	1

Compute AB and state its dimensions. A: see the labelled matrix A; B: see the labelled matrix B; domain: row by rectangular matrix.

A =

2	−1

B =

3	0	4
5	2	−2

Is uv defined, and what size matrix results? domain: column times row vectors; u size: 4×1; v size: 1×2.

AP Precalculus

Matrix Operations and Multiplication

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(AB)_{14}=9$.

Find x so that the indicated row-column product is 6. column: $[3, 1, 4]$; domain: one selected dot product; product entry: 6; row: $[2, x, -1]$.

Your response: ____________

Incoming: The product is $\begin{pmatrix} 8 & 9 & 4 \\ 6 & 13 & 8 \end{pmatrix}$. Rows are Site A and Site B, columns are wood, metal, and fabric, and the shared sum runs over basic and advanced kits.

Multiply the machine-hours matrix by the energy-rate column. Interpret both outputs and explain the unit cancellation. domain: shift energy use; kilowatt hours per hour: $[6, 2, 5]$; machine columns: [press, lathe, dryer]; shift rows: Evening: $[5, 0, 3]$; Morning: $[2, 4, 1]$.

Your response: ____________

Incoming: BA = CA=$\begin{pmatrix} 2 & 0 \\ 4 & 0 \end{pmatrix}$, but $B \neq C$. Right cancellation fails because multiplying by A erases the second column, exactly where B and C differ.

Find $(AB)_{23}$ by recognizing the selector row. A row 2: $[0, 1, 0]$; B column 3: $[-4, 7, 2]$; domain: standard basis row selection; target: $(AB)_2 3$.

Your response: ____________

Incoming: $AA^{\top}$ is 2×2 and $A^{\top}A$ is 3×3, so they cannot be equal as matrices.

State the dimensions of c and give c_{31}.

matrix =

-4
7
2

Your response: ____________

Incoming: The product is (0).

Multiply the unit-count matrix by the mass column and interpret the two outputs. domain: factory package mass; factory rows: East: $[5, 2]$; West: $[1, 6]$; mass per unit: $[3, 8]$; product columns: [box, crate].

Your response: ____________

Incoming: Morning uses 25 kWh and Evening uses 45 kWh. Each dot product sums machine-hours times kWh per machine-hour over the shared machine labels.

Find $(AB)_{12}$. A row 1: $[5, 0]$; B column 2: $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(AB)_1 2$.

Your response: ____________

AP Precalculus

Matrix Operations and Multiplication

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Compute both orders and use them to assess the scalar-style claim that a zero product forces one matrix factor to be zero.

A =

1	−1
1	−1

B =

1	1
1	1

The inner labels match and the numeric dimensions conform. Does quantity-in-kilograms times dollars-per-item yield a valid total cost? State a minimal unit repair.

Matrix Operations and Multiplication

Full Worked Solutions - excerpt

Worked example

Problem. Compute the row-by-column matrix product. column: $[7, -2, 5]$; domain: zero row times a column; row: $[0, 0, 0]$.

Answer. The product is (0).

Explanation and check.

All three row-column contributions are zero.

Worked example

Problem. Find $(AB)_{12}$. A row $1:$ $[5, 0]$; B column $2:$ $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(AB)_{1}2$.

Answer. $(AB)_{12} = -10$.

Explanation and check.

The dot product is $5(-2) + 0 \cdot 3$, so only the first contribution remains.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Determinants and Area

Determinants and Area | APPC-U04-P15

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for dot product, determinant, area, singular set and dependence explanation, singular set, determinant and conclusion, empty real solution set with reason and all parameter values and dependence.

8 PDFs and a README; 55 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Determinants and Area

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the lower-triangular structure to evaluate the determinant without an unnecessary cross-product calculation.

Required response

−3	0
4	2

Bank label: ______

Use the triangular structure to find the determinant of the matrix.

Required response

7	0
−2	−3

Bank label: ______

Evaluate the determinant.

Required response

7	0
0	1

Bank label: ______

Read the determinant from the triangular form.

Required response

1	5
0	3

Bank label: ______

Find the determinant of the coordinate-swap matrix.

Required response

0	1
1	0

Bank label: ______

Find the determinant after the row swap.

Apply this change: interchange row 1 and row 2. The original determinant is 13.

