

Precalculus  
**Tangent Values, Graphs & Transformations | 3-Topic Precalculus Bundle**

3-topic bundle | 288 problems | Four activity formats

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- 01    Tangent Values and Domain**  
View individual product and its full preview
- 02    Tangent Parent Function Graphs**  
View individual product and its full preview
- 03    Tangent Function Transformations**  
View individual product and its full preview
- 

**In every topic**

|                                             |          |
|---------------------------------------------|----------|
| Answer Bank Challenge                       | 24 tasks |
| Grid Rounds                                 | 24 tasks |
| Match & Move                                | 24 cards |
| Error Analysis & Strategy                   | 24 tasks |
| Quick Answer Key and Full Worked Solutions  | included |
| Teacher / Parent Use Guide and Terms of Use | included |

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**Product previews**

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

**24 PDFs and 3 READMEs; 437 buyer PDF pages across the 3 products.**

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

AP Precalculus  
Tangent Values and Domain

Tangent Values and Domain | APPC-U03-P05

96 problems · Four activity formats

|    |                            |              |
|----|----------------------------|--------------|
| 01 | Answer Bank Challenge      | 24 tasks     |
| 02 | Grid Rounds                | 24 tasks     |
| 03 | Match & Move               | 24 cards     |
| 04 | Error Analysis & Strategy  | 24 tasks     |
| 05 | Quick Answer Key           | 96 answers   |
| 06 | Full Worked Solutions      | 96 solutions |
| 07 | Teacher / Parent Use Guide | included     |
| 08 | Terms of Use & License     | included     |

Topic focus

Across 96 tasks this topic asks for classification and decisive evidence, exact slope and sign evidence, classification and earliest error, classification and evidence, exact ordering and earliest error and repair.

8 PDFs and a README; 146 buyer pages.

## AP Precalculus

Tangent Values  
and Domain

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Give exact answers and all requested reasoning; record a bank letter. If bank entries give equivalent final responses for the same quantity and context, either label is valid and may be reused. A shared numerical value alone does not make entries interchangeable.

Order these exact scalar values from least to greatest, grouping ties and using no decimals:  $\tan(2\frac{\pi}{3})$ ,  $\sin(3\frac{\pi}{2})$ ,  $\cos(5\frac{\pi}{6})$ .

Solve the problem and show enough exact work for the requested result.

$$\tan(2\frac{\pi}{3})$$

$$\sin(3\frac{\pi}{2})$$

$$\cos(5\frac{\pi}{6})$$

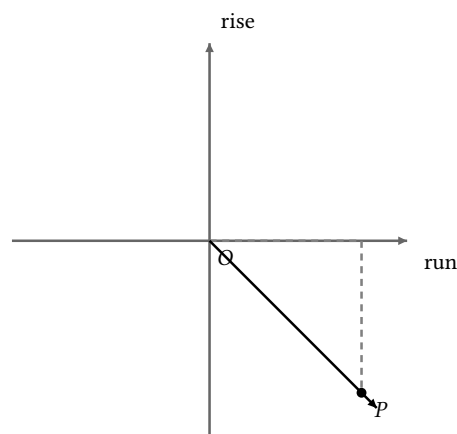
Bank label: \_\_\_\_\_

Given  $\tan(\frac{\pi}{3}) = \sqrt{3}$ , use “negate angle” to infer  $\tan(-\frac{\pi}{3})$ . Explain whether one or both coordinate signs change.

Bank label: \_\_\_\_\_

A terminal ray passes through  $(-4, -4\sqrt{3})$ . Evaluate its exact tangent from rise/run and explain why radius normalization is unnecessary.

A marked ray has signed run 12 and rise  $-12$ . Calculate its exact tangent and explain which geometric datum controls the sign.



Bank label: \_\_\_\_\_

Bank label: \_\_\_\_\_

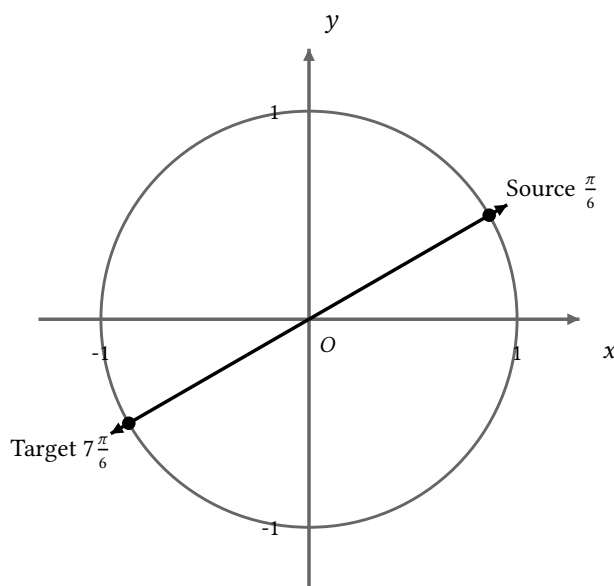
## AP Precalculus

Tangent Values  
and Domain

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Given  $\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$ , use “add  $\pi$ ” to infer  $\tan\left(7\frac{\pi}{6}\right)$ . Explain whether one or both coordinate signs change.



