Sets & Steps
Math Practice & Worked Solutions

Precalculus

Writing & Interpreting Sinusoidal Models | 3-Topic Precalculus Bundle

3-topic bundle | 288 problems | Four activity formats

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01 Writing Sinusoidal Equations from Features

View individual product and its full preview

02 Sinusoidal Modeling

View individual product and its full preview

03 Interpreting Sinusoidal Transformations

View individual product and its full preview

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

24 PDFs and 3 READMEs; 317 buyer PDF pages across the 3 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Writing Sinusoidal Equations from Features

Writing Sinusoidal Equations from Features | APPC-U03-P11

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for existence decision and first contradiction, equivalence decision and decisive normalized evidence, one compatible rule and sufficiency explanation, one complete rule and parameter verification, one example rule or impossibility proof and one sine rule with directional check.

8 PDFs and a README; 152 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Writing Sinusoidal Equations from Features

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$-\frac{5}{2}$
next global maximum	1	$\frac{5}{2}$

Bank label: ______

Determine which sinusoidal parameters are forced by the partial exact features and which remain free. Demonstrate any nonuniqueness by checking the two explicit candidate constructions, or replace either if it fails.

The first proposed rule is $y = 3\cos(2\pi\frac{x+\frac{5}{2}}{6})$.

The second proposed rule is $y = 3\cos(2\pi\frac{x+\frac{5}{2}}{3})$.

The global range is $[-3, 3]$.

The two stated maxima are not specified to be consecutive.

Maxima occur at $x = -\frac{5}{2}$ and $x = \frac{7}{2}$.

Bank label: ______

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

Required response

event	value	x
global minimum	-3	$\frac{3}{2}$
next global maximum	3	$\frac{21}{2}$

Bank label: ______

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 10.

The claimed range is $[-4, 4]$.

The reference input is $x = -\frac{3}{2}$.

The first proposed rule is $y = 4\sin(2\pi\frac{x+\frac{3}{2}}{10})$.

The second proposed rule is $y = 4\cos(2\pi\frac{x-1}{10})$.

Bank label: ______

AP Precalculus

Writing Sinusoidal Equations from Features

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

An exact partial graph is supplied with its global range, labeled events and directions, and the stipulation that the curve continues sinusoidally. Construct one compatible cosine rule and explain why clipping does or does not hide needed information.

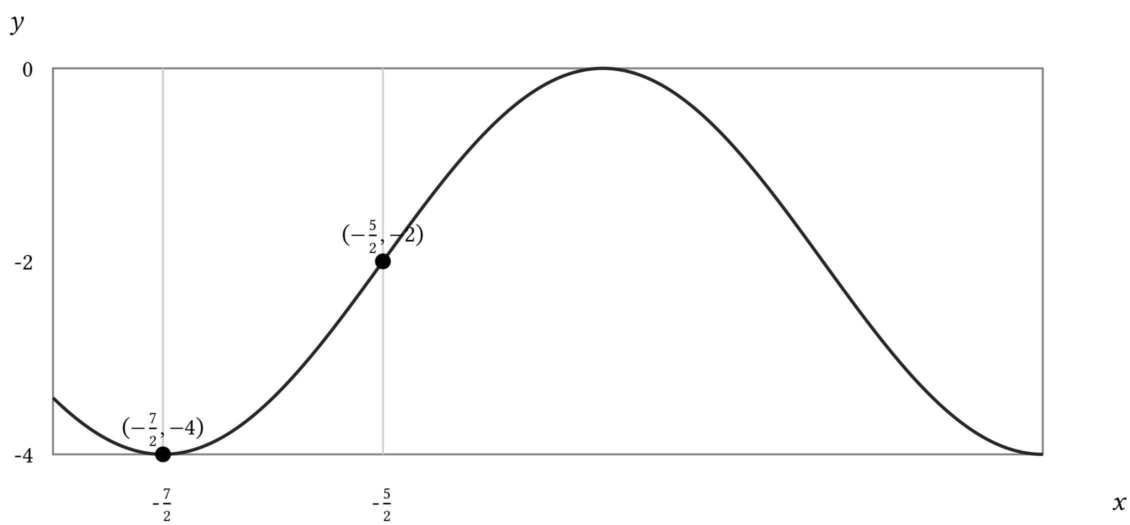
The curve continues sinusoidally beyond the displayed interval.