Bank label: ______

Without reconstructing the entries, find the determinant after the columns are interchanged.

Apply this change: interchange the two columns. The original determinant is −5.

Bank label: ______

Find the determinant after the stated row scaling.

Apply this change: multiply row 2 by $\frac{1}{5}$. The original determinant is 20.

Bank label: ______

AP Precalculus

Determinants and Area

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Find the signed and ordinary areas of the spanned parallelogram. The first column is $(0, 1)$. The second column is $(-3, 0)$.	Find the signed and ordinary areas of the column parallelogram. The first column is $(2, 0)$. The second column is $(0, 5)$.
Compute the signed and ordinary areas and classify the column order. The first column is $(-2, 0)$. The second column is $(0, -7)$.	The second column is a90-degree counterclockwise rotation of the first. Use that structure to determine the signed and ordinary areas of their parallelogram. The first column is $(3, -1)$. The second column is $(1, 3)$.
Find the determinant and ordinary area, then state the exact column relation. The first column is $(-1, -2)$. The second column is $(-3, -6)$.	Find the parallelogram's ordinary area and classify the ordered-column orientation. The first column is $(4, 1)$. The second column is $(1, 3)$. The vertices are $(0, 0)$, $(4, 1)$, $(1, 3)$, $(5, 4)$.

AP Precalculus

Determinants and Area

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The image area is 15 square units, and orientation is preserved.

Verify that the matrices have the same determinant, then show from their columns that they do not define the same map.

Required response

2	0
0	2

1	3
0	4

Your response: ______________

START

Find every real t for which the matrix is singular and explain what happens to its columns then.

Required response

t	2
3	t

Your response: ______________

Incoming: It is singular for every real t because the second column is always twice the first, and the determinant $2t - 2t$ is identically zero.

Verify the common determinant and give a column-based reason the maps differ.

Required response

2	0
0	1

1	1
-1	1

Your response: ______________

Incoming: The determinant is $2(\frac{3}{4}) - 1(-1) = \frac{5}{2}$. Therefore the original area is $\frac{35}{\frac{5}{2}} = 14$.

Find the image area and state whether orientation is preserved.

The determinant is $\frac{1}{4}$. The original area is 20.

Your response: ______________

Incoming: The determinant is $k^2 + 6$, which is positive for every real k. Therefore there are no real singular parameter values.

Find the square's original area and its image area.

The determinant is 2. The square has side length 3.

Your response: ______________

Incoming: The original area is 9 and the image area is 18 square units.

Find all real t that make the matrix singular and explain why the off-diagonal 1 does not change the condition.

Required response

t	1
0	t

Your response: ______________

AP Precalculus

Determinants and Area

Name: ________________ **Date:** ________________ **Class:** ________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Correct the area statement while preserving the sign's geometric information.

The determinant is -7. The student claims: The parallelogram area is -7 square units..

Repair the statement by identifying the actual column relationship and its area consequence.

Use the matrix $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$. The student claims: The determinant is zero, so both columns must be zero..

Repair the claim by stating what equality of determinants does guarantee and one fact showing the maps differ.

Use matrix A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix B = $\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$. The student claims: Both determinants are 4, so A and B are the same map..

Assess the student's claim using the displayed columns. Then replace it with a correct geometric statement.

The proposed first and second columns, respectively, are $(2, -3)$, $(-4, 6)$. The student claims: If $a2 \times 2$ determinant is zero, its two columns must be equal..

Determinants and Area

Full Worked Solutions - excerpt

Worked example

Problem. Give two distinct diagonal matrices with determinant 9 and verify them.

Answer. For example, $\begin{pmatrix} 1 & 0 \\ 0 & 9 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ both have determinant 9, but their diagonal scalings are different.

Explanation and check.

Choose two different factor pairs whose product is 9.

Worked example

Problem. Correct the area statement while preserving the sign's geometric information.

The determinant is -7. The student claims: The parallelogram area is -7 square units..

Answer. The ordinary area is 7 square units, and the negative determinant indicates reversed orientation.

Explanation and check.