## AP Precalculus

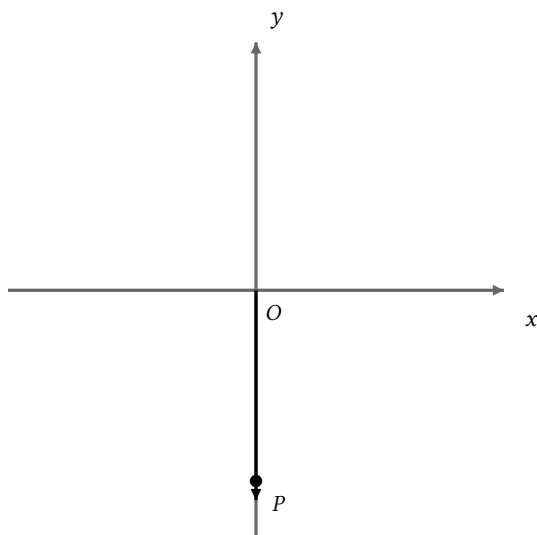
Tangent Values  
and Domain

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

**Incoming:**  $\tan(-5\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$

A ray is represented by the exact point  $(0, -3)$ . Classify its tangent as zero, nonzero, or undefined and cite numerator and denominator roles.



zero

nonzero

undefined

Your response: \_\_\_\_\_

## AP Precalculus

Tangent Values  
and Domain

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the claim “ $\tan(\pi)$  is undefined because sine is zero.” Classify tangent correctly and identify the first quotient or domain error.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

|             |
|-------------|
| denominator |
| permission  |
| sign        |

A student claims: “Because the  $x$ -coordinate contains a radical, only a decimal estimate is possible.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Evidence case: an exact point  $(2\sqrt{3}, 2)$ . Decide whether it determines an exact tangent, only an estimate, or no domain conclusion. State the decisive evidence and any exact datum still needed.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

| Value status | $x$         | $y$ |
|--------------|-------------|-----|
| exact        | $2\sqrt{3}$ | 2   |

## Tangent Values and Domain

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** At  $\theta = \pi$ ,  $(\sin \theta, \cos \theta) = (0, -1)$ . Evaluate  $\tan \theta$  as a signed coordinate quotient and simplify exactly.

**Answer.**  $\tan(\pi) = 0$

#### Explanation and check.

Cosine  $-1$  is the denominator. The quotient  $\frac{0}{-1}$  simplifies to  $0$ .

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#### Worked example

**Problem.** Given  $\tan(3\frac{\pi}{4}) = -1$ , use “add  $\pi$ ” to infer  $\tan(7\frac{\pi}{4})$ . Explain whether one or both coordinate signs change.

**Answer.**  $\tan(7\frac{\pi}{4}) = -1$

#### Explanation and check.

Adding  $\pi$  takes the opposite ray, so both  $x$  and  $y$  change sign. The quotient  $\frac{y}{x}$  is unchanged, giving  $\tan(7\frac{\pi}{4}) = -1$ .

AP Precalculus  
Tangent Parent Function Graphs

Tangent Parent Function Graphs | APPC-U03-P14

96 problems · Four activity formats

|    |                            |              |
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Topic focus

Across 96 tasks this topic asks for signed unbounded behavior without an assigned value, method/conclusion choice, justification, and endpoint-aware output interval, tangent entries and branch conclusion, one disconnected increasing branch, domain, range, and monotonicity and endpoint-aware output interval.

8 PDFs and a README; 140 buyer pages.

## AP Precalculus

Tangent Parent  
Function Graphs

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Select the candidate that can represent  $y = \tan(x)$ . Defend the choice with structural evidence and explain why local steepness or a cropped rising segment, by itself, cannot settle the match.

The axes have exact radian labels: yes.