The global range is $[-4, 0]$.

The labeled minimum is $(-\frac{7}{2}, -4)$.

The labeled upward midline is $(-\frac{5}{2}, -2)$.

The displayed x-interval is $[-4, \frac{1}{2}]$.



event	**x**	**y**
minimum	$-\frac{7}{2}$	-4
upward midline	$-\frac{5}{2}$	-2

AP Precalculus

Writing Sinusoidal Equations from Features

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

A nonconstant sinusoid has the exact global features shown. The timed events are consecutive. Construct one cosine rule, allowing a negative cosine coefficient when convenient, and verify the range, period, and event times.

event	value	x
global maximum	0	$\frac{21}{2}$
next global maximum	0	$\frac{57}{2}$
global minimum	-6	not specified

Your response: ______________

AP Precalculus

Writing Sinusoidal Equations from Features

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the student's multi-parameter reconstruction against every exact feature. Identify the earliest failed parameter relation, retain valid deductions, give a corrected complete rule, and verify the repaired model.

Preserve the valid deductions when repairing the model.

The supplied features are: range $[0, 6]$, period 4, upward midline at $x = 1$.

The student proposes $y = 3 + 3\sin(2\pi \frac{x-1}{8})$.

Two rules are proposed as constructions for the same feature data. Decide whether they define the same function for every real x. Normalize sign, period, family, and phase; agreement at one landmark alone is not sufficient.

The claimed period is 14.

The claimed range is $[-3, 1]$.

The reference input is $x = \frac{11}{2}$.

The first proposed rule is $y = -1 + 2\sin(2\pi \frac{x-\frac{11}{2}}{14})$.

The second proposed rule is $y = -1 + 2\sin(2\pi \frac{x-9}{14})$.

Writing Sinusoidal Equations from Features

Full Worked Solutions - excerpt

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = 1$.

The global range is $[-2, 2]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-2+2}{2} = 0$, not 1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-2+2}{2} = 0$, not 1. Therefore the feature set is inconsistent.

Worked example

Problem. Test whether one nonconstant sinusoidal function can have every exact feature shown. Check range, recurrence, event direction, and single-valuedness before attempting a formula.

The claimed midline is $y = -1$.

The global range is $[-4, 0]$.

Answer. No compatible sinusoidal function exists: the range forces midline $\frac{-4+0}{2} = -2$, not -1.

Explanation and check.

The first decisive contradiction is that the range forces midline $\frac{-4+0}{2} = -2$, not -1. Therefore the feature set is inconsistent.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Sinusoidal Modeling

Sinusoidal Modeling | APPC-U03-P12

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for family decision with evidentiary limit, model recommendation or insufficiency conclusion, invalid claim or first invalid step, corrected conclusion, brief mathematical reason, and the requested underlying result, invalid assumption and actionable repair, plausible fitted rule and uncertainty statement and signed-residual diagnosis and revision recommendation.

8 PDFs and a README; 80 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Sinusoidal Modeling

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 3\sin(2\pi\frac{t+\frac{5}{2}}{8})$. The event level is $y = 10$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[-\frac{31}{6}, \frac{27}{2}]$.

Bank label: ______

The contextual extrema are exact and the cycle is stipulated stationary only on the feasible time domain. Construct one sinusoidal model in the stated time unit, explain which events determine a full or half cycle, and retain the domain restriction. The feasible time domain is $[8, 80]$. Time is measured in days.

Required response

event	time	value
minimum	8	11
next minimum	26	11

Bank label: ______

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 20 + 3\cos(2\pi\frac{t+3}{8})$. The response is measured in units. The sample time window is $[-11, 13]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = -3$. Time is measured in hours.

Bank label: ______

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 3\sin(2\pi\frac{t-\frac{11}{2}}{14})$. The event level is $y = 12$. Find the first upward midline crossing in the window. Time is measured in hours. Use the time window $[\frac{5}{6}, \frac{67}{2}]$.

Bank label: ______

AP Precalculus

Sinusoidal Modeling

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compare the fitted sinusoids for the stated use case. Weigh observed-window error, physical range, prediction horizon, and divergence. If the evidence cannot distinguish them for the intended decision, recommend a specific validation measurement. The intended decision is at $t = 34$. The observed time window is $[0, 13]$. The response tolerance is 1 unit.