Take an absolute value for area but not for the orientation classification.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Matrices and Linear Systems

Inverse Matrices and Linear Systems | APPC-U04-P16

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for values in example and valid general conclusion, conclusion and valid multiplication side, corrected table and verdict, valid and invalid cases, invertible and singular lists and decision and determinant evidence.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Inverse Matrices and Linear Systems

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Find the inverse by undoing the two independent coordinate scalings.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

2	0
0	5

Bank label: ______

Treat the two right-hand sides as columns of one table and use the same inverse once to solve both systems.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Coefficient matrix

2	1
1	1

Right-hand-side column 1

5
3

Right-hand-side column 2

1
−1

Supplied inverse

1	−1
−1	2

Bank label: ______

Use the bottom-right identity entry to find q.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

4	0
0	−2

$\frac{1}{4}$	0
0	q

Bank label: ______

Find the inverse of the three-factor expression ABA^{-1} and verify the result without commuting A and B.

Bank label: ______

AP Precalculus

Inverse Matrices and Linear Systems

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Use the supplied inverse rather than elimination, then substitute the result into both original equations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Coefficient matrix

3	1
1	1

Right-hand-side column

8
4

Supplied inverse

$\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{1}{2}$	$\frac{3}{2}$

Compute the exact inverse, preserving the sign of the determinant throughout the scalar division.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

1	3
2	4

Test the candidate and determine whether a row-column transposition caused the failure.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Proposed inverse

3	-2
-1	1

Given matrix

1	1
2	3

Find the inverse with a single common denominator and verify one diagonal product entry.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

4	3
1	2

AP Precalculus

Inverse Matrices and Linear Systems

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The determinant is $k^2 + 1$, which is positive for every real k. Every matrix in the family is invertible.

Square the matrix to determine whether using the original matrix as its own inverse is valid.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	0
2	-1

1	0
2	-1

Your response: ____________

START

Verify algebraically whether the proposed matrix is an inverse for every real k, rather than checking sample values.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	-1
1-k	k

k	1
$k - 1$	1

Your response: ____________

Incoming: The determinant is $8k - 8 = 8(k - 1)$. Only $k = 1$ is singular; $k = -2, -1, 0, 2, 3$ give invertible matrices.

Use the bottom-right identity constraint to find q.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	2
2	5

5	-2
-2	q

Your response: ____________

Incoming: The inverse is $[[\frac{1}{3}, -\frac{5}{6}], [0, \frac{1}{2}]]$.

Use A’s inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Your response: ____________

AP Precalculus

Inverse Matrices and Linear Systems

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “The proposed matrix cannot be an inverse because its off-diagonal entries are nonzero.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Multiply the matrix by the proposed inverse and decide whether the proposal satisfies the defining identity relation.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

1	-1
-1	2

2	1
1	1

Recompute the product instead of trusting the student’s table. Decide whether the error lies in the candidate or in the verification arithmetic.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

$\frac{1}{3}$	$-\frac{1}{3}$
$\frac{1}{3}$	$\frac{2}{3}$

2	1
-1	1

1	0
1	1

Inverse Matrices and Linear Systems

Full Worked Solutions - excerpt

Worked example

Problem. Find the inverse of the three-factor expression AB A^{-1} and verify the result without commuting A and B.

Answer. The inverse is A B^{-1} A^{-1}. Multiplication gives A B A^{-1} A B^{-1} A^{-1} =I after adjacent cancellations.

Explanation and check.

Reverse all three factors and invert each one; the inverse of A inverse is A itself.

Worked example

Problem. Use A's inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Answer. Multiplying both sides on the right by A^{-1} gives $B = C$.

Explanation and check.

The common factor is on the right, so its inverse must also be placed on the right to cancel it.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Matrices for Linear Transformations

Matrices for Linear Transformations | APPC-U04-P17

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for consistency, uniqueness, and two example matrices, determination and nonlinear compatible rule, determination and two compatible rules, classification and decisive test, classification and matrix and classification.

8 PDFs and a README; 60 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Construct the matrix and use it to map $(2, -1)$ as a check. $T(e_1) = (0,\ 2)$ $T(e_2) = (3,\ 0)$ Bank label: ______	Project the point with the displayed matrix. Use the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Vectors are column vectors. The given point is $(-3,\ 5)$. Bank label: ______
Determine the complete image of the unit disk under the diagonal projection, including exact segment endpoints. Use the matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Vectors are column vectors. Use the disk with center $(0,\ 0)$ and radius 1. Bank label: ______	Solve the two component systems to recover the matrix from these nonbasis correspondences. Assume T is linear. The input-output observations are $(1,\ 2) \mapsto (5,\ 1)$; $(2,\ -1) \mapsto (1,\ 8)$. Bank label: ______

AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the projection matrix by tracking both basis vectors onto the line $y = x$.

