Sets & Steps
Math Practice & Worked Solutions

Precalculus

Parametric Relations, Vectors and Matrices

20-topic bundle | 1,920 problems | Four activity formats

Included topics

The following pages list all 20 included resources in the suggested order, with a link to each product. Each resource retains its complete preview.

Suggested learning sequence

1. Parametric and implicit relations
2. Vector operations and motion
3. Matrices, transformations and transitions

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

160 PDFs and 20 READMEs; 2,078 buyer PDF pages across the 20 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Math Practice & Worked Solutions

Precalculus

Included topics

20-topic bundle | 1,920 problems | Four activity formats

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01 **Parametric Tables and Curve Tracing**

View individual product and its full preview

02 **Eliminating Parameters and Reparameterizing Curves**

View individual product and its full preview

03 **Parametric Motion and Intersections**

View individual product and its full preview

04 **Average Rates and Speed in Parametric Motion**

View individual product and its full preview

05 **Parametric Equations of Lines**

View individual product and its full preview

06 **Parametric Circles**

View individual product and its full preview

07 **Implicit Relations and Curves**

View individual product and its full preview

08 **Conic Sections from Equations and Data**

View individual product and its full preview

09 **Parametric Conic Sections**

View individual product and its full preview

10 **Vector Components and Direction**

View individual product and its full preview

11 **Vector Operations and Magnitude**

View individual product and its full preview

20 included resources | Complete resource previews follow this guide.

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12 **Velocity Vector Applications**

View individual product and its full preview

13 **Vector-Valued Functions: Operations and Motion**

View individual product and its full preview

14 **Matrix Operations and Multiplication**

View individual product and its full preview

15 **Determinants and Area**

View individual product and its full preview

16 **Inverse Matrices and Linear Systems**

View individual product and its full preview

17 **Matrices for Linear Transformations**

View individual product and its full preview

18 **Composing Matrix Transformations**

View individual product and its full preview

19 **Inverse Transformations and Preimages**

View individual product and its full preview

20 **Transition Matrices and Long-Term Behavior**

View individual product and its full preview

20 included resources | Complete resource previews follow this guide.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Parametric Tables and Curve Tracing

Parametric Tables and Curve Tracing | APPC-U04-P01

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for equal x and equal y input pairs, filled table, repeated point, equal-y pair, (t,x,y) table, ordered triples, anchor order and repeated-point times and vertex order and closure.

8 PDFs and a README; 157 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Parametric Tables and Curve Tracing

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

For $x = t + 3$ and $y = t^2 - 1$, complete the (t, x, y) rows for $t = -1, 0, 2$.

Bank label: _____

Find the common parameter domain of $x = \sqrt{t + 2}$ and $y = \frac{1}{t-1}$.

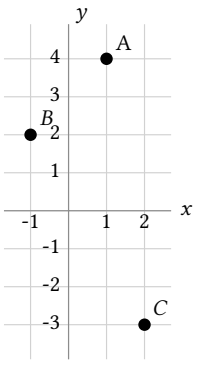
Bank label: _____

For $x = \cos t, y = \cos(2t)$ on $0 \le t \le 2\pi$, find every visit to $(0, -1)$.

Bank label: _____

A unit-scale plane trace labels $t = -2$ at $(1, 4)$, $t = 0$ at $(-1, 2)$, and $t = 3$ at $(2, -3)$. Write the corresponding component table.

Supplied labeled points



Given points only; connections between labels are unspecified here.
A: $t = -2(1, 4)$
B: $t = 0(-1, 2)$
C: $t = 3(2, -3)$

Bank label: _____

AP Precalculus

Parametric Tables and Curve Tracing

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Connect consecutive table points by straight segments for increasing t. List the vertex order and decide whether the path closes.

Required response

t	x	y
0	1	0
1	0	2
2	-1	0
3	0	-2
4	1	0

For $x = t^2 - 1, y = t^2$ on $-2 \le t \le 2$, find every t that visits $(0, 1)$. What causes the repeat?

AP Precalculus

Parametric Tables and Curve Tracing

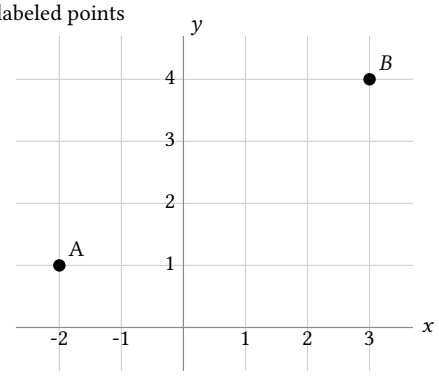
Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

A trace labels $t = 0$ at $(-2, 1)$, $t = 2$ at $(3, 4)$, and $t = 5$ again at $(-2, 1)$. Write the component rows and identify the repeated visit.

Supplied labeled points



Given points only; connections between labels are unspecified here.
A: $t = 0 (-2, 1)$
B: $t = 2 (3, 4)$
A: $t = 5 (-2, 1)$

Your response: _____

Incoming: Rows are $(0, -2, 1), (2, 3, 4), (5, -2, 1)$. The point $(-2, 1)$ is visited at both $t = 0$ and $t = 5$.

Connect consecutive samples in increasing t. Give the point sequence and identify any repeated location.

Required response

t	x	y
0	0	0
1	2	1
2	0	0
3	−1	2

Your response: _____

Incoming: The sequence is $(0, 0) \to (2, 1) \to (0, 0) \to (-1, 2)$. The origin is visited at $t = 0$ and $t = 2$.

Build the paired table for $x = t + 1$ and $y = 2t$ at $t = -2, 0, 3$.

Your response: _____

Incoming: [[−2, −1, −4], [0, 1, 0], [3, 4, 6]]

Determine the joint domain, identify the two denominator holes, and evaluate each attained outer boundary point. Work with -5≤t≤5. Restrict the parameter to [-5,5]. Use x=√((t+1)(t-4))/(t-5). Use y=ln(6-t)/(t+3).

Your response: _____

AP Precalculus

Parametric Tables and Curve Tracing

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

For $x = t^2$ and $y = 2t - 1$, a student writes rows $(0, 0, 1), (1, 1, 3), (2, 4, -1)$. Identify the pairing error and give the correct table.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

t	x	y
0	0	1
1	1	3
2	4	-1

A student claims: "At $t = 0$ record $y = 0$, so all four requested rows are valid." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Build the paired table for $x = t^2 - 3t, y = \frac{4}{t}$ at $t = -2, 0, 2, 4$, marking invalid inputs.

Parametric Tables and Curve Tracing

Full Worked Solutions - excerpt

Worked example

Problem. Build the (t, x, y) table for $x = |t| - 1, y = t^2 - 4$ at $t = -2, -1, 1, 3$. Identify any repeated plane point.

Answer. Rows are $(-2, 1, 0), (-1, 0, -3), (1, 0, -3), (3, 2, 5)$. The point $(0, -3)$ repeats at $t = -1$ and $t = 1$.

Explanation and check.

Both components are even in t on the paired ± 1 inputs, so those two rows have the same x and y outputs.

Worked example

Problem. For $x = (t - 1)(t + 1), y = (t - 2)(t + 1)$, find all visits to $(0, 0)$ on $-2 \le t \le 3$.

Answer. Only t = -1 visits $(0, 0)$.

Explanation and check.

$x = 0$ at $t = -1$ or 1, while $y = 0$ at $t = -1$ or 2. The only common root is -1.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Eliminating Parameters and Reparameterizing Curves | APPC-U04-P02

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for covered old t-values and missing branch, parametric pair and parameter domain, pair and mixed t-domain, pair and parameter domain, relation and exact missing point and two exact set differences.

8 PDFs and a README; 63 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Both pairs eliminate to the same familiar relation. Are their actual images equal?

Work on the domain $t \in \mathbb{R}$; $s \in \mathbb{R} \setminus \{0\}$.

The first parametrization is $x = t$; $y = t^2$.

The second parametrization is $x = \frac{1}{s}$; $y = \frac{1}{s^2}$.

Bank label: _____

Eliminate t from the two affine component rules.

Work on the domain all real t.

The x-coordinate satisfies $x = t - 4$.

The y-coordinate satisfies $y = 2t + 3$.

Bank label: _____

Eliminate t and keep the branch restriction.

Work on the domain $t \geq 0$.

The x-coordinate satisfies $x = t^2$.

The y-coordinate satisfies $y = t - 3$.

Bank label: _____

Eliminate t and describe the traced half of the locus with a Cartesian inequality.

Work on the domain $\frac{\pi}{3} \leq t \leq 4\frac{\pi}{3}$.

