Sets & Steps
Math Practice & Worked Solutions

Precalculus

Polar Radius Graphs & Contextual Models | 3-Topic Precalculus Bundle

3-topic bundle | 288 problems | Four activity formats

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01 Polar Rates of Change and Radial Motion

View individual product and its full preview

02 Radius Graphs vs Polar Traces

View individual product and its full preview

03 Polar Models in Context

View individual product and its full preview

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

24 PDFs and 3 READMEs; 331 buyer PDF pages across the 3 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polar Rates of Change and Radial Motion

Polar Rates of Change and Radial Motion | APPC-U03-P28

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for supported or undetermined with alternative, estimated radius, exact signed rate and interpretation, local closest or farthest, local feature and inward and outward intervals.

8 PDFs and a README; 108 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polar Rates of Change and Radial Motion

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

The average signed-radius rate from $\theta = 0$ to $\theta = 2$ is 3 units per radian, and $r(0) = -1$. Recover $r(2)$. Bank label: _____	The average radial rate over $2 \le \theta \le 4$ is -3, and $r(4) = 1$. Use the secant model to estimate $r(3)$. Bank label: _____
The average radial rate from $\theta = \frac{1}{2}$ to $\theta = \theta_2$ is -4. If $r(\frac{1}{2}) = 6$ and $r(\theta_2) = 2$, find θ_2. Bank label: _____	The average signed-radius rate from 30° to 90° is 2 units per radian, and $r(30^\circ) = 1$. Find $r(90^\circ)$. Bank label: _____

AP Precalculus

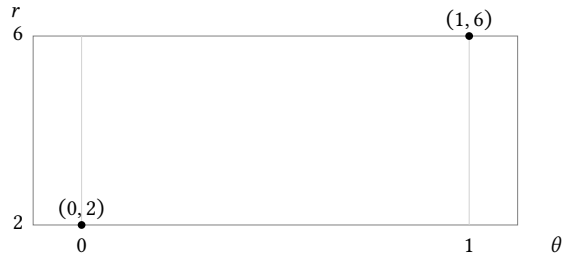
Polar Rates of Change and Radial Motion

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Throughout an angle interval, r is negative and increases from -7 toward -2. Is the point moving inward or outward relative to the pole?

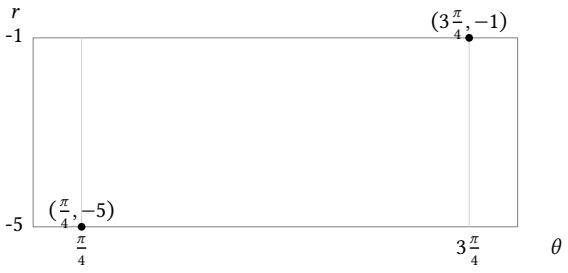
Using the average rate between the two table rows, estimate r at $\theta = \frac{1}{2}$. Is the result necessarily exact?



0	2
1	6

Near $\theta = a$, a radius graph stays positive and has a local minimum $r = 2$. What distance feature does the polar point have at a?

Compute the average signed-radius rate from $\theta = \frac{\pi}{4}$, where $r = -5$, to $\theta = 3\frac{\pi}{4}$, where $r = -1$.



$\frac{\pi}{4}$	-5
$3\frac{\pi}{4}$	-1

AP Precalculus

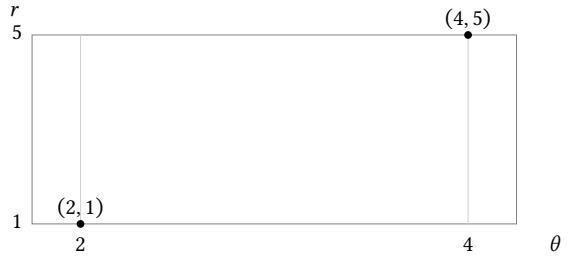
Polar Rates of Change and Radial Motion

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $r(90^\circ) \approx 4$.

A table gives $r(2) = 1$ and $r(4) = 5$. Use the average rate to estimate $r(3)$, and say whether the estimate must be exact.



2	1
4	5

Your response: _____

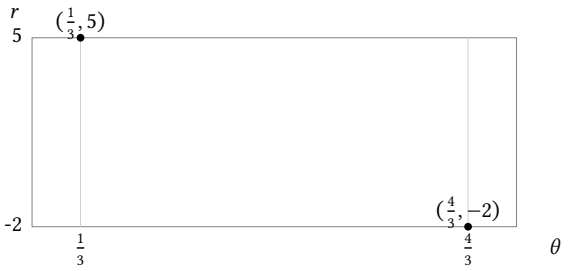
Incoming: Signed-radius rate 2; distance rate $\frac{4}{3}$ radial units per radian.