The map acts by orthogonally project onto the line $y = x$.

Normalize each scaled-axis correspondence to obtain the two basis images.

Assume T is linear.

The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(3,\ 6)$.

Classify the rule using the origin test and distinguish affine from linear behavior.

Use the rule $F(x, y) = (x + 2, y - 1)$.

Do the axis-only samples establish linearity of an unspecified plane map? Exhibit contrasting compatible rules.

The input-output observations are $(0,\ 0)\mapsto(0,\ 0)$; $(1,\ 0)\mapsto(1,\ 0)$; $(2,\ 0)\mapsto(2,\ 0)$.

Construct the matrix and explain geometrically why its image is one-dimensional.

$T(e_1) = (2,\ 1)$

$T(e_2) = (-4,\ -2)$

Map the side generators and determine the image area and orientation change.

Use the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Vectors are column vectors.

The two side vectors are $u = (1,\ 0)$ and $v = (0,\ 2)$.

AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Because $(1, 1) + (1, -1) = (2, 0)$, outputs require $(2 + k, 4) = (5, 4)$, so $k = 3$. Then $T(e1) = (\frac{5}{2}, 2), T(e2) = (-\frac{1}{2}, 2)$, and the matrix is $[[\frac{5}{2}, -\frac{1}{2}], [2, 2]]$.

Construct the exact reflection matrix using the double-angle form.

The map acts by reflect across the line through the origin making a30-degree angle with the x-axis.

Your response: ______________

Incoming: The image generators are $(2, 0)$ and $(1, 2)$. Their determinant has magnitude 4, so the image area is 4.

Use the two directed basis images to assemble the standard matrix.

$T(e_1) = (0,\ -2)$

$T(e_2) = (3,\ 0)$

Your response: ______________

Incoming: Under a linearity assumption they determine matrix $[[2, -1], [1, 3]]$. Without that assumption they do not prove the map is linear, because nonlinear rules can share the same two values.

Map both points and verify that their known scalar relation survives the transformation.

Use the matrix $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$. Vectors are column vectors.

The given points are $P = (2,\ 1)$; $Q = (-6,\ -3)$.

The relation is $Q = -3P$.

Your response: ______________

Incoming: Every member is linear, with matrix $[[a, b], [c, d]]$.

Normalize the two scaled basis correspondences and construct the standard matrix.

Assume T is linear.

The input-output observations are $(2,\ 0) \mapsto (6,\ -2)$; $(0,\ -3) \mapsto (-6,\ -9)$.

Your response: ______________

Incoming: The matrix is $[[0, 1], [-1, 0]]$.

Complete the image table for all three input rows.

Use the matrix $A = \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}$. Vectors are column vectors.

The given points are $(0,\ 1)$, $(2,\ 0)$, $(-1,\ -2)$.

Your response: ______________

Incoming: The first rows give matrix $[[1, 3], [2, 4]]$, which sends $(1, 2)$ to $(7, 10)$, not $(8, 10)$. No linear map fits all rows.

Recover $T(e2)$ from the image of $e1 + e2$ and construct the matrix.

Assume T is linear.

The input-output observations are $(1,\ 0) \mapsto (-1,\ 4)$; $(1,\ 1) \mapsto (2,\ 2)$.

Your response: ______________

AP Precalculus

Matrices for Linear Transformations

Name: ________________ **Date:** ________________ **Class:** ________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The student read the rows as basis images. Correct both images and explain the relevant matrix convention.

Use the matrix $A = \begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix}$. Vectors are column vectors.

Evaluate these student claims: $T(e1) = (2,\ 5)$; $T(e2) = (-1,\ 3)$.

A student claims: "$A90^\circ$ rotation produces a different region because every boundary point moves." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Describe the disk's image and verify the claim with one boundary point.

Use the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Vectors are column vectors.

Use the disk with center $(0,\ 0)$ and radius 2.

Evaluate this student claim: $A90^\circ$ rotation produces a different region because every boundary point moves..

Matrices for Linear Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Write the matrix for a half-turn about the origin.