The endpoints is $((1, \frac{\sqrt{3}}{2}), (-1, -\frac{\sqrt{3}}{2}))$.

The x-coordinate satisfies $x = 2\cos t$.

The y-coordinate satisfies $y = \sin t$.

Required response

1	$\frac{\sqrt{3}}{2}$
-1	$-\frac{\sqrt{3}}{2}$

Bank label: _____

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Use $t = -s$ to reparameterize the curve without changing its point set. Give the required s-domain and compare direction.

Work on the domain $-2 \le t \le 1$.

Use the parameter substitution $t = -s$.

The original parametrization is $x = t$; $y = t^2$.

Compare the point set, orientation, and number of visits for the two descriptions.

Work on the domain $0 \le t \le 2\pi$ and $0 \le s \le 2\pi$.

The first parametrization is $x = \cos t$; $y = \sin t$.

The second parametrization is $x = \cos(2s)$; $y = \sin(2s)$.

Use $t = y$, not $t = x$, to parametrize the restricted graph. Derive $x(t)$ and verify the supplied y-range.

Work on the domain $-1 \le x \le 3$.

The observed y range is $[-2, \frac{2}{5}]$.

The relation is $y = \frac{x-1}{x+2}$.

Eliminate t and identify the algebraic-locus point that is excluded by the strict bound.

Work on the domain $0 < t \le 3$.

The x-coordinate satisfies $x = t^2$.

The y-coordinate satisfies $y = t + 1$.

Eliminate t and state how much of the locus is covered.

Work on the domain $0 \le t \le 2\pi$.

The x-coordinate satisfies $x = 4 \sin t$.

The y-coordinate satisfies $y = 4 \cos t$.

Eliminate t and distinguish the two horizontal-axis endpoint statuses.

Work on the domain $\pi < t \le 2\pi$.

The x-coordinate satisfies $x = 2 \cos t$.

The y-coordinate satisfies $y = 2 \sin t$.

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: They have the same image $y = x^2$ for $-1 \le x \le 1$. The first crosses it once from left to right; the second goes left-to-right and then right-to-left.

Eliminate t from the affine pair and state the x-domain.

Work on the domain all real t.

The x-coordinate satisfies $x = 2t - 1$.

The y-coordinate satisfies $y = t + 4$.

Your response: _____

Incoming: $x^2 + y^2 = 1$ with $x + y > 0$.

Eliminate t and retain the coordinate interval created by the asymmetric parameter bounds.

Work on the domain $-2 \le t \le 4$.

The x-coordinate satisfies $x = |t - 1|$.

The y-coordinate satisfies $y = t + 2$.

Your response: _____

Incoming: $x^2 + y^2 = 1$ with $x \ge 0$ and $y \ge 0$, including $(0, 1)$ and $(1, 0)$.

Do the two descriptions have the same image, and do they trace it in the same way?

Work on the domain $-1 \le t \le 1$ and $-\frac{\pi}{2} \le s \le 3\frac{\pi}{2}$.

The first parametrization is $x = t$; $y = t^2$.

The second parametrization is $x = \sin s$; $y = \sin^2 s$.

Your response: _____

Incoming: $x = -2$ with $-1 < y \le 8$.

Repair the student's $\pm$ choice and test the supplied extra point against the original rules.

Work on the domain $t \ge 0$.

Evaluate this student result: $y = \pm\sqrt{x} - 1$.

Check the point $(4, -3)$.

The x-coordinate satisfies $x = t^2$.

The y-coordinate satisfies $y = t - 1$.

Your response: _____

Incoming: $x = t + 2, y = \frac{1}{t}$ on $1 \le t \le 4$.

Eliminate t and identify the branch restriction inherited from $t \le 1$.

Work on the domain $t \le 1$.

The x-coordinate satisfies $x = (t - 1)^2$.

The y-coordinate satisfies $y = t + 2$.

Your response: _____

Incoming: The new pair is $x = s^3 - s, y = s^3 - s + 1$ and has the same point set. Its t-image is $[-\frac{2}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}]$; $t = 0$ occurs at $s = -1, 0, 1$, and each nonzero interior t-value is attained twice within the interval.

Eliminate t without dropping either sign of the original parameter.

Work on the domain $-2 \le t \le 2$.

The x-coordinate satisfies $x = t^2$.

The y-coordinate satisfies $y = 2t + 1$.

Your response: _____

AP Precalculus

Eliminating Parameters and Reparameterizing Curves

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Repair the student's elimination and use the supplied point to expose the extra branch.

Work on the domain $t < 0$.

The original parametrization is $x = t^2$; $y = t$.

Evaluate this student result: $x = y^2$ with both signs of y.

Check the point $(1, 1)$.

Repair the canceled expression result and use the supplied point to show why the domain matters.

Work on the domain $t \in \mathbb{R}, t \neq 1$.

Evaluate this student result: $y = x + 1$ for every real x.

Check the point $(1, 2)$.

The x-coordinate satisfies $x = t$.

The y-coordinate satisfies $y = \frac{t^2-1}{t-1}$.

Eliminating Parameters and Reparameterizing Curves

Full Worked Solutions - excerpt

Worked example

Problem. Eliminate t from $x = t + 2, y = 3t - 1$ and state the x-domain.

Answer. y = $3x - 7$ for all real x.

Explanation and check.

From $x = t + 2, t = x - 2$. Substitution into y gives $3(x - 2) - 1 = 3x - 7$.

Worked example

Problem. Introduce a parameter for $y = x^2 - 3$ restricted to $-2 \le x \le 1$, and verify coverage.

Answer. x = $t, y = t^2 - 3$ with $-2 \le t \le 1$.

Explanation and check.

Choosing the free x-coordinate as t transfers the x restriction directly to t and reaches each permitted x once.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Parametric Motion and Intersections

Parametric Motion and Intersections | APPC-U04-P03

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for all extrema with times and points, rightmost and highest positions, feasible interval, paired rules, paired model and all times and positions.

8 PDFs and a README; 149 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Parametric Motion and Intersections

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Do the robots collide during the observation window?

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 3$ seconds
Robot A	**X** t **Y** $2t$
Robot B	**X** $6 - 2t$ **Y** $8 - 2t$

Bank label: _____

AP Precalculus

Parametric Motion and Intersections

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Interpret the vehicle's location at $t = 4$ under the stated coordinate convention.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Coordinate direction convention	positive x west, positive y north
Original time range or clock definition	$0 \le t \le 5$ minutes
Time to evaluate	4
Given $x(t)$	$x = 6 - 2t$ kilometers
Given $y(t)$	$y = 1\text{-}t$ kilometers

Where is the object at the final modeled time?

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Coordinate direction convention	positive x east, positive y upward
Original time range or clock definition	$0 \le t \le 5$ seconds
Time to evaluate	5
Given $x(t)$	$x = 1 - 2t$ meters
Given $y(t)$	$y = (t - 3)^2$ meters

Write the position model for the four-second interval.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 4$ seconds
Horizontal change	2 meters west each second
Initial horizontal coordinate	7
Vertical rule	$y = 3 + (t - 1)^2$ meters

List every ground event in the time window and the position at each one.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 5$ seconds
Given $x(t)$	$x = 2t - 3$ meters
Given $y(t)$	$y = (t - 1)(t - 4)$ meters

AP Precalculus

Parametric Motion and Intersections

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: At t = 4 seconds, at $(4, 3)$.

When is the object inside the corridor and at nonnegative height?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Conditions	$-2 \le x \le 4$ $y \ge 0$
Original time range or clock definition	$0 \le t \le 8$ seconds
Given $x(t)$	$x = t - 3$ meters
Given $y(t)$	$y = 12 - 2t$ meters

Your response: _____

Incoming: Leftmost and highest occur at t = 0, 4, both at $(0, 3)$. Rightmost and lowest occur at $t = 2$ at $(4, -1)$.

When does the object cross the x-axis?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 4$ seconds
Given $x(t)$	$x = \frac{1}{t+2}$ meters
Given $y(t)$	$y = t - 1$ meters

Your response: _____

Incoming: $t \in [0, 2] \cup [4, 5]$.

Find the position at $t = 3$ seconds.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 6$ seconds
Time to evaluate	3
Given $x(t)$	$x = t - 4$ meters
Given $y(t)$	$y = 2t + 1$ meters

Your response: _____

Incoming: At t = 1 the position is $(0, 0)$, and at the included endpoint $t = 3$ it is $(0, 2)$.