For $r = 2^\theta$, find the average radial rate from $\theta = 0$ to $\theta = 1$.

Your response: _____

Incoming: Both rates are -3 radial units per radian.

Find the average radial rate from $r(\frac{1}{3}) = 5$ to $r(\frac{4}{3}) = -2$.

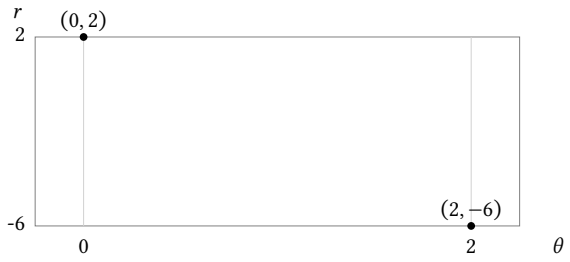


$\frac{1}{3}$	5
$\frac{4}{3}$	-2

Your response: _____

Incoming: The estimate is 10 and the actual radius lies in $[9.5, 10.5]$.

A diagram labels $r = 2$ at $\theta = 0$ and $r = -6$ at $\theta = 2$. Find the average radial and distance rates.



0	2
2	-6

Your response: _____

AP Precalculus

Polar Rates of Change and Radial Motion

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student computes $\frac{5-2}{90-30} = \frac{1}{20}$ and labels it radial units per radian. Repair the interpretation.

Required response

30°	2
90°	5

Polar Rates of Change and Radial Motion

Full Worked Solutions - excerpt

Worked example

Problem. A signed radius rises from 2 to 12 at average rate 5. What is the angular interval width?

Answer. $\theta_2 - \theta_1 = 2$ radians.

Explanation and check.

The radius change is 10. Dividing change by rate gives a width of 2 radians.

Worked example

Problem. For $r = 2^{\theta}$, find the average radial rate from $\theta = 0$ to $\theta = 1$.

Answer. 1 radial unit per radian.

Explanation and check.

The radius changes from 1 to 2 over one radian, so the secant quotient is 1.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Radius Graphs vs Polar Traces

Radius Graphs vs Polar Traces | APPC-U03-P29

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for graph sets, qualification and least point repeat, change descriptions, opposite trends, qualification about least period and two graph descriptions.

8 PDFs and a README; 167 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Radius Graphs vs Polar Traces

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

For $r = \sin(3\theta)$, determine the scalar period and the least positive shift that repeats the same parameterized polar point for every generic θ. Explain why they differ.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given parameter range or context	all real θ
Given half-turn relation	$r(\theta + \pi) = -r(\theta)$
Original rule	$r = \sin(3\theta)$

Bank label: _____

A table gives $r = 1, 2, 1$ at $\theta = 0, \frac{\pi}{2}, \pi$. State the plotted samples in the scalar and polar views.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

θ	Given r
0	1
$\frac{\pi}{2}$	2
π	1

Condition or evidence	Supplied value
Given parameter range or context	three sampled angles

Bank label: _____

AP Precalculus

Radius Graphs vs Polar Traces

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A complete scalar radius graph has highest value 5 and lowest value −7. Repair the claim that its top point must correspond to the farthest polar point.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given parameter range or context	complete scalar graph
Claim to examine	The top of the scalar graph is always the farthest polar point.
Marked values	−7 5

