Sets & Steps
Math Practice & Worked Solutions

Precalculus

Matrix Models, Inverses & Long-Term States | 3-Topic Precalculus Bundle

3-topic bundle | 288 problems | Four activity formats

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01 Matrix Operations and Multiplication

View individual product and its full preview

02 Inverse Matrices and Linear Systems

View individual product and its full preview

03 Transition Matrices and Long-Term Behavior

View individual product and its full preview

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

24 PDFs and 3 READMEs; 221 buyer PDF pages across the 3 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Matrix Operations and Multiplication

Matrix Operations and Multiplication | APPC-U04-P14

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for constraint comparison and verdict, numeric and label judgments, constraint and uniqueness conclusion, compatibility verdict and repair, valid and invalid labels with reason and intermediate and final sizes.

8 PDFs and a README; 66 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

State the dimensions of A and give a_{23}.

matrix =

4	0	7
−2	5	−1

Bank label: _____

Compute Ax and confirm the result's dimensions. A: see the labelled matrix A; domain: matrix-vector multiplication; x: $[4, 5]$.

A =

2	−1
0	3

Bank label: _____

Recover x and y from both product entries and explain why neither condition is redundant. B: see the labelled matrix B; domain: row vector times invertible-looking coefficient matrix; product: $[5, 1]$; row: $[x, y]$.

B =

1	2
1	−1

Bank label: _____

Use a row-column product to find the meal's total calories. calories per serving: $[80, 120, 50]$; domain: meal calorie total; foods: [fruit, grain, milk]; servings: $[2, 1, 3]$.

Bank label: _____

AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compute AB and BA and identify the entry that disproves commutativity. A: see the labelled matrix A; *B*: see the labelled matrix *B*; domain: diagonal matrix and shear.

A =

2	0
0	1

B =

1	3
0	1

Determine all possible x and y. Explain why the two output entries do not give two independent constraints. *B*: see the labelled matrix *B*; domain: row vector product with repeated columns; product: $[5, 5]$; row: $[x, y]$.

B =

1	1
1	1

Compute AB and state its dimensions. A: see the labelled matrix A; *B*: see the labelled matrix *B*; domain: row by rectangular matrix.

A =

2	-1

B =

3	0	4
5	2	-2

Is uv defined, and what size matrix results? domain: column times row vectors; u size: 4×1; v size: 1×2.

AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(AB)_{14}$=9.

Find x so that the indicated row-column product is 6. column: $[3, 1, 4]$; domain: one selected dot product; product entry: 6; row: $[2, x, -1]$.

Your response: _____

Incoming: The product is $\begin{pmatrix} 8 & 9 & 4 \\ 6 & 13 & 8 \end{pmatrix}$. Rows are Site A and Site B, columns are wood, metal, and fabric, and the shared sum runs over basic and advanced kits.

Multiply the machine-hours matrix by the energy-rate column. Interpret both outputs and explain the unit cancellation. domain: shift energy use; kilowatt hours per hour: $[6, 2, 5]$; machine columns: [press, lathe, dryer]; shift rows: Evening: $[5, 0, 3]$; Morning: $[2, 4, 1]$.

Your response: _____

Incoming: BA = CA=$\begin{pmatrix} 2 & 0 \\ 4 & 0 \end{pmatrix}$, but $B \neq C$. Right cancellation fails because multiplying by A erases the second column, exactly where B and C differ.

Find $(AB)_{23}$ by recognizing the selector row. A row 2: $[0, 1, 0]$; B column 3: $[-4, 7, 2]$; domain: standard basis row selection; target: $(AB)_{2}3$.

Your response: _____

Incoming: AA^{T} is 2×2 and $A^{\mathsf{T}}A$ is 3×3, so they cannot be equal as matrices.

State the dimensions of c and give c_{31}.

matrix =

-4
7
2

Your response: _____

Incoming: The product is $\begin{pmatrix} 0 \end{pmatrix}$.

Multiply the unit-count matrix by the mass column and interpret the two outputs. domain: factory package mass; factory rows: East: $[5, 2]$; West: $[1, 6]$; mass per unit: $[3, 8]$; product columns: [box, crate].

Your response: _____

Incoming: Morning uses 25 kWh and Evening uses 45 kWh. Each dot product sums machine-hours times kWh per machine-hour over the shared machine labels.

Find $(AB)_{12}$. A row 1: $[5, 0]$; B column 2: $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(AB)_{1}2$.

Your response: _____

AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Compute both orders and use them to assess the scalar-style claim that a zero product forces one matrix factor to be zero.