When is the object both on or right of the y-axis and at nonnegative height?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Conditions	$x \ge 0$ $y \ge 0$
Original time range or clock definition	$0 \le t \le 6$ seconds
Given $x(t)$	$x = t^2 - 1$ meters
Given $y(t)$	$y = 5\text{-}t$ meters

Your response: _____

AP Precalculus

Parametric Motion and Intersections

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Repair the student's claim using the two supplied times.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Comparison times	2 3
Original time range or clock definition	$0 \le t \le 6$ seconds
Student claim to examine	At $t = 2$ the cart is moving left because $x(2) < 0$.
Given $x(t)$	$x = -5 + t$ meters
Given $y(t)$	$y = 2$ meters

A student claims the motion never reaches the y-axis during $0 \le t \le 3$ because the event would occur only after the interior of the interval. Identify the error, then give the correct event time and position.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 3$ seconds
Event	y-axis means $x = 0$
Given $x(t)$	$x = 3\text{-}t$ meters
Given $y(t)$	$y = t^2 + 1$ meters

Parametric Motion and Intersections

Full Worked Solutions - excerpt

Worked example

Problem. When does the object cross the y-axis?

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 4$ seconds
Given $x(t)$	$x = 3t - 6$ meters
Given $y(t)$	$y = t^2$ meters

Answer. At t = 2 seconds, at $(0, 4)$ meters.

Explanation and check.

Set $x = 0$ to get $t = 2$, then evaluate y at that same time.

Worked example

Problem. List every ground event in the time window and the position at each one.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original time range or clock definition	$0 \le t \le 5$ seconds
Given $x(t)$	$x = 2t - 3$ meters
Given $y(t)$	$y = (t - 1)(t - 4)$ meters

Answer. At t = 1 the position is $(-1, 0)$, and at $t = 4$ it is $(5, 0)$.

Explanation and check.

The height factors vanish at $t = 1$ and $t = 4$; the horizontal coordinate must be evaluated separately at each event time.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Average Rates and Speed in Parametric Motion

Average Rates and Speed in Parametric Motion | APPC-U04-P04

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for classification and slope, classification, slope, and orientation, slope status, slope status and direction, two ordered pairs and two rate pairs and scale factor.

8 PDFs and a README; 141 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Average Rates and Speed in Parametric Motion

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Find the average horizontal and vertical rates over the interval.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

t	x	y
1	2	−1
3	8	5

Condition or evidence	Supplied value
Original interval or clock definition	$1 \le t \le 3$ seconds
Units	meters

Bank label: _____

AP Precalculus

Average Rates and Speed in Parametric Motion

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Find average speed and average-velocity magnitude.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

t	x	y
0	0	0
2	8	0
5	3	0

Condition or evidence	Supplied value
Original interval or clock definition	$0 \le t \le 5$ seconds
Segments	straight between rows
Units	meters

Find the endpoint after the stated interval.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given component averages	-4 5
Original interval or clock definition	three-parameter-unit interval
Start	8 -1

Compute both exact average coordinate rates.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original interval or clock definition	$3 \le t \le 8$ seconds
X rule	$x = \sqrt{t+1}$ meters
Y rule	$y = \frac{1}{t}$ meters

Determine the endpoint-secant slope from the exact rates.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given average x-rate	$\frac{1}{3}$
Given average y-rate	0
Original interval or clock definition	common nonzero interval

AP Precalculus

Average Rates and Speed in Parametric Motion

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Average speed 2 meters per second; average-velocity magnitude $\frac{1}{2}$ meter per second.

Reconstruct the initial position that is compatible with the endpoint and both average rates.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Given component averages	-2 3
Original interval or clock definition	four-second observation window
End	5 8

Your response: _____

AP Precalculus

Average Rates and Speed in Parametric Motion

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Diagnose and repair the student's calculation.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Original interval or clock definition	$2 \le t \le 5$ seconds
End point	7 11
Start point	1 2
Student claim to examine	The average vertical rate is $\frac{\Delta y}{\Delta x} = \frac{9}{6} = \frac{3}{2}$ meters per second.

A student claims: "The endpoint displacement is right and down, so the object moves right and down throughout the sampled interval." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Use all rows, not just the endpoints, to describe net direction and any observed reversals.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

t	x	y
0	−2	6
1	−5	4
3	−1	4
4	2	1

Condition or evidence	Supplied value
Original interval or clock definition	$0 \le t \le 4$ seconds
Student claim to examine	The endpoint displacement is right and down, so the object moves right and down throughout the sampled interval.

Average Rates and Speed in Parametric Motion

Full Worked Solutions - excerpt

Worked example

Problem. Determine the endpoint-secant slope from the exact rates.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given average x-rate	$\frac{1}{3}$
Given average y-rate	0
Original interval or clock definition	common nonzero interval

Answer. 0

Explanation and check.

The nonzero horizontal rate permits division, and 0 divided by $\frac{1}{3}$ is 0.

Worked example

Problem. Determine the two finite coordinate rates across the entire parameter interval.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original interval or clock definition	$1 \le t \le 4$
X	$3t - 2$
Y	$t^2 - 5$

Answer. $(3, 5)$

Explanation and check.

The endpoints change from $(1, -4)$ to $(10, 11)$, giving changes 9 and 15 over an interval of length 3.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Parametric Equations of Lines

Parametric Equations of Lines | APPC-U04-P05

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for domain, interval notation, closed interval, interval, inequality and sufficiency statement and family.

8 PDFs and a README; 125 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Parametric Equations of Lines

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use a normal direction of the reference line to parametrize the perpendicular line through the point. point: $[3, -2]$; reference line: $x - 2y = 7$; required relation: perpendicular.

Bank label: _____

AP Precalculus

Parametric Equations of Lines

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Write parametric equations for the line through the point with the given direction. direction: $[3, 4]$; point: $[2, -1]$.	State the parameter interval that produces the full supporting line.
Determine whether changing both the anchor and direction scale changed the geometric line. domains: t and s real; first: $(x, y) = (-2 + t, 4 + 3t)$; second: $(x, y) = (1 + 2s, 13 + 6s)$.	Use the given direction angle and scale to parametrize the full line from the marked point. coordinate speed: 2; direction angle degrees: 150; point: $[1, 1]$.

AP Precalculus

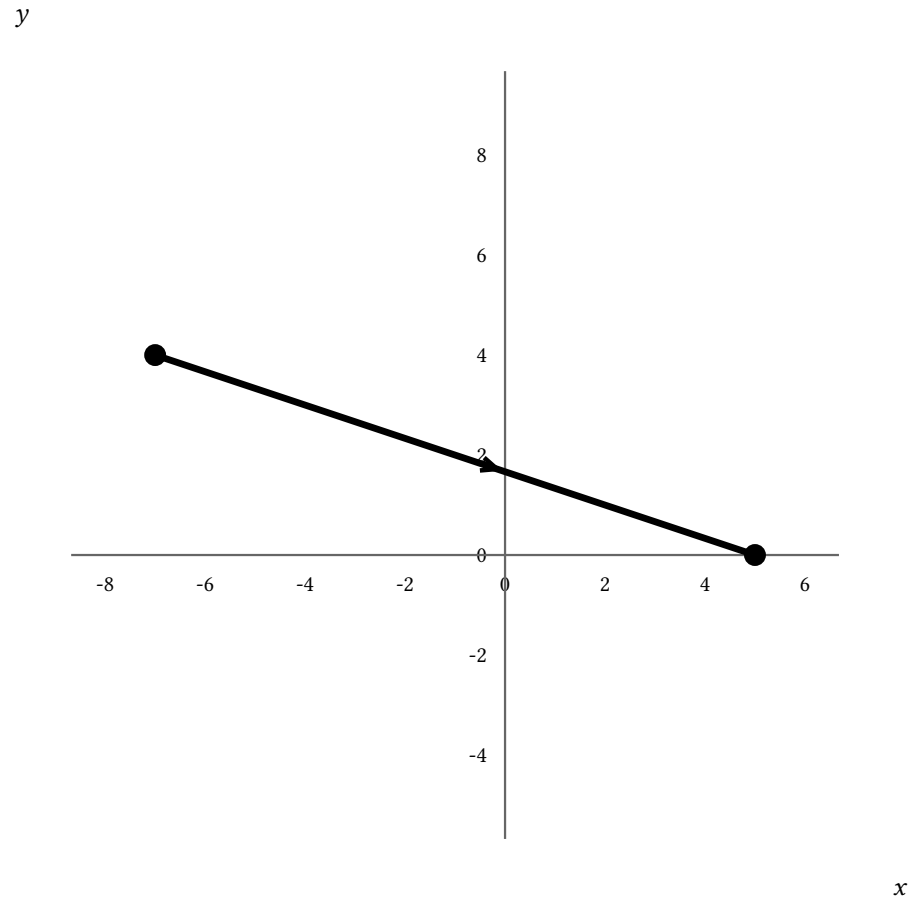
Parametric Equations of Lines

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: a = 2 and $b = 4$

Choose the t-values that trace exactly the highlighted closed segment. endpoint membership: both included; endpoint parameters: $[-2, 2]$; map: $x = 3t - 1, y = 2\text{-}t$; domain: to be selected.