Compare these candidates:  $A$  = decreasing branches between the same cosine-zero gaps;  $B$  = a continuous bounded wave with range  $[-1, 1]$ ;  $C$  =  $U$ -shaped reciprocal branches with no  $x$ -intercepts;  $D$  = an odd graph through  $(0, 0)$  whose central component approaches  $-\infty$  and  $+\infty$  at its excluded ends.

Apply this decision constraint: Distinguish increasing tangent branches from decreasing cotangent branches.

Some candidate cards are cropped: no.

Work on the domain structural identification of the tangent parent.

Bank label: \_\_\_\_\_

Complete the tangent column using  $\tan(x) = \frac{\sin(x)}{\cos(x)}$ . Mark a row undefined rather than dividing by zero, then use the ordered rows to state one graph conclusion; do not connect points across an undefined input.

Required response

| $\cos(x)$ | $\sin(x)$ | $x$            |
|-----------|-----------|----------------|
| -1.000    | 0.025     | $-\pi - 0.025$ |
| -1.000    | 0.000     | $-\pi$         |
| -1.000    | -0.025    | $-\pi + 0.025$ |

Bank label: \_\_\_\_\_

## AP Precalculus

Tangent Parent  
Function Graphs

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

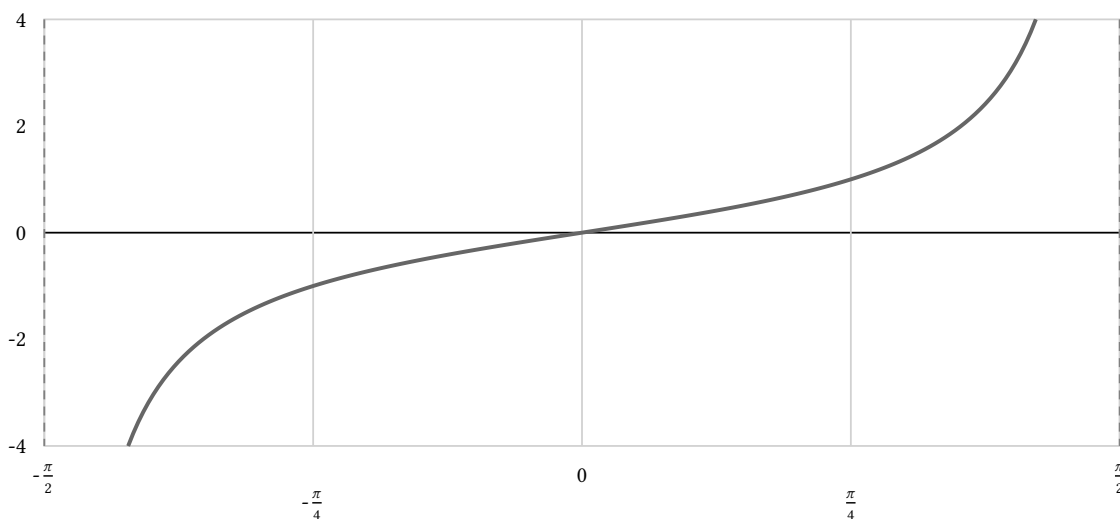
Reconstruct  $y = \tan(x)$  on the displayed window using half-turn translations of the supplied central branch. List every visible zero and vertical asymptote, keep the branches disconnected, and state how many connected branch pieces meet the interior of the window.

The source branch is  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

The target window is display endpoints included is yes; left is  $-9\frac{\pi}{4}$ ; right is  $3\frac{\pi}{4}$ .

The translation rule is  $(x, y)$  maps to  $(x + k\pi, y)$ .

Work on the domain half-turn repetition of the tangent parent.



## AP Precalculus

Tangent Parent  
Function Graphs

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

**Incoming:** the claim is false on  $[5\frac{\pi}{4}, 7\frac{\pi}{4})$ . The component ranges are  $[1, \infty)$  and  $(-\infty, -1)$ ; tangent increases on each component, but the gap at  $3\frac{\pi}{2}$  prevents a single increasing interval claim.

Complete the tangent column using  $\tan(x) = \frac{\sin(x)}{\cos(x)}$ . Mark a row undefined rather than dividing by zero, then use the ordered rows to state one graph conclusion; do not connect points across an undefined input.

Required response

| $\cos(x)$ | $\sin(x)$ | $x$            |
|-----------|-----------|----------------|
| 0.999     | -0.050    | $-2\pi - 0.05$ |
| 1.000     | 0.000     | $-2\pi$        |
| 0.999     | 0.050     | $-2\pi + 0.05$ |

Your response: \_\_\_\_\_

## AP Precalculus

**Tangent Parent  
Function Graphs**

**Name:** \_\_\_\_\_ **Date:** \_\_\_\_\_ **Class:** \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Identify the defining contradiction in the student's tangent graph or statement. Give the smallest mathematical repair and support it with the quotient definition or branch structure; do not redraw unrelated valid features.