Required response

future prediction	model	observed window RMSE	period
21	A	0.43	11
26	B	0.45	14

The dataset and fully specified candidate regression outputs are supplied. Select every usable model in the stated angle mode. Verify at least one observed or held-out landmark and do not infer software behavior that is not shown. Use radians mode on the calculator.

Required response

formula	label
$y = 3 + 2\cos(2\pi\frac{t-10}{8})$	A
$y = 3 + 2\sin(2\pi\frac{t-8}{8})$	B

t	y
10	5
14	1
18	5

Use the signed residual definition shown. Assess both the pattern across phases and the held-out observation; a small total error does not excuse a systematic pattern. Recommend retention, revision, or investigation. The absolute tolerance is 0.75. At held-out time $t = 5$, the signed residual is 0.11.

Required response

observed minus predicted	t
0.11	0
−0.11	1
0.0	2
0.11	3
−0.11	4

Use several measurements rather than a single high or low value to estimate a stationary sinusoid. Give a plausible rule, identify the approximate baseline, amplitude, period, and phase, and state what the finite noisy sampling cannot establish exactly. The measured response is in units. Time is measured in hours.

Required response

measurement	time
24.2	$\frac{13}{2}$
19.8	$\frac{23}{2}$
20.2	$\frac{43}{2}$
23.8	$\frac{53}{2}$

AP Precalculus

Sinusoidal Modeling

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Evaluate the accepted fitted rule at the target time, then classify the use relative to the sample window. Report the contextual unit and state whether stationarity evidence supports the prediction or only makes it conditional. The supplied fitted formula is $y = 23 + 4\cos(2\pi\frac{t-4}{12})$. The response is measured in units. The sample time window is $[-8, 28]$. Stationarity has been verified only within the sample window. Evaluate the requested prediction at $t = 13$. Time is measured in hours.

Your response: ______________________

Incoming: Prediction 8: 23 units; interpolation inside the sampled window.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 12 + 4\sin(2\pi\frac{t-\frac{5}{2}}{8})$. The event level is $y = 12$. Find the second occurrence of the midline level in the window, counting either direction. Time is measured in hours. Use the time window $[\frac{5}{2}, \frac{21}{2}]$.

Your response: ______________________

Incoming: Event case 6: $t = \frac{13}{2}$ (downward).

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 9. Initially the point is initial vertical velocity zero, then downward. Use seconds as the time unit for t. The operating window is $\frac{1}{2} \le t \le \frac{85}{2}$. The radius is 6. The starting position is top. The starting time is $t = \frac{1}{2}$. One uniform rotation takes 14 seconds.

Your response: ______________________

Incoming: $y = 9 + 6\cos(2\pi\frac{t-\frac{1}{2}}{14})$

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Treat negative depth as physically valid. A fitted water-depth sinusoid has range $[-2, 8]$ meters.

Your response: ______________________

Incoming: Water depth in $[-2, 8]$ meters includes negative depth. Restrict use to nonnegative depths or refit with a nonnegative constrained model.

Use the accepted model to locate the requested contextual event. Screen solutions by the closed time window and crossing direction; the second occurrence of a level is not automatically the first upward occurrence. Use the accepted model $y = 10 + 6\sin(2\pi\frac{t-\frac{1}{2}}{14})$. The event level is $y = 10$. Find the all upward midline crossings in the window. Time is measured in hours. Use the time window $[\frac{1}{2}, \frac{57}{2}]$.

Your response: ______________________

Incoming: Event case 4: $t = \frac{1}{2}, \frac{29}{2}, \frac{57}{2}$.

A point rotates uniformly on the mechanism described. Model its height above the stated datum over the operating window. Use the start position and motion direction to choose the sign and reference family, and verify the geometric range. The center height above the datum is 7. Initially the point is initial vertical velocity zero, then upward. Use seconds as the time unit for t. The operating window is $-\frac{5}{2} \le t \le \frac{43}{2}$. The radius is 3. The starting position is bottom. The starting time is $t = -\frac{5}{2}$. One uniform rotation takes 8 seconds.

Your response: ______________________

AP Precalculus

Sinusoidal Modeling

Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Combine times without a common clock origin. Tidal observations are in hours after midnight on different dates.

Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Repair only the phase of a stationary sinusoid. A sensor baseline rises steadily while residuals alternate.