The map acts by rotate 180 degrees about the origin.

Answer. The matrix is $[[-1, 0], [0, -1]]$.

Explanation and check.

Both basis vectors reverse direction under a half-turn.

Worked example

Problem. Classify the rule and give a matrix representation if one exists.

Use the rule $F(x, y) = (2x - y, 3y)$.

Answer. The rule is linear with matrix $[[2, -1], [0, 3]]$.

Explanation and check.

Each output coordinate is a constant linear combination of the inputs with no added constant or nonlinear term.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Composing Matrix Transformations

Composing Matrix Transformations | APPC-U04-P18

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for projection and product, second matrix and product, all valid ordered pairs and verification principle, all valid pairs, all sequences and exclusions and all valid sequences and exclusion.

8 PDFs and a README; 156 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Composing Matrix Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Compute both orders and decide whether these maps commute.

Use the reflection matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Use the scaling matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.

Bank label: ______

AP Precalculus

Composing Matrix Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the ordered composite and verify it stage by stage. First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The original input vector is $(1, 2)$.	Classify all allowed two-stage sequences that produce reflection across $y = x$. The allowed transformations are exactly one 90-degree counterclockwise rotation and one coordinate-axis reflection. The target matrix is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

AP Precalculus

Composing Matrix Transformations

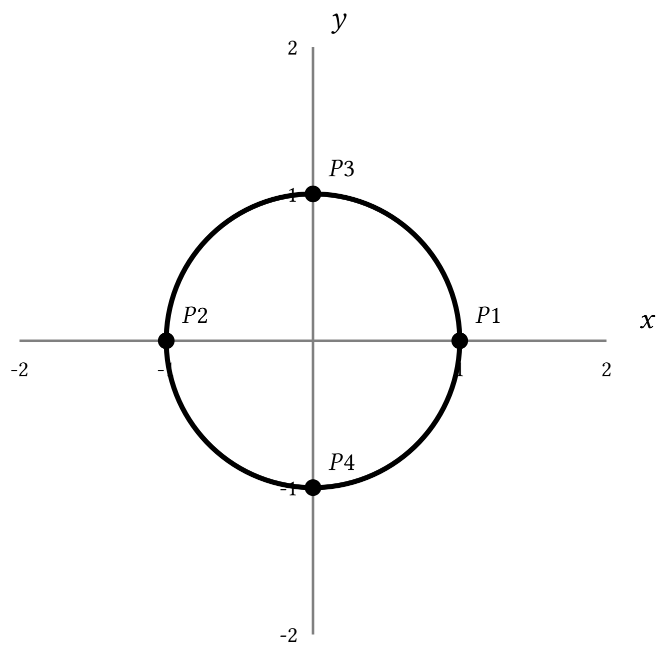
Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The first two stages form a half-turn; the final product is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, reflection across the y-axis.

Track the four key boundary points and state the major-axis direction after each stage.

The boundary points are $(1, 0)$, $(-1, 0)$, $(0, 1)$, $(0, -1)$. First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.



Your response: ______________________________

AP Precalculus

Composing Matrix Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student multiplies the two matrices in the written transformation order and claims the composite is the first matrix times the second. Identify the order error, construct the correct composite, and verify it on the test vector.

First apply $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Then apply $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. The original input vector is $(2, 1)$.

Composing Matrix Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare both orders and identify their common net action.

Use matrix A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix B = $\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}$.

Answer. Both products equal -I, so either order is a half-turn with no change in length.

Explanation and check.

Scalar matrices commute, and the scale factors multiply to -1.

Worked example

Problem. Complete the table and explain why the final column returns to the original data.

First apply $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Use the points A: $(1, 2)$; B: $(-2, 1)$; C: $(0, -3)$. Then apply $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

Answer. Intermediate: $A = (5, 2), B = (0, 1), C = (-6, -3)$. Final: $A = (1, 2), B = (-2, 1), C = (0, -3)$, the original points.

Explanation and check.

The second shear subtracts the same vertical-coordinate multiple that the first added.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Inverse Transformations and Preimages

Inverse Transformations and Preimages | APPC-U04-P19

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for inverse existence and two inverse images, conclusion and example matrices, qualified conclusion, existence and uniqueness qualifications, all preimages and all inputs and geometric set.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Inverse Transformations and Preimages

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Recover the input from the mixed-sign image and verify it with the forward map.