Evaluate claim 1.

Limit your correction to change only the contradicted feature.

Evaluate this student work: The graph labels  $x = \frac{\pi}{2}$  as an  $x$ -intercept at  $(\frac{\pi}{2}, 0)$ .

Work on the domain diagnosis of a proposed tangent graph.

Identify the defining contradiction in the student's tangent graph or statement. Give the smallest mathematical repair and support it with the quotient definition or branch structure; do not redraw unrelated valid features.

Evaluate claim 2.

Limit your correction to change only the contradicted feature.

Evaluate this student work: The graph's  $y$ -axis states that the range of tangent is  $[-1, 1]$ .

Work on the domain diagnosis of a proposed tangent graph.

## Tangent Parent Function Graphs

### Full Worked Solutions - excerpt

#### Worked example

**Problem.** Use the supplied sine and cosine sign evidence to determine the one-sided behavior of  $\tan(x)$  at the excluded input. Justify each quotient sign and explicitly state that the function has no value at the boundary.

The excluded input is  $-5\frac{\pi}{2}$ .

The requested order is both sides.

The supplied sign information is cos from left is -; cos from right is +; sin near boundary is -.

Work on the domain one-sided quotient behavior.

**Answer.** as  $x \rightarrow -5\frac{\pi}{2}^-$ ,  $\tan(x) \rightarrow +\infty$ ; as  $x \rightarrow -5\frac{\pi}{2}^+$ ,  $\tan(x) \rightarrow -\infty$ ;  $\tan(-5\frac{\pi}{2})$  is undefined.

#### Explanation and check.

On the left, numerator and small denominator have the same sign, so the quotient is large positive. On the right their signs differ, so it is large negative. A one-sided trend does not define a value where cosine equals zero.

#### Worked example

**Problem.** Use the supplied sine and cosine sign evidence to determine the one-sided behavior of  $\tan(x)$  at the excluded input. Justify each quotient sign and explicitly state that the function has no value at the boundary.

The excluded input is  $-15\frac{\pi}{2}$ .

The requested order is left side then right side.

The supplied sign information is cos changes sign is + to -; sin at boundary is +1.

Work on the domain one-sided quotient behavior.

**Answer.** as  $x \rightarrow -15\frac{\pi}{2}^-$ ,  $\tan(x) \rightarrow +\infty$ ; as  $x \rightarrow -15\frac{\pi}{2}^+$ ,  $\tan(x) \rightarrow -\infty$ ;  $\tan(-15\frac{\pi}{2})$  is undefined.

#### Explanation and check.

On the left, numerator and small denominator have the same sign, so the quotient is large positive. On the right their signs differ, so it is large negative. A one-sided trend does not define a value where cosine equals zero.

AP Precalculus

Tangent Function Transformations

Tangent Function Transformations | APPC-U03-P15

96 problems · Four activity formats

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Topic focus

Across 96 tasks this topic asks for compatible pattern or precise contradiction, invariants, center outputs, and orientation, period, zero family, and asymptote family, failed relation and corrected feature statement, invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result and included zeros and asymptote locations.

8 PDFs and a README; 151 buyer pages.

## AP Precalculus

## Tangent Function Transformations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

List every zero and every vertical-asymptote location inside the specified window. Solve integer bounds for the two families separately and honor each open or closed display endpoint exactly.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

| Condition                       | Supplied value                         |
|---------------------------------|----------------------------------------|
| Given vertical-asymptote family | $x = \frac{\pi}{2} + k(\frac{\pi}{2})$ |
| Required window                 | $(-3\frac{\pi}{4}, 5\frac{\pi}{4}]$    |
| Given zero family               | $x = \frac{\pi}{4} + k(\frac{\pi}{2})$ |

Bank label: \_\_\_\_\_

AP Precalculus

Tangent Function Transformations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Compare the two supplied tangent models without solving a general tangent equation. State what happens to period, asymptotes, branch-center outputs, former zeros, and branch orientation after the vertical translation, with special attention to the required evidence focus. The supplied models are  $f(x) = 3 \tan(\frac{1}{2}(x))$  and  $g(x) = 3 \tan(\frac{1}{2}(x)) + 1$ . The branch-center family is  $x = 0 + k(2\pi)$ . The required evidence focus is to state exactly which invariants come from the inner map.  
Solve the problem and show enough exact work for the requested result.