Your response: _____

AP Precalculus

Parametric Equations of Lines

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The student interchanged point coordinates and direction components. Repair the parametrization. required direction: $[3, 1]$; required point: $[4, -2]$; student map: $x = 3 + 4t, y = 1 - 2t$.

The rates are right but the clock origin is not. Repair the student's timed segment. required: $(1, 4)$ at $t = 2$ and $(7, 1)$ at $t = 5$; student map: $x = 1 + 2t, y = 4 - t, 0 \le t \le 3$.

Parametric Equations of Lines

Full Worked Solutions - excerpt

Worked example

Problem. Find a parameter-free equation for the line image. map: $x = -3 + 2t, y = 5 + 5t$; domain: t real.

Answer. $5x - 2y = -25$

Explanation and check.

The combination $5(-3 + 2t) - 2(5 + 5t)$ is constantly -25.

Worked example

Problem. Give an affine parametrization of this horizontal line. equation: $y = -7$; domain: full line.

Answer. x = $t, y = -7$, with t real.

Explanation and check.

Let x vary freely while y remains equal to -7.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Parametric Circles

Parametric Circles | APPC-U04-P06

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for center, radius, start, orientation, exact set-builder list, interval, motion comparison and times, set, direction, parameter-rate comparison and locus, start, period, orientation.

8 PDFs and a README; 139 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Parametric Circles

Name: _____ **Date:** _____ **Class:** _____

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Read the geometric and traversal features from the paired rule.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$3 + 4\cos(t)$
Given $y(t)$	$-2 + 4\sin(t)$

Bank label: _____

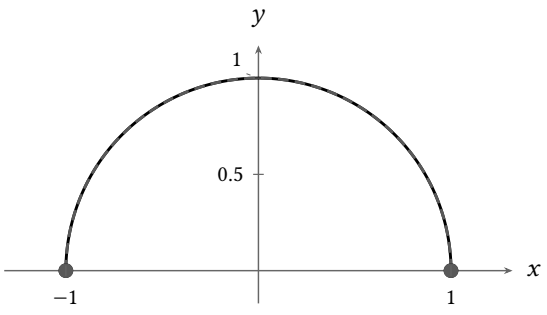
Model the ninety-second clockwise revolution from the delayed leftmost start.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given center	1 2
Original parameter range or clock definition	time t in seconds
Given direction	clockwise
Period seconds	90
Given radius	3
Start point	−2 2
Start time	30

Bank label: _____

Compare the two restricted maps as arc sets and as directed traversals.



Condition or evidence	Supplied value
Original parameter range or clock definition	$0 \le t \le \pi$ and $-\pi \le s \le 0$
First motion rule	$(\cos t, \sin t)$
Second motion rule	$(\cos(-s), \sin(-s))$

Bank label: _____

Account for the doubled angle when choosing the lower half-circle parameter window.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Given arc	lower semicircle
Original parameter range or clock definition	to be selected
Given ending position	1 0
Given endpoint membership	both included
Given direction	counterclockwise
Given coordinate rule	$x = \cos(2t), y = \sin(2t)$
Given starting position	−1 0

Bank label: _____

AP Precalculus

Parametric Circles

Name: _____ **Date:** _____ **Class:** _____

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Read the circle geometry and zero-time position from the rule.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$2 + 3\cos(t)$
Given $y(t)$	$4 + 3\sin(t)$

Construct the clockwise motion from the $3 - 4 - 5$ radius position without using a decimal phase.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Given center	2 -3
Original parameter range or clock definition	t real
Given direction	clockwise
Given radius	5
Given starting position	5 1

AP Precalculus

Parametric Circles

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Incoming: x = 3 sin(t), $y = 2 - 3\cos(t)$.

Construct a rule that begins at the top and initially moves right.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Given center	−3 2
Original parameter range or clock definition	t real
Given direction	clockwise
Given radius	4
Given starting position	−3 6

Your response: _____

Incoming: x = −4-cos(t), $y = \sin(t)$.

Can any increasing parameter interval satisfy the endpoint and sweep requirements simultaneously?

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Original parameter range or clock definition	to be selected
Given ending position	1 0
Required sweep	exactly one-half revolution
Given coordinate rule	$x = \cos(t), y = \sin(t)$
Given starting position	1 0

Your response: _____

AP Precalculus

Parametric Circles

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The radius, phase, and orientation are correct, but one center coordinate is not. Repair it.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Student rule to examine	$x = 1 + 3\cos(t), y = 2 + 3\sin(t)$
Given target requirements	**Given center** $1, -2$ **Given direction** counterclockwise **Given radius** 3 **Given starting position** $4, -2$

Parametric Circles

Full Worked Solutions - excerpt

Worked example

Problem. Read the basic circle geometry from the constants and amplitudes.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$-2 + 6\cos(t)$
Given $y(t)$	$4 + 6\sin(t)$

Answer. Center $(-2, 4)$, radius 6.

Explanation and check.

The coordinate midlines are -2 and 4, and both components have amplitude 6.

Worked example

Problem. Identify the circle's center and radius from the paired rule.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Original parameter range or clock definition	t real
Given $x(t)$	$5 + 2\cos(t)$
Given $y(t)$	$1 + 2\sin(t)$

Answer. Center $(5, 1)$ and radius 2.

Explanation and check.

The additive constants locate the center, and the equal sinusoidal amplitudes give the radius.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Implicit Relations and Curves

Implicit Relations and Curves | APPC-U04-P07

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for consistency judgment, possible entries, member set, invalid row, inequality and rule and restriction, rule, y-domain.

8 PDFs and a README; 104 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Implicit Relations and Curves

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Exactly one row misses the relation. Which one?

Given: domain: x, y real; relation: $(x-2)(y+1)=6$; rows: $(5,1)$; $(-1,-3)$; $(4,2)$; $(3,4)$

Required response

x	y
5	1
-1	-3
4	2
3	4

Bank label: _____

Identify every coordinate pair that satisfies the scaled relation.

Given: domain: x, y real; relation: $\frac{x^2}{4}+\frac{y^2}{9}=1$; rows: $(0,3)$; $(2,0)$; $(1,2)$; $(\sqrt{2}, 3\frac{\sqrt{2}}{2})$

Required response

x	y
0	3
2	0
1	2
$\sqrt{2}$	$3\frac{\sqrt{2}}{2}$

Bank label: _____

How many rows lie on the relation, and which are they?

Given: domain: x, y real; relation: $2x-y=5$; rows: $(3,1)$; $(\frac{7}{2}, 2)$; $(\frac{1}{2}, -4)$; $(-1,-7)$

Required response

x	y
3	1
$\frac{7}{2}$	2
$\frac{1}{2}$	-4
-1	-7

Bank label: _____

Mark the valid rows, then explain what they show about y-values at $x=10$.

Given: domain: x, y real; relation: $y^2=x-1$; rows: (x: 10; y: 3); (x: 10; y: -3); (x: 10; y: 0)

Required response

x	y
10	3
10	-3
10	0

Bank label: _____

AP Precalculus

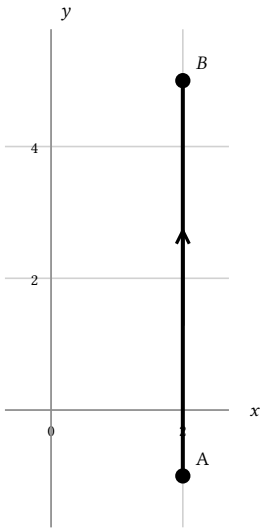
Implicit Relations and Curves

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Describe both coordinate changes along the arrow.

Given graph



A: $(2, -1)$

B: $(2, 5)$

AP Precalculus

Implicit Relations and Curves

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Write every real y-branch and state the x-values for which it exists.

Given: domain: x, y real; isolated variable: y; relation: $y^2 = 4\text{-}x$

Your response: _____

Incoming: $y = \sqrt{4 - x}$ or $y = -\sqrt{4 - x}$, with $x \leq 4$.

Express the relation as a rule with x as input.

Given: domain: x, y real; isolated variable: y; relation: $2x + 3y = 12$

Your response: _____

Incoming: $y = 4 - (\frac{2}{3})x$ for every real x.

List every relation point with the prescribed first coordinate.