AP Precalculus

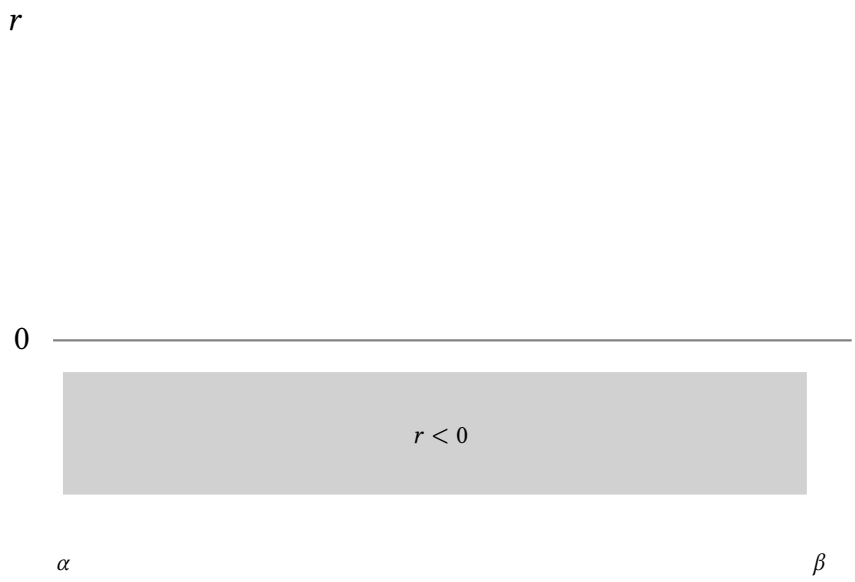
Radius Graphs vs Polar Traces

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The scalar value decreases, but the distance increases from 1 to 3.

A scalar $\theta - r$ graph is negative only on $\alpha < \theta < \beta$ and is zero at both endpoints. Translate this entire feature into a statement about the polar trace.



Condition or evidence	Supplied value
Boundary values	$r(\alpha) = r(\beta) = 0$
Condition	$r(\theta) < 0$ for $\alpha < \theta < \beta$
Given parameter range or context	interval (α, β)

Your response: _____

AP Precalculus

Radius Graphs vs Polar Traces

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student says every negative value on $a\theta - r$ graph becomes a polar point below the x-axis. Use $r = -2$ at $\theta = 3\frac{\pi}{2}$ to repair the claim.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Given parameter range or context	$\theta = 3\frac{\pi}{2}$
Polar pair	-2 $3\frac{\pi}{2}$
Student claim to examine	negative r means the plotted point has $y < 0$

A student claims: “Because the scalar radius repeats after π, the same polar point also repeats after π.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: The scalar rule $\cos(2\theta)$ has period π. Does shifting θ by π repeat the same polar point?

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Given parameter range or context	all real θ
Original rule	$r = \cos(2\theta)$
Given scalar period	π
Student claim to examine	Because the scalar radius repeats after π, the same polar point also repeats after π.

Radius Graphs vs Polar Traces

Full Worked Solutions - excerpt

Worked example

Problem. On an interval, r rises from -6 to -1. Compare the scalar graph's change with the polar point's distance change.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given parameter range or context	an interval with $r < 0$
Scalar behavior	r increases from -6 to -1

Answer. The scalar graph increases, while the polar point moves inward because its distance decreases from 6 to 1.

Explanation and check.

Scalar order uses signed outputs; polar distance uses absolute radius. Those measures move in opposite directions in this negative regime.

Worked example

Problem. For constant radius $r = -4$ while θ varies, compare scalar output change, polar distance change, and polar position change.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given parameter range or context	θ varies over an interval
Radius	constant -4

Answer. Scalar output and distance are both constant, but the polar position changes around the circle on opposite rays.

Explanation and check.

Neither r nor $|r|$ changes. The angle still changes the direction, so the point is not stationary in the plane.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polar Models in Context

Polar Models in Context | APPC-U03-P30

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for feasibility decision, two model choices and noninterchangeability, supported model, admissibility decision, feasibility and repair and decision.

8 PDFs and a README; 56 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Polar Models in Context

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

A rotating sensor obeys $r = 2 + \cos\theta$ and $\theta = \pi t$, with t in seconds. Write its radius as a function of time.

Bank label: _____

Held-out radii are 4.1 and 2.2 at $\theta = \frac{\pi}{3}$ and $2\frac{\pi}{3}$. Model A predicts 4.0 and 2.0; Model B predicts 3.6 and 2.7. With tolerance 0.25 at each angle, which model is supported?

Required response

$\frac{\pi}{3}$	4.1
$2\frac{\pi}{3}$	2.2

Bank label: _____

Engineers measure the maximum audible distance from a siren separately for each compass bearing. Choose a model that directly supports drawing the warning boundary.

Bank label: _____

A radius graph has maximum 9 at $\theta = \pi$ and minimum 3 at $\theta = 0$. Select the compatible model from $r = 6 - 3\cos\theta$, $r = 6 + 3\cos\theta$, and $r = 3 + 6\sin\theta$.