Sinusoidal Modeling

Full Worked Solutions - excerpt

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Use $T = 3$ with t measured in seconds. The fitted period is 3 minutes but t is entered in seconds.

Answer. Assumption repair 3: convert the period to 180 seconds, or convert every input to minutes.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to convert the period to 180 seconds, or convert every input to minutes.

Worked example

Problem. Audit the modeling claim separately from algebraic curve fit. Identify the physical, unit, or evidence assumption that fails and state a revision that changes the model or restricts its use. Evaluate this claim: Extrapolate the fitted cycle indefinitely. Seasonal data cover one year during an unusual shutdown.

Answer. Assumption repair 4: limit claims to the observed window and gather ordinary-year validation data.

Explanation and check.

The claim fails because the stated context does not support its physical or unit assumption. The actionable revision is to limit claims to the observed window and gather ordinary-year validation data.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Interpreting Sinusoidal Transformations

Interpreting Sinusoidal Transformations | APPC-U03-P13

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for changed and invariant physical features, relevant common-scale comparison, first misleading claim and corrected quantitative statement, phase event and initial-value qualification, contextual attained range and boundary status and parameter meanings and derived features.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Interpreting Sinusoidal Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes and $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 230 + 40\cos(2\pi\frac{t-90}{300})$; output unit before: centimeters. Bank label: _______	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 11 + 3\cos(2\pi\frac{t-3}{16})$; valid window: $0, 64$. Bank label: _______
Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $R = \frac{Q}{100}$ meters; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 210 + 40\cos(2\pi\frac{t-30}{360})$; output unit before: centimeters. Bank label: _______	Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: degrees; supplied formula: $Q(t) = 9 + 5\cos(2\pi\frac{t-4}{8})$; valid window: $0, 32$. Bank label: _______

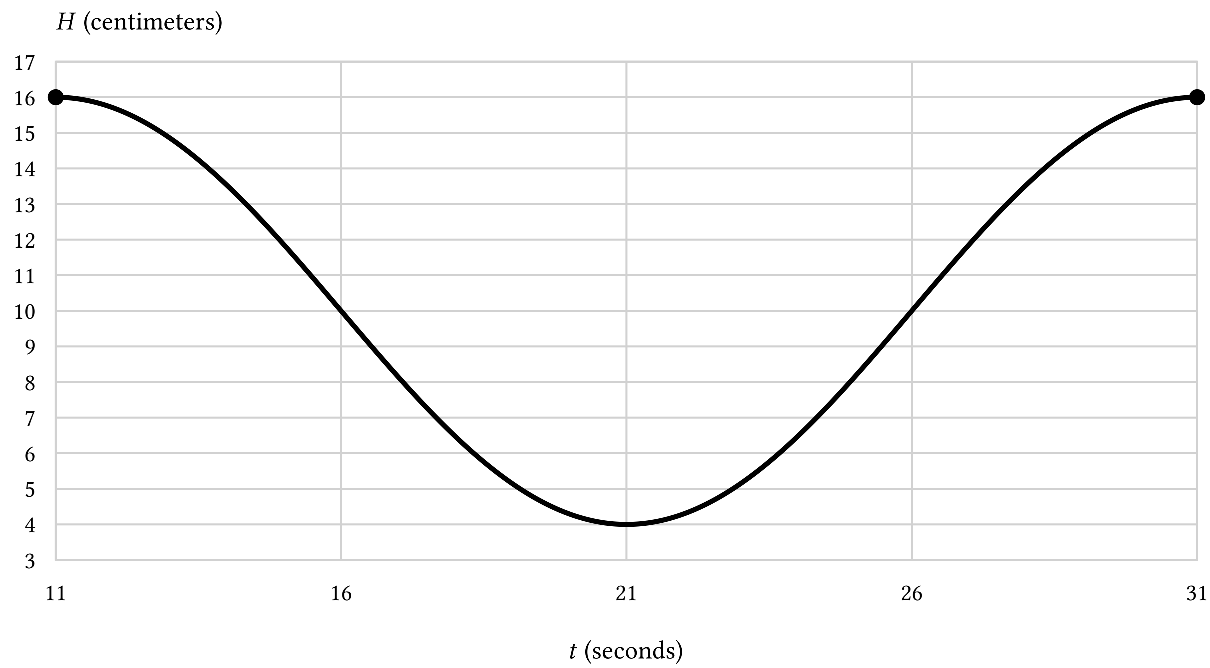
AP Precalculus

Interpreting Sinusoidal Transformations

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Interpret the supplied graph only on its physical operating window. State the attainable output interval, midline meaning, and whether displayed boundaries are attained; do not substitute the full mathematical range when the window clips an extremum.