A =

1	−1
1	−1

B =

1	1
1	1

The inner labels match and the numeric dimensions conform. Does quantity-in-kilograms times dollars-per-item yield a valid total cost? State a minimal unit repair.

Matrix Operations and Multiplication

Full Worked Solutions - excerpt

Worked example

Problem. Compute the row-by-column matrix product. column: $[7, -2, 5]$; domain: zero row times a column; row: $[0, 0, 0]$.

Answer. The product is $\begin{pmatrix} 0 \end{pmatrix}$.

Explanation and check.

All three row-column contributions are zero.

Worked example

Problem. Find $(\mathrm{AB})_{12}$. A row 1: $[5, 0]$; B column 2: $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(\mathrm{AB})_{12}$.

Answer. $(\mathrm{AB})_{12} = -10$.

Explanation and check.

The dot product is $5(-2) + 0 \cdot 3$, so only the first contribution remains.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Matrices and Linear Systems

Inverse Matrices and Linear Systems | APPC-U04-P16

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for values in example and valid general conclusion, conclusion and valid multiplication side, corrected table and verdict, valid and invalid cases, invertible and singular lists and decision and determinant evidence.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Find the inverse by undoing the two independent coordinate scalings.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

2	0
0	5

Bank label: _____

Treat the two right-hand sides as columns of one table and use the same inverse once to solve both systems.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Coefficient matrix

2	1
1	1

Right-hand-side column 1

5
3

Right-hand-side column 2

1
−1

Supplied inverse

1	−1
−1	2

Bank label: _____

Use the bottom-right identity entry to find q.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

4	0
0	−2

$\frac{1}{4}$	0
0	q

Bank label: _____

Find the inverse of the three-factor expression $AB{A}^{-1}$ and verify the result without commuting A and B.

Bank label: _____

AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Use the supplied inverse rather than elimination, then substitute the result into both original equations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Coefficient matrix

3	1
1	1

Right-hand-side column

8
4

Supplied inverse

$\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{1}{2}$	$\frac{3}{2}$

Compute the exact inverse, preserving the sign of the determinant throughout the scalar division.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

1	3
2	4

Test the candidate and determine whether a row-column transposition caused the failure.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Proposed inverse

3	−2
−1	1

Given matrix

1	1
2	3

Find the inverse with a single common denominator and verify one diagonal product entry.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

4	3
1	2

AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The determinant is $k^2 + 1$, which is positive for every real k. Every matrix in the family is invertible.

Square the matrix to determine whether using the original matrix as its own inverse is valid.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	0
2	-1

1	0
2	-1

Your response: _____

START

Verify algebraically whether the proposed matrix is an inverse for every real k, rather than checking sample values.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	-1
1-k	k

k	1
$k-1$	1

Your response: _____

Incoming: The determinant is $8k - 8 = 8(k - 1)$. Only $k = 1$ is singular; $k = -2, -1, 0, 2, 3$ give invertible matrices.

Use the bottom-right identity constraint to find q.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	2
2	5

5	-2
-2	q

Your response: _____

Incoming: The inverse is $[[\frac{1}{3}, -\frac{5}{6}], [0, \frac{1}{2}]]$.

Use A's inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Your response: _____

AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “The proposed matrix cannot be an inverse because its off-diagonal entries are nonzero.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Multiply the matrix by the proposed inverse and decide whether the proposal satisfies the defining identity relation.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

1	-1
-1	2

2	1
1	1

Recompute the product instead of trusting the student’s table. Decide whether the error lies in the candidate or in the verification arithmetic.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

$\frac{1}{3}$	$-\frac{1}{3}$
$\frac{1}{3}$	$\frac{2}{3}$

2	1
-1	1

1	0
1	1

Inverse Matrices and Linear Systems

Full Worked Solutions - excerpt

Worked example

Problem. Find the inverse of the three-factor expression AB A^{-1} and verify the result without commuting A and B.

Answer. The inverse is A B^{-1} A^{-1}. Multiplication gives A B A^{-1} A B^{-1} A^{-1} =I after adjacent cancellations.

Explanation and check.

Reverse all three factors and invert each one; the inverse of A inverse is A itself.

Worked example

Problem. Use A's inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Answer. Multiplying both sides on the right by A^{-1} gives $B = C$.

Explanation and check.