Bank label: _____

AP Precalculus

Polar Models in Context

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A boundary circle centered at $(3, 0)$ with radius 3 satisfies $r^2 = 6r\cos\theta$. Find its nonzero radial model and the angles where it gives a forward boundary point.	An accepted model $r = 5 - \sin\theta$ is explicitly valid only for $0 \le \theta \le \pi$. A report requests the modeled location at $\theta = 7\frac{\pi}{6}$. What should be reported?
Models A and B have equal held-out RMSE 0.18. In two safety-critical directions A's residuals are 0.08 and 0.11, while B's are 0.32 and 0.29; tolerance there is 0.15. Which model is validated for the safety use?	A signed radial signal is $r = 5\cos\theta$, and $\theta = \pi\frac{t-2}{4}$ so that the polar axis is reached at $t = 2s$. Express r as a function of t.
A radius graph has maxima at 0 and π and minima at $\frac{\pi}{2}$ and $3\frac{\pi}{2}$. Which candidate fits: $r = 5 + 2\cos\theta, r = 5 + 2\cos(2\theta)$, or $r = 5 + 2\sin(2\theta)$?	Radar gives the storm edge's distance from a station for every bearing at one instant. Choose a representation for drawing the current edge, then decide whether the same data can predict when the edge reaches a town.

AP Precalculus

Polar Models in Context

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The matching model is $r = 4 - 2\sin\theta$.

A scanner starts at angle $\frac{\pi}{6}$ and turns counterclockwise at 3 revolutions per minute. For $r = 4 + \cos\theta$ and t in minutes, write $\theta(t)$ and $r(t)$.

Your response: _____

Incoming: $\theta(t) = \frac{\pi}{6} + 6\pi t$ and $r(t) = 4 + \cos(\frac{\pi}{6} + 6\pi t)$.

A scanner follows $r = 3 + 2\sin\theta$ and turns according to $\theta = \pi\frac{t}{6}$. Write $r(t)$ and find the radius at $t = 3$ seconds.

Your response: _____

Incoming: $r(\theta) = \min(\frac{4}{|\cos\theta|}, \frac{3}{|\sin\theta|})$, with the usual convention that division by zero is ignored. A corner is hit when $\frac{4}{|\cos\theta|} = \frac{3}{|\sin\theta|}$, or $|\tan\theta| = \frac{3}{4}$.

Radii at $0, \frac{\pi}{2}, \pi, 3\frac{\pi}{2}$ are $3, 5, 7, 5$. Which candidate fits: $r = 5 - 2\cos\theta, r = 5 + 2\cos\theta$, or $r = 5 - 2\sin\theta$?

Required response

0	3
$\frac{\pi}{2}$	5
π	7
$3\frac{\pi}{2}$	5

Your response: _____

Incoming: $r(t) = 7 + \sin(\pi\frac{t-1}{4})$, and the scan covers $0 \le \theta \le \pi$.

For the signed rule $r = 1 + 3\cos\theta$, evaluate at $\theta = 4\frac{\pi}{3}$ and report the signed radius, distance from the origin, and Cartesian point.

Your response: _____

AP Precalculus

Polar Models in Context

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A device claims that $d(\theta) = 2\cos\theta$ is its nonnegative distance to an obstacle for every $0 \le \theta \le 2\pi$. Is the model physically admissible as stated?

An unlabeled polar loop is used to claim that a vehicle moved more slowly where the curve bends sharply. Diagnose the claim.

A student claims: “Model B has the smaller mean absolute error because it exactly matches the last observation more closely.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Held-out radii are $5.0, 4.2, 3.1, 4.1$. Models A and B predict $(4.9, 4.0, 3.2, 4.3)$ and $(5.4, 4.1, 2.7, 4.0)$. Which has the smaller mean absolute radial error?

A scanner can measure at most 10 m, but a proposed all-bearing model is $r = 7 + 4\cos\theta$. Is the model within the hardware range?

Polar Models in Context

Full Worked Solutions - excerpt

Worked example

Problem. A rotating sensor obeys $r = 2 + \cos\theta$ and $\theta = \pi t$, with t in seconds. Write its radius as a function of time.

Answer. $r(t) = 2 + \cos(\pi t)$

Explanation and check.

Substitute the supplied angle law $\theta = \pi t$ into the radial rule.

Worked example

Problem. A circular safety boundary has model $r = 6$. What boundary point corresponds to $\theta = \pi$?

Answer. $(-6, 0)$

Explanation and check.

At angle π, cosine is -1 and sine is 0, so $(r\cos\theta, r\sin\theta) = (-6, 0)$.