AP Precalculus

Interpreting Sinusoidal Transformations

Name: ________________ **Date:** ________________ **Class:** ________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{16})$; only changed category: period; original formula: $P(t) = 7 + 4\sin(2\pi\frac{t-\frac{7}{2}}{8})$; output unit: units; time unit: hours.

Your response: ________________

Incoming: period doubles from 8 to 16 time units, so recurrence halves; range and the anchor at $t = \frac{7}{2}$ stay fixed.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: new datum is 52 cm above the old datum, so $R = Q - 52$; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 220 + 60\cos(2\pi\frac{t-60}{240})$; output unit before: centimeters.

Your response: ________________

Incoming: $R(t) = 168 + 60\cos(2\pi\frac{t-60}{240})$; event times are unchanged while every height label decreases by 52 cm.

Interpret every relevant coefficient of the supplied model in complete contextual sentences. Normalize an expanded argument before reading phase, distinguish signed coefficient from amplitude, and attach output or time units correctly. argument form: factored argument; input unit: hours; output unit: meters; supplied formula: $Q(t) = 10 + 4\sin(2\pi\frac{t+\frac{3}{2}}{10})$; valid window: 0, 40.

Your response: ________________

Incoming: amplitude 4 meters; midline 10 meters; period 10 hours; frequency $\frac{1}{10}$ cycle per hour; $h = -\frac{3}{2}$ hours labels an upward midline crossing.

Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 6 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12}), B(t) = 8 + 5\sin(2\pi\frac{t-\frac{3}{2}}{12})$; practical measure: same-period.

Your response: ________________

Incoming: same period 12 hours and same spread 10, but B's baseline and range are 2 units higher.

Compare the supplied original and changed models. Explain the physical consequence of the one parameter category that changed and explicitly name the range, recurrence, or event features that remain invariant. changed formula: $P_2(\tau) = 8 + 5\sin(2\pi\frac{\tau}{10})$; only changed category: clock; original formula: $P(t) = 8 + 5\sin(2\pi\frac{t-4}{10})$; output unit: units; time unit: hours.

Your response: ________________

Incoming: with t = $\tau + 4$, the same physical anchor is labeled $\tau = 0$ rather than $t = 4$; range and physical period are unchanged.

Rewrite the supplied model in the requested input/output units or datum. Convert the input coordinate inside the argument and the output coordinate outside it, then verify one named physical event in both systems. conversion: $u = \frac{t}{60}$ minutes; input unit before: seconds; named event: cosine maximum; original model: $Q(t) = 200 + 40\cos(2\pi\frac{t}{360})$; output unit before: centimeters.

Your response: ________________

AP Precalculus

Interpreting Sinusoidal Transformations

Name: ______________ **Date:** ______________ **Class:** ______________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Test the student's contextual parameter statement against the supplied model. Identify the first misleading claim and replace it with a quantitatively correct statement including units or window limits where relevant. context units: stated in formula or prompt; student statement: The period is 4π .; supplied model: $Q(t) = 4 + 3\sin(4\pi(t-2))$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Interpreting Sinusoidal Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 2\sin(2\pi\frac{t+2}{8}), B(t) = 5 + 2\sin(2\pi\frac{t+2}{16})$; practical measure: same-range.

Answer. same range $[3,7]$, but B recurs every 16 rather than 8 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[3,7]$, but B recurs every 16 rather than 8 hours.

Worked example

Problem. Compare the two supplied contextual models on a common unit scale. Address only the feature relevant to the stated practical measure—range, recurrence, baseline, or phase equivalence—and identify what cannot distinguish the models. common units: input: hours; output: units; formula pair: $A(t) = 5 + 4\sin(2\pi\frac{t-1}{10}), B(t) = 5 + 4\sin(2\pi\frac{t-1}{20})$; practical measure: same-range.

Answer. same range $[1,9]$, but B recurs every 20 rather than 10 hours.

Explanation and check.

Normalize the ranges, periods, and reference families. The decisive result is: same range $[1,9]$, but B recurs every 20 rather than 10 hours.