Given: domain: x, y real; input x: 3; relation: $y^2 = 12\text{-}x$

Your response: _____

Incoming: $(3, 3)$ and $(3, -3)$.

Determine whether the relation has any real point above this x-value.

Given: domain: x, y real; input x: 3; relation: $(y - 1)^2 = x - 4$

Your response: _____

Incoming: It has none.

Solve for x, keeping every real branch and the output-variable domain.

Given: domain: x, y real; isolated variable: x; relation: $(x - 2)^2 = y + 1$

Your response: _____

Incoming: $x = 2+\sqrt{y + 1}$ or $x = 2-\sqrt{y + 1}$, for $y \geq -1$.

Make y explicit and state the excluded vertical line.

Given: domain: x, y real; isolated variable: y; relation: $(x + 3)(y - 2) = 4$

Your response: _____

AP Precalculus

Implicit Relations and Curves

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Which proposed points belong to the relation?

Given: candidates: $(5, 3)$; $(5, -3)$; $(5, 1)$; domain: x, y real; relation: $y^2 = x + 4$

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

x	y
5	3
5	−3
5	1

A student claims: "All four listed rows satisfy $x^2 - y = 3$." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: Three rows satisfy the relation. Identify the one that does not.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

x	y
−1	−2
0	−3
2	1
3	5

Implicit Relations and Curves

Full Worked Solutions - excerpt

Worked example

Problem. Determine whether the relation has any real point above this x-value.

Given: domain: x, y real; input $x: \ 3$; relation: $(y-1)^2 = x - 4$

Answer. It has none.

Explanation and check.

Substitution would make the square $(y-1)^2$ equal -1, impossible over the reals.

Worked example

Problem. Give a complete explicit description of the relation.

Given: domain: x, y real; relation: $y^2 = (x-1)^2$

Answer. y = $x - 1$ or $y = 1 - x$, for every real x.

Explanation and check.

Equality of the squares means $y = \pm(x-1)$; the two lines meet at $(1, 0)$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Conic Sections from Equations and Data

Conic Sections from Equations and Data | APPC-U04-P08

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for axis, direction, x-intercepts, vertex, axis, direction, sample point, exact points, normalized form, equation or contradiction and equation or impossibility.

8 PDFs and a README; 158 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Conic Sections from Equations and Data

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the two labeled rows to determine the vertex-form coefficient.

Required response

role	x	y
vertex	1	−2
curve point	3	6

Bank label: _____

Normalize the equation and determine whether any real point remains. domain: x, y real; equation: $x^2 + y^2 - 4x + 2y + 9 = 0$.

Bank label: _____

AP Precalculus

Conic Sections from Equations and Data

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

List the graph-defining features of the hyperbola. domain: x, y real; equation: $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

AP Precalculus

Conic Sections from Equations and Data

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Complete both squares and normalize the constant to 1. domain: x, y real; equation: $4x^2 + 9y^2 - 16x + 18y - 11 = 0$.

Your response: _____

Incoming: $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$.

Use the labeled extreme-point table to construct the ellipse.

Required response

label	x	y
right	6	0
left	-6	0
top	0	2
bottom	0	-2

Your response: _____

AP Precalculus

Conic Sections from Equations and Data

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Find the constant error in the work and repair the conic description. domain: x, y real; equation: $4x^2 - 16x + y^2 + 6y - 11 = 0$; student work: $4(x-2)^2 + (y+3)^2 = 20$. Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

A student claims: “The focus $(0, 2)$ gives $p = 1$, so the equation is $x^2 = 4y$ and $(4, 2)$ is not on the parabola.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Build the focal equation and test the proposed point. candidate point: $4, 2$; domain: vertical parabola; focus: $0, 2$; rule: $x^2 = 4py$; student claim: The focus $(0, 2)$ gives $p = 1$, so the equation is $x^2 = 4y$ and $(4, 2)$ is not on the parabola.; vertex: $0, 0$. Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

Conic Sections from Equations and Data

Full Worked Solutions - excerpt

Worked example

Problem. State the two defining geometric features. domain: x, y real; equation: $x^2 + y^2 = 81$.

Answer. Center $(0, 0)$, radius 9.

Explanation and check.

The equation is a sum of squared displacements equal to 9^2.

Worked example

Problem. Write the exact equation without converting the radius to a decimal. center: $-3, 1$; domain: circle; radius: $\sqrt{11}$.

Answer. $(x + 3)^2 + (y - 1)^2 = 11$.

Explanation and check.

Coordinate differences measure displacement from $(-3, 1)$, and the equation uses radius squared.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Parametric Conic Sections

Parametric Conic Sections | APPC-U04-P09

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for coverage and orientation, image, number of traversals, start visits, coverage, start, direction, branches and parameter pieces, parametric pair and parametric pair and domain.

8 PDFs and a README; 49 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Parametric Conic Sections

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Parametrize one counterclockwise traversal from the stated start. direction: counterclockwise; domain: $0 \le t \le 2\pi$; equation: $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1$; start: rightmost point. Bank label: _____	Parametrize only the component above the center. desired: upper branch only; domain: $-\frac{\pi}{2} < t < \frac{\pi}{2}$; equation: $\frac{(y+3)^2}{25} - \frac{(x-2)^2}{4} = 1$. Bank label: _____
Separate curve membership from traversal multiplicity. domain: $0 \le t \le 2\pi$; rules: x: $5\cos(2t)$; y: $2\sin(2t)$; target: $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Bank label: _____	Let the vertical coordinate be the free parameter. curve: $x = 3y - 5$; domain: y real. Bank label: _____
The formulas are correct but reach only one branch on the student's domain. Repair only the domain so both branches are covered. desired: both complete branches; domain: student used $-\frac{\pi}{2} < t < \frac{\pi}{2}$; implicit curve: $\frac{x^2}{9} - \frac{y^2}{4} = 1$; rules: x: $3\sec t$; y: $2\tan t$. Bank label: _____	Use a free horizontal parameter while retaining the branch endpoints. curve: $y = \sqrt{(9 - x^2)}$; domain: $y \ge 0$; x interval: $-3 \le x \le 3$. Bank label: _____
Use the supplied identity to parametrize the hyperbola. domain: $\cos t \neq 0$; equation: $x^2 - y^2 = 1$; identity: $\sec^2 t - \tan^2 t = 1$. Bank label: _____	Trace the specified lower half exactly once. desired arc: left endpoint to right endpoint through bottom; domain: $\pi \le t \le 2\pi$; equation: $\frac{x^2}{36} + \frac{(y-1)^2}{4} = 1$. Bank label: _____

AP Precalculus

Parametric Conic Sections

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Decide whether the pairs trace the same ellipse and relate their parameters. domain: $0 \le$parameter$\le 2\pi$; pair A: x: $3\cos t$; y: $2\sin t$; pair B: x: $3\cos u$; y: $-2\sin u$.	Trace only the component below the center. desired: lower branch; domain: $\frac{\pi}{2} < t < 3\frac{\pi}{2}$; equation: $\frac{(y+1)^2}{16} - \frac{(x-3)^2}{9} = 1$.
Determine whether the enlarged parameter interval changes the branch image. domain: $-\frac{\pi}{2} <$parameter$< \frac{\pi}{2}$; pair A: x: $3\sec t$; y: $2\tan t$; pair B: domain: $-\pi < u < \pi$; x: $3\sec(\frac{u}{2})$; y: $2\tan(\frac{u}{2})$.	Parametrize only the specified quarter arc with both endpoints included. desired arc: from $(7,-2)$ to $(3,1)$ in quadrant relative to center; domain: $0 \le t \le \frac{\pi}{2}$; equation: $\frac{(x-3)^2}{16} + \frac{(y+2)^2}{9} = 1$.
Use y as the free coordinate and preserve its forbidden value. curve: $x = \frac{y+2}{y-1}$; domain: $y \ne 1$.	Describe the traversal produced by this nonstandard one-period interval. domain: $\frac{\pi}{2} \le t \le 5\frac{\pi}{2}$; rules: x: $2 + 6\cos t$; y: $1 + 3\sin t$; target: $\frac{(x-2)^2}{36} + \frac{(y-1)^2}{9} = 1$.

AP Precalculus

Parametric Conic Sections

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The first covers the whole parabola; the second covers only the upper half $y \geq 0$, with each nonvertex point reached by u and $-u$.

Test the proposed assignment against the signed equation. candidate: x: $3\tan t$; y: $4\sec t$; domain: $\cos t \neq 0$; implicit curve: $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

Your response: _____

Incoming: The open arc with angles $\frac{\pi}{3} < t < 2\frac{\pi}{3}$ is omitted; both boundary points at angles $\frac{\pi}{3}$ and $2\frac{\pi}{3}$ are included.

