Sets & Steps
Math Practice & Worked Solutions

Precalculus

Matrix Transformations, Composition & Preimages | 3-Topic Precalculus Bundle

3-topic bundle | 288 problems | Four activity formats

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01 Matrices for Linear Transformations

View individual product and its full preview

02 Composing Matrix Transformations

View individual product and its full preview

03 Inverse Transformations and Preimages

View individual product and its full preview

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

24 PDFs and 3 READMEs; 286 buyer PDF pages across the 3 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Matrices for Linear Transformations

Matrices for Linear Transformations | APPC-U04-P17

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for consistency, uniqueness, and two example matrices, determination and nonlinear compatible rule, determination and two compatible rules, classification and decisive test, classification and matrix and classification.

8 PDFs and a README; 60 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Construct the matrix and use it to map $(2, -1)$ as a check.

$T(e_1) = (0,\ 2)$

$T(e_2) = (3,\ 0)$

Bank label: ______

Project the point with the displayed matrix.

Use the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Vectors are column vectors.

The given point is $(-3,\ 5)$.

Bank label: ______

Determine the complete image of the unit disk under the diagonal projection, including exact segment endpoints.

Use the matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Vectors are column vectors.

Use the disk with center $(0,\ 0)$ and radius 1.

Bank label: ______

Solve the two component systems to recover the matrix from these nonbasis correspondences.

Assume T is linear.

The input-output observations are $(1,\ 2) \mapsto (5,\ 1)$; $(2,\ -1) \mapsto (1,\ 8)$.

Bank label: ______

AP Precalculus

Matrices for Linear Transformations

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the projection matrix by tracking both basis vectors onto the line $y = x$.

The map acts by orthogonally project onto the line $y = x$.

Normalize each scaled-axis correspondence to obtain the two basis images.

Assume T is linear.

The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(3,\ 6)$.

Classify the rule using the origin test and distinguish affine from linear behavior.

Use the rule $F(x, y) = (x + 2, y - 1)$.

Do the axis-only samples establish linearity of an unspecified plane map? Exhibit contrasting compatible rules.

The input-output observations are $(0,\ 0)\mapsto(0,\ 0)$; $(1,\ 0)\mapsto(1,\ 0)$; $(2,\ 0)\mapsto(2,\ 0)$.

Construct the matrix and explain geometrically why its image is one-dimensional.

$T(e_1) = (2,\ 1)$

$T(e_2) = (-4,\ -2)$

Map the side generators and determine the image area and orientation change.

Use the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Vectors are column vectors.

The two side vectors are $u = (1,\ 0)$ and $v = (0,\ 2)$.

AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Because $(1,1)+(1,-1)=(2,0)$, outputs require $(2+k,4)=(5,4)$, so $k=3$. Then $T(e1)=(\frac{5}{2},2), T(e2)=(-\frac{1}{2},2)$, and the matrix is $[[\frac{5}{2},-\frac{1}{2}],[2,2]]$.

Construct the exact reflection matrix using the double-angle form.

The map acts by reflect across the line through the origin making a30-degree angle with the x-axis.

Your response: ______________

Incoming: The image generators are $(2,0)$ and $(1,2)$. Their determinant has magnitude 4, so the image area is 4.

Use the two directed basis images to assemble the standard matrix.

$T(e_1) = (0,\ -2)$

$T(e_2) = (3,\ 0)$

Your response: ______________

Incoming: Under a linearity assumption they determine matrix $[[2,-1],[1,3]]$. Without that assumption they do not prove the map is linear, because nonlinear rules can share the same two values.

Map both points and verify that their known scalar relation survives the transformation.

Use the matrix $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$. Vectors are column vectors.

The given points are $P = (2,\ 1)$; $Q = (-6,\ -3)$.

The relation is $Q = -3P$.

Your response: ______________

Incoming: Every member is linear, with matrix $[[a,b],[c,d]]$.

Normalize the two scaled basis correspondences and construct the standard matrix.

Assume T is linear.

The input-output observations are $(2,\ 0) \mapsto (6,\ -2)$; $(0,\ -3) \mapsto (-6,\ -9)$.

Your response: ______________

Incoming: The matrix is $[[0,1],[-1,0]]$.

Complete the image table for all three input rows.

Use the matrix $A = \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}$. Vectors are column vectors.

The given points are $(0,\ 1)$, $(2,\ 0)$, $(-1,\ -2)$.

Your response: ______________

Incoming: The first rows give matrix $[[1,3],[2,4]]$, which sends $(1,2)$ to $(7,10)$, not $(8,10)$. No linear map fits all rows.

Recover $T(e2)$ from the image of $e1+e2$ and construct the matrix.