The image point is $(-2, 3)$.

Required response

Supplied inverse matrix

1	−1
0	1

Forward matrix

1	1
0	1

Bank label: ______

Identify the inverse map without recomputing an adjugate.

Required response

0	1
1	0

Bank label: ______

Apply the inverse turn to recover every labeled original vertex.

The image vertex A' is $(0, 1)$.

The image vertex B' is $(-2, 1)$.

The image vertex C' is $(-2, 3)$.

The image vertex D' is $(0, 3)$.

Required response

Forward matrix

0	−1
1	0

Bank label: ______

Find every input mapped to the target and describe the fiber.

The target point is $(3, -6)$.

Required response

Forward matrix

2	−1
−4	2

Bank label: ______

AP Precalculus

Inverse Transformations and Preimages

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recover the original rectangle and state its width and height.

Required response

2	0
0	$\frac{1}{2}$

-4	-1
4	-1
4	1
-4	1

Determine whether the target has any preimage.

The target point is $(4, 1)$.

Required response

Forward matrix

1	0
0	0

Decide separately whether the map is determined and whether it has an inverse.

Use $T(e_1) = (1, 2)$.

Use $T(e_2) = (1, 2)$.

Assume that T is linear.

Explain why these four checks are sufficient for mutual inverses.

Assume that F and G are linear.

The supplied checks are $G(F(e_1)) = e_1$; $G(F(e_2)) = e_2$; $F(G(e_1)) = e_1$; $F(G(e_2)) = e_2$.

AP Precalculus

Inverse Transformations and Preimages

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The preimage vertices are $(2, 1), (-1, 0), (0, -3)$.

Use the supplied inverse to reconstruct the labeled triangle.

Required response

Forward matrix

2	1
1	1

Image vertices (one coordinate pair per row)

0	0
2	1
2	2

Supplied inverse matrix

1	-1
-1	2

Your response: ____________________

Incoming: Map (X, Y) to $(X, Y + 2X)$, using [[1, 0], [2, 1]].

Find when the target lies on the image line and then parameterize its fiber.

The target point is $(k, 9)$.

Required response

Forward matrix

1	2
3	6

Your response: ____________________

Incoming: Reachability requires k = 3. Then $x + 2y = 3$, so the preimages are $(3 - 2t, t)$.

Compute both compositions on their proper domains and classify the relationship.

Let $F(x, y) = (x, 0)$ from R^2 to the x-axis.

Let $G(t, 0) = (t, 0)$ from the x-axis to R^2.

Your response: ____________________

Incoming: The preimage is $x^2 + y^2 \geq 1$ with $y > 0$. The circle is included, while the x-axis is excluded.

Prove the candidate is an inverse and explain exactly where the nonzero condition is needed.

Required response

1	a
0	b

1	$-\frac{a}{b}$
0	$\frac{1}{b}$

Your response: ____________________

AP Precalculus

Inverse Transformations and Preimages

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “A shear matrix cannot be invertible unless it is diagonal.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Verify the inverse-function claim on all vectors by checking both compositions.

Required response

1	1
0	1

1	−1
0	1

Repair the claim that one recovered input proves a global inverse function.

Use the map $F(x, y) = (x, 0)$.

The student proposes the preimage $(3, 0)$.

The target point is $(3, 0)$.

Inverse Transformations and Preimages

Full Worked Solutions - excerpt

Worked example

Problem. Prove the product-inverse rule by checking recovery in both directions and naming the undoing order.

Answer. Undo A first and B second. Composing this undoing map before or after the original product cancels adjacent inverse pairs and gives the identity in both directions.

Explanation and check.

A composite inverse reverses the original factor order so that each action meets its own undoing action.

Worked example

Problem. Prove that the inverse function of B is the original matrix function A.
Assume that $B = A^{-1}$.

Answer. AB = BA=I, so A satisfies both defining equations for B^{-1}. Therefore $(A^{-1})^{-1} = A$.

Explanation and check.