The common factor is on the right, so its inverse must also be placed on the right to cancel it.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Transition Matrices and Long-Term Behavior

Transition Matrices and Long-Term Behavior | APPC-U04-P20

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for matrix family and parameter interval, terminal states and decision, switch counts, terminal states, and decision, complete set, fixed states for t=0 and t>0 and all possible priors.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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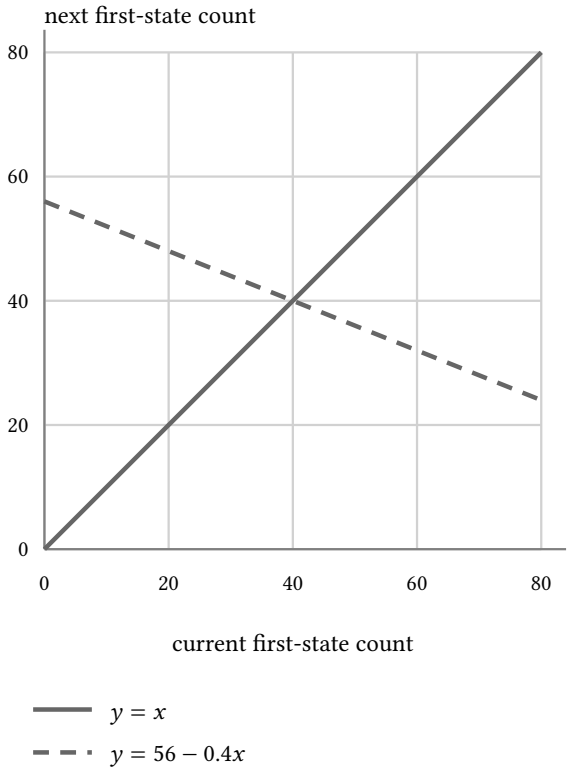
AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the graph equations to find the fixed distribution.



0.3	0.7
0.7	0.3

Bank label: _____

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Compare subscribed counts after two months and recommend the policy for that horizon. The components are ordered subscribed, then paused. The horizon in cycles is 2.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

100
0

Policy A

0.8	0.3
0.2	0.7

Policy B

0.6	0.5
0.4	0.5

Compute tomorrow's proportions and identify both contributions to the online state. The components are ordered online, then offline.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

0.7
0.3

Transition matrix

0.85	0.4
0.15	0.6

Classify all fixed distributions as t ranges from 0 through 0.5. The conserved total is 90. The allowed parameter interval is $0 \le t \le 0.5$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Transition matrix

1-t	$2t$
t	$1 - 2t$

Determine the two unknown first-row probabilities from the observations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Matrix pattern

p	q
1-p	1-q

Observation 1: initial state

50
50

Observation 1: later state

55
45

Observation 2: initial state

80
20

Observation 2: later state

70
30

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[80, 20]$

Convert the shares to counts and calculate the next state. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Initial proportions

0.25
0.75

Transition matrix

0.5	0.1
0.5	0.9

Your response: _____

Incoming: $[28, 72]$

Find the equilibrium counts. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Transition matrix

0.9	0.3
0.1	0.7

Your response: _____

Incoming: $[[0, 0], [1, 1]]$

Translate the conditional branches into a transition matrix. The components are ordered A, then *B*. The conditional A-to-*B* probability is 0.4; the *B*-to-A probability is 0.15.

Your response: _____

Incoming: $[16, 64]$

Find the fixed distribution with total 120.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

0.7	0.3
0.3	0.7

Your response: _____

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: "Because every displayed row rounds to $(50.0, 50.0)$, the state sequence converges to $(50, 50)$." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: A rounded table displays $(50.0, 50.0)$ repeatedly. Decide whether that proves convergence.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

49.96
50.04

0	1
1	0

Find the state after three periods using the supplied power, and explain why multiplying the one-period matrix by 3 would be invalid.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

40
60

0.8	0.3
0.2	0.7

0.65	0.525
0.35	0.475

Transition Matrices and Long-Term Behavior

Full Worked Solutions - excerpt

Worked example

Problem. Translate the conditional branches into a transition matrix. The components are ordered A, then B. The conditional A-to-B probability is 0.4; the B-to-A probability is 0.15.

Answer. $[[0.6, 0.15], [0.4, 0.85]]$

Explanation and check.

Each tree root is a current-state source, so its two destination branches become one matrix column.

Worked example

Problem. Encode the deterministic switching rule. The components are ordered left, then right. The transition rule is: every member switches states each cycle.

Answer. $[[0, 1], [1, 0]]$

Explanation and check.

Each source sends probability one to the opposite destination and zero to itself.