Assume T is linear.

The input-output observations are $(1,\ 0) \mapsto (-1,\ 4)$; $(1,\ 1) \mapsto (2,\ 2)$.

Your response: ______________

AP Precalculus

Matrices for Linear Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The student read the rows as basis images. Correct both images and explain the relevant matrix convention.

Use the matrix $A = \begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix}$. Vectors are column vectors.

Evaluate these student claims: $T(e1) = (2,\, 5)$; $T(e2) = (-1,\, 3)$.

A student claims: "$A90^\circ$ rotation produces a different region because every boundary point moves." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Describe the disk's image and verify the claim with one boundary point.

Use the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Vectors are column vectors.

Use the disk with center $(0,\, 0)$ and radius 2.

Evaluate this student claim: $A90^\circ$ rotation produces a different region because every boundary point moves..

Matrices for Linear Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Write the matrix for a half-turn about the origin.

The map acts by rotate 180 degrees about the origin.

Answer. The matrix is $[[-1, 0], [0, -1]]$.

Explanation and check.

Both basis vectors reverse direction under a half-turn.

Worked example

Problem. Classify the rule and give a matrix representation if one exists.

Use the rule $F(x, y) = (2x - y, 3y)$.

Answer. The rule is linear with matrix $[[2, -1], [0, 3]]$.

Explanation and check.

Each output coordinate is a constant linear combination of the inputs with no added constant or nonlinear term.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Composing Matrix Transformations

Composing Matrix Transformations | APPC-U04-P18

96 problems · Four activity formats

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06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for projection and product, second matrix and product, all valid ordered pairs and verification principle, all valid pairs, all sequences and exclusions and all valid sequences and exclusion.

8 PDFs and a README; 156 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Composing Matrix Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Compute both orders and decide whether these maps commute.

Use the reflection matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Use the scaling matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.

Bank label: ______

AP Precalculus

Composing Matrix Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the ordered composite and verify it stage by stage. First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The original input vector is $(1, 2)$.	Classify all allowed two-stage sequences that produce reflection across $y = x$. The allowed transformations are exactly one 90-degree counterclockwise rotation and one coordinate-axis reflection. The target matrix is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

AP Precalculus

Composing Matrix Transformations

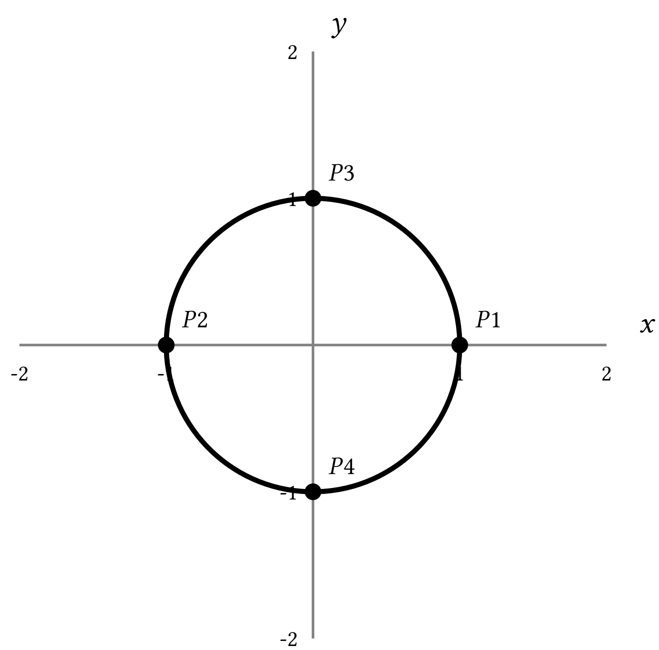
Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The first two stages form a half-turn; the final product is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, reflection across the y-axis.

Track the four key boundary points and state the major-axis direction after each stage.

The boundary points are $(1,0)$, $(-1,0)$, $(0,1)$, $(0,-1)$. First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.



Your response: ____________

AP Precalculus

Composing Matrix Transformations

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student multiplies the two matrices in the written transformation order and claims the composite is the first matrix times the second. Identify the order error, construct the correct composite, and verify it on the test vector.

First apply $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Then apply $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. The original input vector is $(2, 1)$.

Composing Matrix Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare both orders and identify their common net action.

Use matrix A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix B = $\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}$.

Answer. Both products equal -I, so either order is a half-turn with no change in length.

Explanation and check.

Scalar matrices commute, and the scale factors multiply to -1.

Worked example

Problem. Complete the table and explain why the final column returns to the original data.