Identify the relative quadrant and endpoint membership. domain: $\frac{\pi}{2} < t < \pi$; rules: x: $2 + 4\cos t$; y: $-1 + 3\sin t$; target: $\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1$.

Your response: _____

Incoming: x = $3 + 7\cos t, y = -2 - 4\sin t, 0 \leq t \leq \pi$.

Parametrize both branches across one trigonometric period. domain: $0 \leq t \leq 2\pi$ excluding poles; equation: $\frac{(x-1)^2}{9} - \frac{(y+4)^2}{4} = 1$.

Your response: _____

Incoming: x = $-1 + 2\cos t, y = 2 + 5\sin t, 0 \leq t \leq \frac{\pi}{2}$.

Give one rule that follows the specified right-bottom-left route without tracing the upper half. direction: clockwise; domain: half arc; end: $[-4, -2]$; implicit curve: $\frac{(x-3)^2}{49} + \frac{(y+2)^2}{16} = 1$; start: $[10, -2]$; via: $[3, -6]$.

Your response: _____

Incoming: x = $1 + 3\sec t, y = -4 + 2\tan t; 0 \leq t \leq 2\pi$ with $t \neq \frac{\pi}{2}, 3\frac{\pi}{2}$.

Place secant on the coordinate with the positive normalized square. domain: $\cos t \neq 0$; equation: $\frac{y^2}{16} - \frac{x^2}{9} = 1$; identity: $\sec^2 t - \tan^2 t = 1$.

Your response: _____

Incoming: Invalid; it gives $\tan^2 t - \sec^2 t = -1$ instead of 1.

Use the horizontal coordinate while preserving the real-power branch. curve: $y = (x-1)^{\frac{3}{2}}$; domain: $x \geq 1, y \geq 0$.

Your response: _____

AP Precalculus

Parametric Conic Sections

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Change the minimum necessary component to fit the ellipse. domain: t real; student pair: $x:\ 5\cos t$; $y:\ 5\sin t$; target: $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Fix both the semiaxis error and the excessive trace domain. domain: $0\le t\le 2\pi$; student domain: $0\le t\le 2\pi$; student pair: $x:\ 1+4\cos t$; $y:\ -2+2\sin t$; target: $\frac{(x-1)^2}{25}+\frac{(y+2)^2}{4}=1$, upper half only.

Decide whether the proposed pair remains on the ellipse for every parameter. candidate: $x:\ 3\cos t$; $y:\ 3\sin t$; domain: t real; implicit curve: $\frac{x^2}{9}+\frac{y^2}{4}=1$.

The formulas lie on the ellipse, but the student's interval selects the wrong half. Change only the interval. desired: lower arc from left vertex to right vertex; domain: student used $0\le t\le \pi$; implicit curve: $\frac{(x-1)^2}{25}+\frac{(y-2)^2}{4}=1$; rules: $x:\ 1+5\cos t$; $y:\ 2+2\sin t$.

Parametric Conic Sections

Full Worked Solutions - excerpt

Worked example

Problem. Use a direct horizontal free parameter. curve: $y = -3x + 7$; domain: x real.

Answer. x = t, $y = -3t + 7$, for all real t.

Explanation and check.

Each real input is represented once and its output follows the explicit rule.

Worked example

Problem. Factor the relation and use the coordinate that keeps the formula simplest. curve: $xy + y = 4$; domain: $y \neq 0$.

Answer. x = $\frac{4}{t} - 1$, $y = t$, with $t \neq 0$.

Explanation and check.

The relation is $y(x + 1) = 4$, so choosing $y = t$ gives the stated reciprocal and excludes zero.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Vector Components and Direction

Vector Components and Direction | APPC-U04-P10

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for classification with decisive evidence, vector, components, relations with components, equal, opposite, and non-equivalent relations and components and relation.

8 PDFs and a README; 146 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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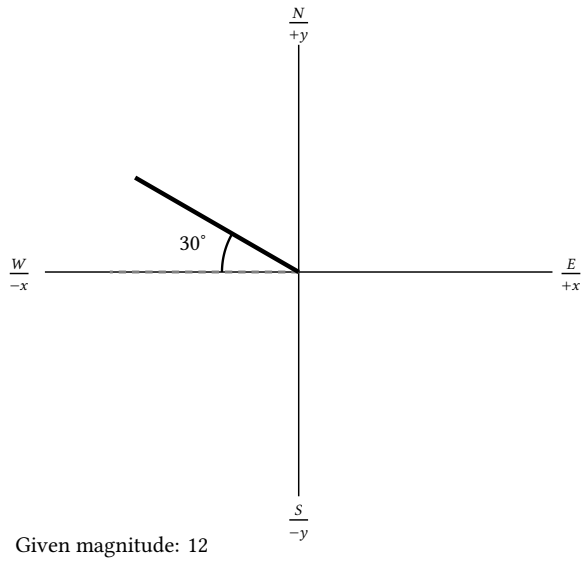
AP Precalculus

Vector Components and Direction

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Resolve the west-based heading into east and north components.



Given magnitude: 12

Bank label: _____

Find the directed component changes.

Domain: Cartesian plane; Head: $(2, 1)$; Tail: $(5, 5)$

Bank label: _____

AP Precalculus

Vector Components and Direction

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Do the two directed segments represent the same free vector even though they start at different points?

Required response

Arrow	Tail (x, y)	Head (x, y)
A	$1, 2$	$5, 5$
B	$-3, 0$	$1, 3$

State the magnitude and standard-position direction of the vector.

Components: $(0, 7)$; Domain: Cartesian plane; angles in degrees when stated

AP Precalculus

Vector Components and Direction

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Find the vector directed from the stated tail to the stated head.

Domain: Cartesian plane; angles in degrees when stated; Head: $(7, 1)$; Tail: $(2, 3)$

Your response: _____

AP Precalculus

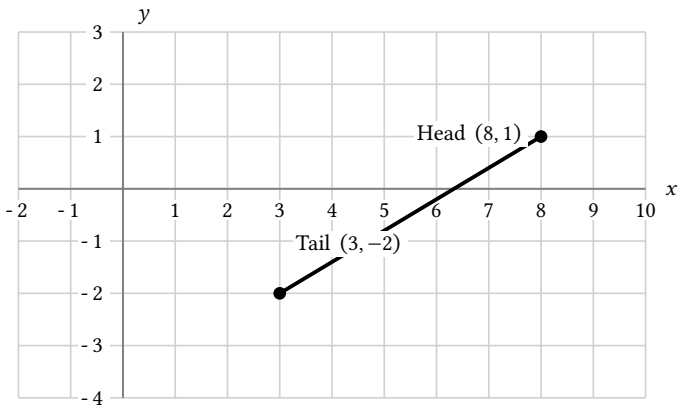
Vector Components and Direction

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Explain why the student's ordered pair is not the vector and give the correct components.

Domain: Cartesian plane; angles in degrees when stated; Head: $(8, 1)$; Student claim: The vector is $\langle 8, 1 \rangle$.; Tail: $(3, -2)$



Given arrow

Vector Components and Direction

Full Worked Solutions - excerpt

Worked example

Problem. Find the vector directed from the stated tail to the stated head.

Domain: Cartesian plane; angles in degrees when stated; Head: $(7, 1)$; Tail: $(2, 3)$

Answer. $\langle 5, -2 \rangle$

Explanation and check.

Subtracting tail coordinates from head coordinates gives horizontal change 5 and vertical change -2.

Worked example

Problem. Find the terminal point of the directed segment.

Domain: Cartesian plane; angles in degrees when stated; Tail: $(-2, 5)$; Vector: $(4, -3)$

Answer. The head is $(2, 2)$.

Explanation and check.

The head equals the tail plus the displacement, giving $(-2 + 4, 5 - 3) = (2, 2)$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Vector Operations and Magnitude

Vector Operations and Magnitude | APPC-U04-P11

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for components, components of u+v, resultant and endpoint, resultant and final head, squared norms and longer expression and longer derived vector and exact squared norms.