| Condition                        | Supplied value                                         |
|----------------------------------|--------------------------------------------------------|
| Given branch-center input family | $x = 0 + k(2\pi)$                                      |
| Required evidence focus          | state exactly which invariants come from the inner map |
| Given output translation         | 1                                                      |

AP Precalculus

Tangent Function Transformations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

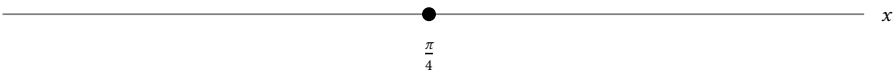
**Incoming:** zeros are  $-15\frac{\pi}{4}, -7\frac{\pi}{4}, \frac{\pi}{4}, 9\frac{\pi}{4}, 17\frac{\pi}{4}$ ; asymptote locations are  $-19\frac{\pi}{4}, -11\frac{\pi}{4}, -3\frac{\pi}{4}, 5\frac{\pi}{4}, 13\frac{\pi}{4}, 21\frac{\pi}{4}$ .

Recover the horizontal period and a phase-anchor class from the graph evidence. Decide whether the asymptote spacing is one period or several, and state exactly what sign ambiguity remains; use orientation only when it is supplied.

Observed asymptote locations



Marked branch-center input



| Feature                      | Exact x-location(s)              |
|------------------------------|----------------------------------|
| Observed asymptote locations | $-\frac{\pi}{4}, 3\frac{\pi}{4}$ |
| Marked branch-center input   | $\frac{\pi}{4}$                  |

| Condition                   | Supplied value            |
|-----------------------------|---------------------------|
| Given adjacency information | certified consecutive     |
| Given branch direction      | not supplied              |
| Given outer coefficient     | not orientation-resolving |

Your response: \_\_\_\_\_

**AP Precalculus****Tangent Function Transformations****Name:** \_\_\_\_\_ **Date:** \_\_\_\_\_ **Class:** \_\_\_\_\_

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

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A student claims: “Omit  $x = -7\frac{\pi}{3}$  and  $x = 7\frac{\pi}{6}$  because boundary locations can never appear in requested zero or asymptote lists.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Use this route: audit both endpoint flags. Enumerate zeros from  $x = -\frac{\pi}{3} + k(\pi)$  and poles from  $x = \frac{\pi}{6} + k(\pi)$  inside  $[-7\frac{\pi}{3}, 7\frac{\pi}{6}]$ , honoring every endpoint.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

| Condition                       | Supplied value                      |
|---------------------------------|-------------------------------------|
| Given vertical-asymptote family | $x = \frac{\pi}{6} + k(\pi)$        |
| Required window                 | $[-7\frac{\pi}{3}, 7\frac{\pi}{6}]$ |
| Given zero family               | $x = -\frac{\pi}{3} + k(\pi)$       |

# Tangent Function Transformations

## Full Worked Solutions - excerpt

### Worked example

**Problem.** Find the first invalid relation in the worked tangent-feature calculation, repair only that relation, and verify the correction with one representative zero, asymptote, or spacing check.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

| Condition       | Supplied value                                                   |
|-----------------|------------------------------------------------------------------|
| Step to examine | calls every branch center an $x$ -intercept after adding $d = 4$ |

**Answer.** centers have output 4 and need not be  $x$ -intercepts.

### Explanation and check.

Check the tangent-specific period  $\pi$ , the separate inner-angle conditions for zeros and poles, and the role of any vertical offset. Preserve steps that remain valid after the single correction. At an old center, the inner angle is 0 and  $\tan(0) = 0$ . After the stated translation by 4, the output there is 4, not 0. This direct substitution verifies that the old center is not an  $x$ -intercept.

### Worked example

**Problem.** Find the first invalid relation in the worked tangent-feature calculation, repair only that relation, and verify the correction with one representative zero, asymptote, or spacing check.

Compare the stated objects or approaches and justify the conclusion with exact evidence.

| Condition       | Supplied value                            |
|-----------------|-------------------------------------------|
| Step to examine | reports $T = -\frac{\pi}{3}$ for $b = -3$ |

**Answer.** period is positive  $\frac{\pi}{3}$ .

### Explanation and check.

Check the tangent-specific period  $\pi$ , the separate inner-angle conditions for zeros and poles, and the role of any vertical offset. Preserve steps that remain valid after the single correction. With  $b = -3$  and  $T = \frac{\pi}{3}$ , the inner-angle change is  $-3(\frac{\pi}{3}) = -\pi$ . Tangent has the same value after that change, while  $T$  is positive. The claimed negative value is not a period length.