The two-sided inverse equations are symmetric in A and B.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Transition Matrices and Long-Term Behavior

Transition Matrices and Long-Term Behavior | APPC-U04-P20

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for matrix family and parameter interval, terminal states and decision, switch counts, terminal states, and decision, complete set, fixed states for t=0 and t>0 and all possible priors.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

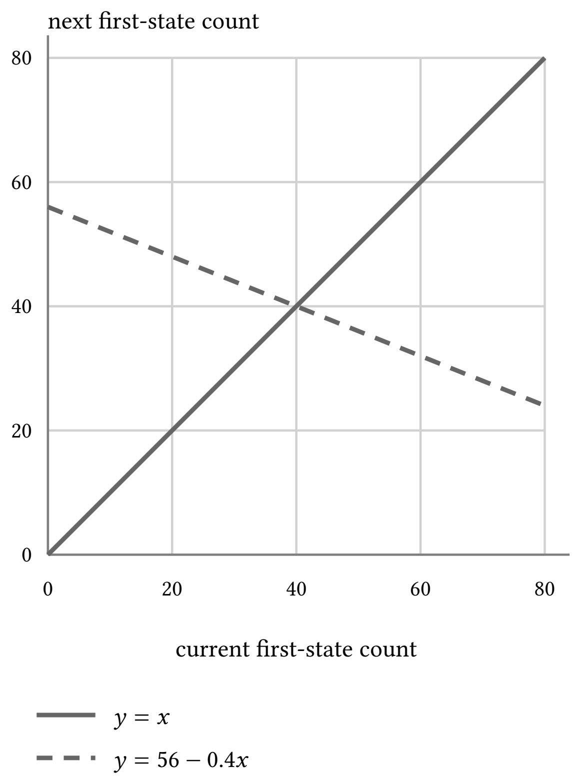
AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the graph equations to find the fixed distribution.



0.3	0.7
0.7	0.3

Bank label: ______

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Compare subscribed counts after two months and recommend the policy for that horizon. The components are ordered subscribed, then paused. The horizon in cycles is 2.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

100
0

Policy A

0.8	0.3
0.2	0.7

Policy B

0.6	0.5
0.4	0.5

Compute tomorrow's proportions and identify both contributions to the online state. The components are ordered online, then offline.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

0.7
0.3

Transition matrix

0.85	0.4
0.15	0.6

Classify all fixed distributions as t ranges from 0 through 0.5. The conserved total is 90. The allowed parameter interval is $0 \leq t \leq 0.5$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Transition matrix

1-t	2t
t	$1 - 2t$

Determine the two unknown first-row probabilities from the observations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Matrix pattern

p	q
1-p	1-q

Observation 1: initial state

50
50

Observation 1: later state

55
45

Observation 2: initial state

80
20

Observation 2: later state

70
30

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[80, 20]$

Convert the shares to counts and calculate the next state. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Initial proportions

0.25
0.75

Transition matrix

0.5	0.1
0.5	0.9

Your response: ______________

Incoming: $[28, 72]$

Find the equilibrium counts. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Transition matrix

0.9	0.3
0.1	0.7

Your response: ______________

Incoming: $[[0, 0], [1, 1]]$

Translate the conditional branches into a transition matrix. The components are ordered A, then B. The conditional A-to-B probability is 0.4; the B-to-A probability is 0.15.

Your response: ______________

Incoming: $[16, 64]$

Find the fixed distribution with total 120.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

0.7	0.3
0.3	0.7

Your response: ______________

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: "Because every displayed row rounds to $(50.0, 50.0)$, the state sequence converges to $(50, 50)$." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: A rounded table displays $(50.0, 50.0)$ repeatedly. Decide whether that proves convergence.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

49.96
50.04

0	1
1	0

Find the state after three periods using the supplied power, and explain why multiplying the one-period matrix by 3 would be invalid.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

40
60

0.8	0.3
0.2	0.7

0.65	0.525
0.35	0.475

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Full Worked Solutions - excerpt

Worked example

Problem. Translate the conditional branches into a transition matrix. The components are ordered A, then B. The conditional A-to-B probability is 0.4; the B-to-A probability is 0.15.

Answer. $[[0.6, 0.15], [0.4, 0.85]]$

Explanation and check.

Each tree root is a current-state source, so its two destination branches become one matrix column.

Worked example

Problem. Encode the deterministic switching rule. The components are ordered left, then right. The transition rule is: every member switches states each cycle.

Answer. $[[0, 1], [1, 0]]$

Explanation and check.

Each source sends probability one to the opposite destination and zero to itself.