

Precalculus
**Radians, Exact Trig Values & Reference Angles | 3-Topic
Precalculus Bundle**

3-topic bundle | 288 problems | Four activity formats

01 Periodic Functions, Radians, and the Unit Circle

[View individual product and its full preview](#)

02 Exact Trig Values at Special Angles

[View individual product and its full preview](#)

03 Trig Signs and Reference Angles

[View individual product and its full preview](#)

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

24 PDFs and 3 READMEs; 366 buyer PDF pages across the 3 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Periodic Functions, Radians, and the Unit Circle | APPC-U03-COMB-P01-P02

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
02	Grid Rounds	24 tasks
03	Match & Move	24 cards
04	Error Analysis & Strategy	24 tasks
05	Quick Answer Key	96 answers
06	Full Worked Solutions	96 solutions
07	Teacher / Parent Use Guide	included
08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for directed radian measure, supported periodicity claim, recurrence rates and range comparison, exact degrees, exact radians and exact ordering.

8 PDFs and a README; 131 buyer pages.

AP Precalculus

**Periodic Functions, Radians, and
the Unit Circle**

Name: _____ Date: _____ Class: _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Express $-2\frac{\pi}{9}$ radians as an exact directed angle in degrees.

Bank label: _____

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ Date: _____ Class: _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

On a circle of radius 4 m, a point rotates through $\frac{3}{8}$ radians. Find the directed arc displacement s and state whether the motion is clockwise or counterclockwise. _____ _____	For the directed angle $13\frac{\pi}{6}$, describe its terminal ray and list every coterminal representative in the interval $(\pi, 5\pi]$. Respect the endpoint symbols. _____ _____
An exact periodic graph has period 9. A cycle marker occurs at $t = 5$; the graph reaches a specified decreasing phase 4 time units after every marker. Is $t = 38$ that same phase? Give an equivalent cycle window beginning at the marker. _____ _____	For the directed angle $15\frac{\pi}{8}$, describe its terminal ray and list every coterminal representative in the interval $(0, 4\pi)$. Respect the endpoint symbols. _____ _____

AP Precalculus

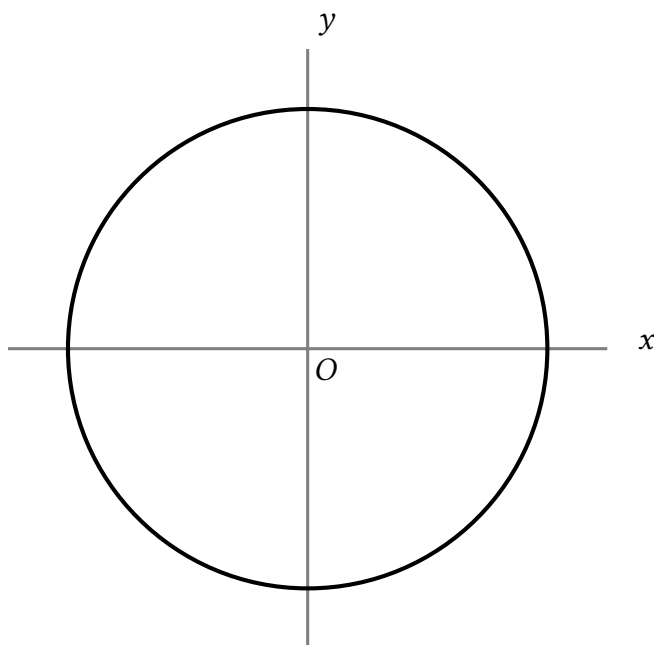
Periodic Functions, Radians, and the Unit Circle

Name: _____ Date: _____ Class: _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: 105°

Point $P = (\frac{8}{17}, y)$ lies on the unit circle and is below the x -axis. Find y and verify the point.



Your response: _____

AP Precalculus

Periodic Functions, Radians, and the Unit Circle

Name: _____ Date: _____ Class: _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims that the smallest period is 7 time units for these exact repeating (sample index, value) pairs: (0, 78); (1, 71); (2, 76); (3, 73); (4, 72); (5, 73); (6, 82); (7, 78); (8, 71); (9, 76); (10, 73); (11, 72); (12, 73); (13, 82).

Consecutive sample indices are 3 time units apart. Diagnose the claim, state the corrected smallest period, and cite one decisive comparison.