First apply $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Use the points A: $(1, 2)$; B: $(-2, 1)$; C: $(0, -3)$. Then apply $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

Answer. Intermediate: $A = (5, 2), B = (0, 1), C = (-6, -3)$. Final: $A = (1, 2), B = (-2, 1), C = (0, -3)$, the original points.

Explanation and check.

The second shear subtracts the same vertical-coordinate multiple that the first added.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Transformations and Preimages

Inverse Transformations and Preimages | APPC-U04-P19

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
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Topic focus

Across 96 tasks this topic asks for inverse existence and two inverse images, conclusion and example matrices, qualified conclusion, existence and uniqueness qualifications, all preimages and all inputs and geometric set.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Inverse Transformations and Preimages

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Recover the input from the mixed-sign image and verify it with the forward map.

The image point is $(-2, 3)$.

Required response

Supplied inverse matrix

1	−1
0	1

Forward matrix

1	1
0	1

Bank label: ______

Identify the inverse map without recomputing an adjugate.

Required response

0	1
1	0

Bank label: ______

Apply the inverse turn to recover every labeled original vertex.

The image vertex A' is $(0, 1)$.

The image vertex B' is $(-2, 1)$.

The image vertex C' is $(-2, 3)$.

The image vertex D' is $(0, 3)$.

Required response

Forward matrix

0	−1
1	0

Bank label: ______

Find every input mapped to the target and describe the fiber.

The target point is $(3, -6)$.

Required response

Forward matrix

2	−1
−4	2

Bank label: ______

AP Precalculus

Inverse Transformations and Preimages

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recover the original rectangle and state its width and height.

Required response

2	0
0	$\frac{1}{2}$

-4	-1
4	-1
4	1
-4	1

Determine whether the target has any preimage.

The target point is $(4, 1)$.

Required response

Forward matrix

1	0
0	0

Decide separately whether the map is determined and whether it has an inverse.

Use $T(e_1) = (1, 2)$.

Use $T(e_2) = (1, 2)$.

Assume that T is linear.

Explain why these four checks are sufficient for mutual inverses.

Assume that F and G are linear.

The supplied checks are $G(F(e_1)) = e_1$; $G(F(e_2)) = e_2$; $F(G(e_1)) = e_1$; $F(G(e_2)) = e_2$.

AP Precalculus

Inverse Transformations and Preimages

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The preimage vertices are $(2, 1), (-1, 0), (0, -3)$.

Use the supplied inverse to reconstruct the labeled triangle.

Required response

Forward matrix

2	1
1	1

Image vertices (one coordinate pair per row)

0	0
2	1
2	2

Supplied inverse matrix

1	-1
-1	2

Your response: ____________________

Incoming: Map (X, Y) to $(X, Y + 2X)$, using $[[1, 0], [2, 1]]$.

Find when the target lies on the image line and then parameterize its fiber.

The target point is $(k, 9)$.

Required response

Forward matrix

1	2
3	6

Your response: ____________________

Incoming: Reachability requires k = 3. Then $x + 2y = 3$, so the preimages are $(3 - 2t, t)$.

Compute both compositions on their proper domains and classify the relationship.

Let $F(x, y) = (x, 0)$ from R^2 to the x-axis.

Let $G(t, 0) = (t, 0)$ from the x-axis to R^2.

Your response: ____________________

Incoming: The preimage is $x^2 + y^2 \geq 1$ with $y > 0$. The circle is included, while the x-axis is excluded.

Prove the candidate is an inverse and explain exactly where the nonzero condition is needed.

Required response

1	a
0	b

1	$-\frac{a}{b}$
0	$\frac{1}{b}$

Your response: ____________________

AP Precalculus

Inverse Transformations and Preimages

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “A shear matrix cannot be invertible unless it is diagonal.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Verify the inverse-function claim on all vectors by checking both compositions.

Required response

1	1
0	1

1	-1
0	1

Repair the claim that one recovered input proves a global inverse function.

Use the map $F(x, y) = (x, 0)$.

The student proposes the preimage $(3, 0)$.

The target point is $(3, 0)$.

Inverse Transformations and Preimages

Full Worked Solutions - excerpt

Worked example

Problem. Prove the product-inverse rule by checking recovery in both directions and naming the undoing order.

Answer. Undo A first and B second. Composing this undoing map before or after the original product cancels adjacent inverse pairs and gives the identity in both directions.

Explanation and check.

A composite inverse reverses the original factor order so that each action meets its own undoing action.

Worked example

Problem. Prove that the inverse function of B is the original matrix function A.
Assume that $B = A^{-1}$.

Answer. AB = BA=I, so A satisfies both defining equations for B^{-1}. Therefore $(A^{-1})^{-1} = A$.

Explanation and check.

The two-sided inverse equations are symmetric in A and B.