8 PDFs and a README; 97 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Vector Operations and Magnitude

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Combine the three displacements u, v, and w. Use the expression $u + v + w$. $u = \langle 1,\ 4 \rangle$ $v = \langle -3,\ 2 \rangle$ $w = \langle 5,\ -1 \rangle$ Bank label: _____	Multiply each component by the scalar. Use the expression $3u$. $u = \langle -2,\ 3 \rangle$ Bank label: _____
Compute the vector obtained by reversing u and multiplying its length by four. Use the expression $-4u$. $u = \langle -1,\ 2 \rangle$ Bank label: _____	Construct the unit vector preserving the direction. The given vector is $\langle -7,\ 24 \rangle$. Bank label: _____
Scale the direction to length 5 without rounding. The required magnitude is 5. Use the same direction as the given vector. The given vector is $\langle 12,\ -5 \rangle$. Bank label: _____	Construct the requested length without changing the downward direction. The required magnitude is 4. Use the same direction as the given vector. The given vector is $\langle 0,\ -9 \rangle$. Bank label: _____
Undo the subtraction to recover u. Use the equation $u - \langle 2, 7 \rangle = \langle -5, 1 \rangle$. Bank label: _____	Find the missing second summand. Use the equation $u + v =$ resultant. The resultant is $\langle 7,\ 2 \rangle$. $u = \langle -4,\ 9 \rangle$ Bank label: _____

AP Precalculus

Vector Operations and Magnitude

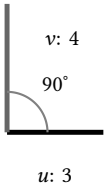
Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

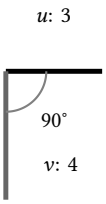
Use the right triangle formed by the two vectors to find the resultant magnitude.

The included angle, in degrees, is 90.

The vector magnitudes are $|u| = 3$ and $|v| = 4$.



One permitted placement



Reflected permitted placement

The two panels show alternative placements of the same data.

No absolute coordinate orientation is specified.

AP Precalculus

Vector Operations and Magnitude

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Vector v points in the same direction as u and has magnitude $2\sqrt{13}$. Find v.

The magnitude of v is $2\sqrt{13}$.

The relation is v has the same direction as u.

$u = \langle 2,\ -3 \rangle$

Your response: _____

Incoming: v = $\langle 4, -6 \rangle$.

Isolate and determine v from the given scaled difference.

The given linear combination equals $\langle 1,\ -14 \rangle$.

Use the equation $3u - v = \langle 1, -14 \rangle$.

$u = \langle 2,\ -3 \rangle$

Your response: _____

Incoming: v = $\langle 5, 5 \rangle$.

Given $|u| = 5, |v| = 10$, and $u \cdot v = 20$, determine the smaller angle between the vectors.

The dot product $u \cdot v$ is 20.

The magnitude of u is 5.

The magnitude of v is 10.

Your response: _____

Incoming: $\cos\theta = \frac{2}{5}$, so $\theta \approx 66.4^\circ$.

Scale the direction to length 9.

The required magnitude is 9.

Use the same direction as the given vector.

The given vector is $\langle -8,\ -15 \rangle$.

Your response: _____

Incoming: $\langle -\frac{72}{17}, -\frac{135}{17} \rangle$

Undo the vector subtraction and verify the recovered minuend.

Use the equation $u - v = \langle 4, -7 \rangle$.

$v = \langle 2,\ 5 \rangle$

Your response: _____

Incoming: u = $\langle 6, -2 \rangle$; subtracting v returns $\langle 4, -7 \rangle$.

Isolate the unknown vector and check the original equation.

Use the equation $2u - v = w$.

$v = \langle 1,\ 5 \rangle$

$w = \langle 7,\ -3 \rangle$

Your response: _____

AP Precalculus

Vector Operations and Magnitude

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Determine whether the scalar relationship gives angle 0° or 180°.

$u = \langle -2,\ 5 \rangle$

$v = \langle -6,\ 15 \rangle$

Vector Operations and Magnitude

Full Worked Solutions - excerpt

Worked example

Problem. Solve the two vector equations simultaneously for u and v.

Use these conditions: $u + v = \langle 1, 9 \rangle,\ 2u - v = \langle 8, 3 \rangle$.

Answer. u = $\langle 3, 4 \rangle$ and $v = \langle -2, 5 \rangle$.

Explanation and check.

Adding the equations eliminates v and gives $3u = \langle 9, 12 \rangle$; substitution into the sum then determines v.

Worked example

Problem. Construct the unit vector preserving the direction.

The given vector is $\langle -7,\ 24 \rangle$.

Answer. $\langle -\frac{7}{25}, \frac{24}{25} \rangle$

Explanation and check.

The component pair has norm 25, so divide both entries by 25.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Velocity Vector Applications

Velocity Vector Applications | APPC-U04-P12

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for elapsed time and endpoint, required component and impossibility proof, final coordinates, exact coordinates, displacement and exact path length and net displacement and exact distance.

8 PDFs and a README; 123 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Velocity Vector Applications

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An aircraft flies due east at $120\frac{km}{h}$ relative to the air while $a50\frac{km}{h}$ wind blows due north. Find its ground velocity, speed, and course.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Air velocity ($\frac{km}{h}$)	$\langle 120, 0\rangle$
Coordinate convention	east-north
Given context	aircraft motion
Wind velocity ($\frac{km}{h}$)	$\langle 0, 50\rangle$

Bank label: _____

A boat covers displacement $\langle 24, 8\rangle$ km in 2 hours while its water-relative velocity is $\langle 10, 1\rangle$ $\frac{km}{h}$. Assuming a constant current, find it.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Condition or evidence	Supplied value
Coordinate convention	east-north
Boat velocity relative water ($\frac{km}{h}$)	$\langle 10, 1\rangle$
Given context	boat and current
Duration (hours)	2
Observed displacement (km)	$\langle 24, 8\rangle$

Bank label: _____

AP Precalculus

Velocity Vector Applications

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

A plane flies at airspeed $200\,\frac{km}{h}$ on heading $N30^\circ E$ while the wind is $\langle -40, 20\rangle\ \frac{km}{h}$. Find the ground velocity and approximate ground course.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Airspeed ($\frac{km}{h}$)	200
Coordinate convention	east-north
Given context	aircraft navigation
Heading	$N30^\circ E$
Wind ($\frac{km}{h}$)	$\langle -40, 20\rangle$

A person walks at $\langle 1, 2\rangle\ \frac{m}{s}$ relative to a walkway moving at $\langle 3, 0\rangle\ \frac{m}{s}$ relative to the floor. Describe the person's ground motion.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Condition or evidence	Supplied value
Coordinate convention	forward-left
Given context	person on moving walkway
Person velocity relative walkway ($\frac{m}{s}$)	$\langle 1, 2\rangle$
Walkway velocity relative ground ($\frac{m}{s}$)	$\langle 3, 0\rangle$

Infer the current independently from each observation row and decide whether the constant-current model is supported.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

interval	duration h	displacement km
1	2	$\langle 12, 6\rangle$
2	0.5	$\langle 3, 1.5\rangle$

Condition or evidence	Supplied value
Coordinate convention	east-north
Boat velocity relative water ($\frac{km}{h}$)	$\langle 8, 0\rangle$
Given context	repeated boat observations

Convert every duration to seconds before using the velocity table. Then find displacement and distance.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

stage	duration	velocity
1	30 s	$\langle 2, 0\rangle$
2	1 min	$\langle 0, 1\rangle$
3	0.5 min	$\langle -1, 0\rangle$

Condition or evidence	Supplied value
Given context	stages with mixed time units
Velocity unit	$\frac{m}{s}$

AP Precalculus

Velocity Vector Applications

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The stage displacements sum to $\langle 0, 4\rangle$ *km*, whose magnitude is 4 km; the total distance is 20 km.

Find the current that changes water-relative velocity $\langle 10, 0\rangle$ $\frac{m}{s}$ into ground velocity $\langle 6, 8\rangle$ $\frac{m}{s}$.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Condition or evidence	Supplied value
Coordinate convention	east-north
Boat velocity relative water ($\frac{m}{s}$)	$\langle 10, 0\rangle$
Given context	boat velocity frames
Ground velocity ($\frac{m}{s}$)	$\langle 6, 8\rangle$

Your response: _____

AP Precalculus

Velocity Vector Applications

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student adds the two perpendicular speeds as scalars. Explain the error and correct the ground-speed calculation.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Coordinate convention	east-north
Given context	drone in wind
Drone air velocity ($\frac{m}{s}$)	$\langle 40, 0\rangle$
Student claim to examine	The ground speed is $40 + 30 = 70\frac{m}{s}$.
Wind ($\frac{m}{s}$)	$\langle 0, 30\rangle$

Infer the current from the two velocities and decide whether it can be due east as claimed.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

Condition or evidence	Supplied value
Coordinate convention	east-north
Boat velocity relative water ($\frac{m}{s}$)	$\langle 3, 4\rangle$
Claimed current direction to test	due east
Given context	boat observation with directional assumption
Ground velocity ($\frac{m}{s}$)	$\langle 8, 5\rangle$

Velocity Vector Applications

Full Worked Solutions - excerpt

Worked example

Problem. The velocity does not change between the two table rows. Find displacement and distance after all 5 minutes.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Duration (min)	Given velocity (m/min)
2	$\langle 3, 0 \rangle$
3	$\langle 3, 0 \rangle$

Condition or evidence	Supplied value
Given context	collinear stages
Units	**Time** min **Given velocity** m/min

Answer. The displacement is $\langle\, 15, 0 \,\rangle\ m$ and the distance is 15 m.