Identify the error or first invalid claim, give the corrected conclusion, and justify the correction.

sample index	value
0	78
1	71
2	76
3	73
4	72
5	73
6	82

sample index	value
7	78
8	71
9	76
10	73
11	72
12	73
13	82

A circle centered at O has radius 18 cm. A point moves counterclockwise along the circle through an arc of length $\frac{162}{5}$ cm. Find the directed angle in radians and explain how the sign records the sweep direction. Before solving, choose and justify the valid method. Method A: Use radius divided by arc length. Method B: Use signed arc length divided by radius.

Periodic Functions, Radians, and the Unit Circle

Full Worked Solutions - excerpt

Worked example

Problem. Convert the directed rotation 225° to an exact radian measure. Preserve its direction.

Answer. $5\frac{\pi}{4}$ radians

Explanation and check.

Multiply by $\frac{\pi}{180}^\circ$: $225(\frac{\pi}{180}) = 5\frac{\pi}{4}$. The sign is preserved.

Worked example

Problem. Convert the directed rotation -24° to an exact radian measure. Preserve its direction.

Answer. $-2\frac{\pi}{15}$ radians

Explanation and check.

Multiply by $\frac{\pi}{180}^\circ$: $-24(\frac{\pi}{180}) = -2\frac{\pi}{15}$. The sign is preserved.

AP Precalculus
Exact Trig Values at Special Angles

Exact Trig Values at Special Angles | APPC-U03-P04

96 problems · Four activity formats

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08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for correct equality and approximation, mode diagnosis and exact value, valid exact and approximate relations, exact equality groups and ordering, all permitted angle labels and completed exact row.

8 PDFs and a README; 128 buyer pages.

AP Precalculus

Exact Trig Values
at Special Angles

Name: _____ Date: _____ Class: _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

A special-angle table row has $(\sin \theta, \cos \theta) = (1, 0)$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate.

Required response

Angle	$\sin \theta$	$\cos \theta$
_____	1	0

Bank label: _____

On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos \theta = \frac{1}{2}$. Use the unit-circle line intersection and check both endpoint flags.

Bank label: _____

AP Precalculus

Exact Trig Values
at Special Angles

Name: _____ Date: _____ Class: _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

The pair at $\frac{\pi}{3}$ is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. The target ray $\frac{\pi}{6}$ is related by reverse complementary transfer. Track coordinate exchange and any signs to infer its exact pair.

Audit this special-angle table row: $\theta = \frac{\pi}{3}, \sin \theta = \frac{1}{2}, \cos \theta = \frac{1}{2}$.
Identify the earliest exact inconsistency and replace the row with the correct pair.
Required response

Angle	sin θ	cos θ
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{1}{2}$

AP Precalculus

Exact Trig Values
at Special Angles

Name: _____ Date: _____ Class: _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START A unit-circle table row for $\theta = 7\frac{\pi}{6}$ shows $\cos \theta$ missing and the other coordinate equal to $-\frac{1}{2}$. Complete the row exactly and use the angle's location to justify the square-root sign. Required response <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th style="width: 33%;">Angle</th> <th style="width: 33%;">sin θ</th> <th style="width: 33%;">cos θ</th> </tr> </thead> <tbody> <tr> <td>$7\frac{\pi}{6}$</td> <td>$-\frac{1}{2}$</td> <td>_____</td> </tr> </tbody> </table> Your response: _____	Angle	sin θ	cos θ	$7\frac{\pi}{6}$	$-\frac{1}{2}$	_____	Incoming: $\sin \theta = -\frac{1}{2}, \cos \theta = -\frac{\sqrt{3}}{2}$ A special-angle table row has $(\sin \theta, \cos \theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ but its angle label is missing. Find every label allowed by $[0, 2\pi)$; treat included endpoints as distinct labels when appropriate. Required response <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th style="width: 33%;">Angle</th> <th style="width: 33%;">sin θ</th> <th style="width: 33%;">cos θ</th> </tr> </thead> <tbody> <tr> <td>_____</td> <td>$\frac{1}{2}$</td> <td>$\frac{\sqrt{3}}{2}$</td> </tr> </tbody> </table> Your response: _____	Angle	sin θ	cos θ	_____	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
Angle	sin θ	cos θ											
$7\frac{\pi}{6}$	$-\frac{1}{2}$	_____											
Angle	sin θ	cos θ											
_____	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$											
Incoming: $\theta = \frac{\pi}{6}$ On the finite one-turn window $(-\pi, \pi]$, locate every special-angle ray satisfying $\cos \theta = 1$. Use the unit-circle line intersection and check both endpoint flags. Your response: _____	Incoming: $\theta = 0$ On the finite one-turn window $[0, 2\pi)$, locate every special-angle ray satisfying $\cos \theta = -1$. Use the unit-circle line intersection and check both endpoint flags. Your response: _____												

AP Precalculus

**Exact Trig Values
at Special Angles****Name:** _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A calculator in degree mode evaluates the radian expression $\cos(\frac{\pi}{3})$ and displays 0.9998. Diagnose the mode error, give the correct exact value, and explain why the display is not evidence of equality.

Exact Trig Values at Special Angles

Full Worked Solutions - excerpt

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{3}$ on the unit circle and evaluate the ordered pair $(\sin \theta, \cos \theta)$ exactly.

Answer. $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{3}$ has $x = \frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$. Therefore $(\sin \theta, \cos \theta) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$.

Worked example

Problem. Locate the terminal ray of $\theta = \frac{\pi}{6}$ on the unit circle and evaluate the ordered pair $(\sin \theta, \cos \theta)$ exactly.

Answer. $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

Explanation and check.

The terminal point at $\frac{\pi}{6}$ has $x = \frac{\sqrt{3}}{2}$ and $y = \frac{1}{2}$. Therefore $(\sin \theta, \cos \theta) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

AP Precalculus
Trig Signs and Reference Angles

Trig Signs and Reference Angles | APPC-U03-P06

96 problems · Four activity formats

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08	Terms of Use & License	included

Topic focus

Across 96 tasks this topic asks for three signs with coordinate evidence, all allowed angles, terminal location and reference angle, correct boundary classification, possible location or inconsistency and all consistent pairings or contradiction.

8 PDFs and a README; 107 buyer pages.

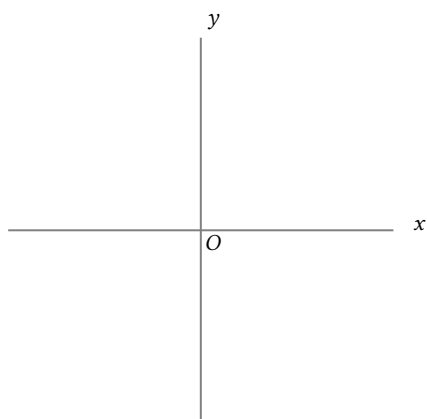
AP Precalculus

Trig Signs and Reference Angles

Name: _____ Date: _____ Class: _____

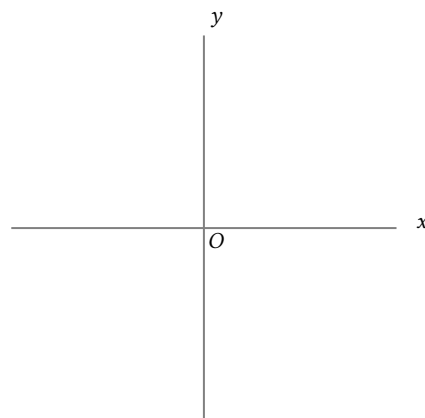
Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Intersect the constraints $\sin \theta > 0$, $\cos \theta \neq 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.



Bank label: _____

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(2\pi - \alpha)$ using $\cos \alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α .



Bank label: _____

Intersect the constraints $\cos \theta < 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent.

Construct every angle in $[0, 2\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant IV. Respect both endpoint flags and include all admitted turns.

Bank label: _____

Bank label: _____

AP Precalculus

Trig Signs and Reference Angles

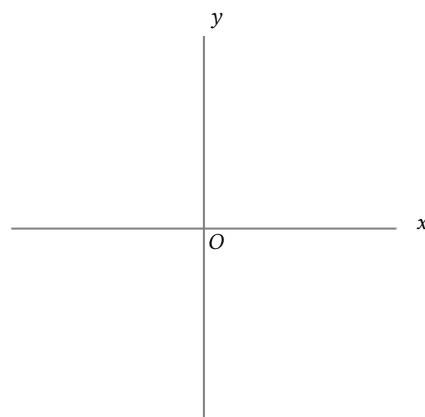
Name: _____ Date: _____ Class: _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For quadrant IV coordinate box with $x > 0, y < 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x , and $\frac{y}{x}$, including zero or undefined when required.

Given evidence	Sine	Cosine	Tangent
$x > 0, y < 0$	_____	_____	_____

For positive y -axis ray with $x = 0, y > 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x , and $\frac{y}{x}$, including zero or undefined when required.



Reduce the standard-position rotation $\theta = 7\frac{\pi}{6}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

Reduce the standard-position rotation $\theta = 11\frac{\pi}{4}$ only as needed, then determine its terminal quadrant and acute reference angle. If it is an axis ray, say so explicitly.

AP Precalculus

Trig Signs and Reference Angles

Name: _____ Date: _____ Class: _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $\theta = 2\frac{\pi}{3}$ For negative x-axis ray with $x < 0, y = 0$, classify the signs of sine, cosine, and tangent. Derive them from y, x, and $\frac{y}{x}$, including zero or undefined when required. <table border="1" style="width: 100%; border-collapse: collapse; margin: 10px 0;"> <thead> <tr> <th style="width: 20%;">Given evidence</th> <th style="width: 20%;">Sine</th> <th style="width: 20%;">Cosine</th> <th style="width: 20%;">Tangent</th> </tr> </thead> <tbody> <tr> <td>$x < 0, y = 0$</td> <td>_____</td> <td>_____</td> <td>_____</td> </tr> </tbody> </table> Your response: _____	Given evidence	Sine	Cosine	Tangent	$x < 0, y = 0$	_____	_____	_____	Incoming: $\theta = 7\frac{\pi}{6}$ Intersect the constraints $\sin \theta = 0, \cos \theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent. Your response: _____
Given evidence	Sine	Cosine	Tangent						
$x < 0, y = 0$	_____	_____	_____						
Incoming: $5\frac{\pi}{6}$ Intersect the constraints $\sin \theta > 0, \cos \theta > 0$. State every possible terminal location, distinguishing axes from open quadrants, or prove the conditions inconsistent. Your response: _____	Incoming: quadrant I Construct every angle in $[0, 4\pi)$ whose acute reference angle is $\frac{\pi}{3}$ and whose terminal ray is in quadrant III. Respect both endpoint flags and include all admitted turns. Your response: _____								

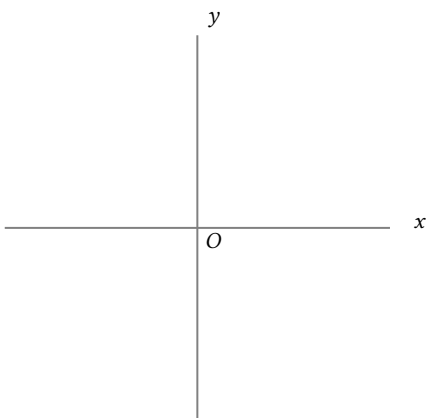
AP Precalculus

Trig Signs and Reference Angles

Name: _____ Date: _____ Class: _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\cos(3\pi - \alpha)$ using $\cos \alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α . First select and justify the valid method. Method A: Reduce $3\pi - \alpha$ to $\pi - \alpha$ and use the reflected x -coordinate. Method B: Cancel 3π as though it were an integer multiple of a full turn.



Trig Signs and Reference Angles

Full Worked Solutions - excerpt

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\sin(3\pi - \alpha)$ using $\sin \alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α .

Answer. $\sin(3\pi - \alpha) = \sin \alpha$

Explanation and check.

The reference magnitude is $\sin \alpha$; reduce to $\pi - \alpha$ in quadrant II. Therefore $\sin(3\pi - \alpha) = \sin \alpha$.

Worked example

Problem. Assume $0 < \alpha < \frac{\pi}{2}$. Rewrite $\tan(2\pi - \alpha)$ using $\tan \alpha$ only, and justify the sign from the terminal coordinates rather than evaluating α .

Answer. $\tan(2\pi - \alpha) = -\tan \alpha$

Explanation and check.

The reference magnitude is $\tan \alpha$; quadrant IV has opposite coordinate signs. Therefore $\tan(2\pi - \alpha) = -\tan \alpha$.