Explanation and check.

The durations combine because the velocity is unchanged, so $5\langle\, 3, 0 \,\rangle$ gives the entire trip.

Worked example

Problem. Find the one-vector displacement and the scalar path length for the three stages.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Duration (s)	Given velocity (m/s)
2	$\langle 4, 0 \rangle$
1	$\langle 0, 3 \rangle$
1	$\langle -4, 0 \rangle$

Condition or evidence	Supplied value
Given context	three-sided path
Units	**Time** s **Given velocity** $\frac{m}{s}$

Answer. The net displacement is $\langle\, 4, 3 \,\rangle\ m$, and the distance is 15 m.

Explanation and check.

The displacements $\langle\, 8, 0 \,\rangle$, $\langle\, 0, 3 \,\rangle$, and $\langle\, -4, 0 \,\rangle$ add to the net; their lengths add to 15.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Vector-Valued Functions: Operations and Motion

Vector-Valued Functions: Operations and Motion | APPC-U04-P13

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for direction intervals, axis event, and stop, basis form and vector value, i-j notation, ordered-pair rule, vector table and insufficiency statement and formal component sum and interpretation verdict.

8 PDFs and a README; 65 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Use the supplied velocity value to describe the motion at $t = 3s$. Use $v(3) = \langle -2, 0 \rangle$. Velocity components are measured in meters per second.

Bank label: _____

Determine the discrete domain of $3u - 2w$ and evaluate the function everywhere it is defined. The table with inputs $0, 1, 2$ gives $u(t)$. The table with inputs $1, 2, 3$ gives $w(t)$.

Required response

0	$1, 0$
1	$2, 3$
2	$-1, 4$

1	$5, -2$
2	$0, 6$
3	$7, 1$

Bank label: _____

AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Describe the velocity direction on the interval and identify every stationary time. Use $v(t) = \langle t - 2, (t-2)(t+1) \rangle \frac{m}{s}$. Use the time restriction $0 \le t \le 4$.	Combine the radical interval with the denominator exclusion, then evaluate at the left endpoint. Use $r(t) = \langle \sqrt{(}9\text{-}t^2), \frac{1}{t-1} \rangle$.
Find when the object is at the origin and compare its distances from the origin at $t = 0$ and $t = 2$. Use $p(t) = \langle t - 1, 2t - 2 \rangle$. Position components are measured in meters. Use the time restriction $0 \le t \le 2$.	Derive the speed function, then find the speeds at $t = -1$ and $t = 1$. Use $v(t) = \langle t + 1, -(t+1) \rangle$. Use the times $t = -1, 1$.
Form the resultant velocity and describe its direction for every real t. Use $v_1(t) = \langle t, t^2 \rangle$. Use $v_2(t) = \langle -t, 1 \rangle$. The stated time domain is all real t. Velocity components are measured in meters per second.	Write $d(t)$ in angle-bracket notation and find $d(-1)$. Use $d(t) = -(t-1)i + (2-3t)j$. The specified input is $t = -1$.

AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Describe the motion implied by this supplied velocity function. Use $v(t) = \langle 0, 5 \rangle$. The stated time domain is all t. Velocity components are measured in meters per second.

Your response: _____

Incoming: The object always moves straight upward at $5\,\frac{m}{s}$, with no horizontal motion.

Use both the time restriction and radical condition to find the domain, then evaluate the boundary input. Use $r(t) = \langle \sqrt{(5-t)}, t + 2 \rangle$. Use the time restriction $t \geq 0$.

Your response: _____

Incoming: The domain is $[0, 5]$, and $r(5) = \langle 0, 7 \rangle$.

Build $2u\text{-}v$ componentwise and state every allowed real input. Use $u(t) = \langle \sqrt{t}, t \rangle, t \geq 0$. Use $v(t) = \langle t, \frac{1}{t-1} \rangle, t \neq 1$.

Your response: _____

Incoming: $(2u\text{-}v)(t) = \langle 2\sqrt{t}\text{-}t, 2t - \frac{1}{t-1} \rangle$ on $[0, 1) \cup (1, \infty)$.

Express p in i-j notation and evaluate the same vector function at $t = \frac{\pi}{2}$. Use $p(t) = \langle \cos t, \sin t \rangle$. The specified input is $t = \frac{\pi}{2}$.

Your response: _____

Incoming: $p(t) = (\cos t)i + (\sin t)j$, and $p(\frac{\pi}{2}) = j = \langle 0, 1 \rangle$.

Find the speed function and determine its least value. Use $v(t) = \langle t + 1, t - 1 \rangle$. The stated time domain is all real t. Velocity components are measured in meters per second.

Your response: _____

Incoming: $s(t) = \sqrt{(t+1)^2 + (t-1)^2} = \sqrt{2t^2 + 2}$; its minimum is $\sqrt{2}\,\frac{m}{s}$ at $t = 0$.

Combine the supplied component functions into a single basis-vector rule. Use $x(t) = \sin t$. Use $y(t) = \cos t$.

Your response: _____

AP Precalculus

Vector-Valued Functions: Operations and Motion

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Explain what the position value actually tells and why the student's motion claim is unsupported. Use $p(4) = \langle -3, 4 \rangle$ meters. The student claims: The object is moving left and up at $t = 4$.

Vector-Valued Functions: Operations and Motion

Full Worked Solutions - excerpt

Worked example

Problem. Combine the supplied component functions into a single basis-vector rule. Use $x(t) = \sin t$. Use $y(t) = \cos t$.

Answer. $r(t) = (\sin t)i + (\cos t)j$.

Explanation and check.

Both scalar functions use the same input and become the two basis coefficients.

Worked example

Problem. Rewrite $q(t)$ in ordered-pair notation. Use $q(t) = (2 - t)i + (3t + 1)j$.

Answer. $q(t) = \langle 2 - t, 3t + 1 \rangle$.

Explanation and check.

The coefficient of i is the first component and the coefficient of j is the second.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Matrix Operations and Multiplication

Matrix Operations and Multiplication | APPC-U04-P14

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for constraint comparison and verdict, numeric and label judgments, constraint and uniqueness conclusion, compatibility verdict and repair, valid and invalid labels with reason and intermediate and final sizes.

8 PDFs and a README; 66 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

State the dimensions of A and give a_{23}.

matrix =

4	0	7
−2	5	−1

Bank label: _____

Compute Ax and confirm the result's dimensions. A: see the labelled matrix A; domain: matrix-vector multiplication; x: $[4, 5]$.

A =

2	−1
0	3

Bank label: _____

Recover x and y from both product entries and explain why neither condition is redundant. B: see the labelled matrix B; domain: row vector times invertible-looking coefficient matrix; product: $[5, 1]$; row: $[x, y]$.

B =

1	2
1	−1

Bank label: _____

Use a row-column product to find the meal's total calories. calories per serving: $[80, 120, 50]$; domain: meal calorie total; foods: [fruit, grain, milk]; servings: $[2, 1, 3]$.

Bank label: _____

AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Compute AB and BA and identify the entry that disproves commutativity. A: see the labelled matrix A; *B*: see the labelled matrix *B*; domain: diagonal matrix and shear.

A =

2	0
0	1

B =

1	3
0	1

Determine all possible x and y. Explain why the two output entries do not give two independent constraints. *B*: see the labelled matrix *B*; domain: row vector product with repeated columns; product: $[5, 5]$; row: $[x, y]$.

B =

1	1
1	1

Compute AB and state its dimensions. A: see the labelled matrix A; *B*: see the labelled matrix *B*; domain: row by rectangular matrix.

A =

2	-1

B =

3	0	4
5	2	-2

Is uv defined, and what size matrix results? domain: column times row vectors; u size: 4×1; v size: 1×2.

AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $(AB)_{14}$=9.

Find x so that the indicated row-column product is 6. column: $[3, 1, 4]$; domain: one selected dot product; product entry: 6; row: $[2, x, -1]$.

Your response: _____

Incoming: The product is $\begin{pmatrix} 8 & 9 & 4 \\ 6 & 13 & 8 \end{pmatrix}$. Rows are Site A and Site B, columns are wood, metal, and fabric, and the shared sum runs over basic and advanced kits.

Multiply the machine-hours matrix by the energy-rate column. Interpret both outputs and explain the unit cancellation. domain: shift energy use; kilowatt hours per hour: $[6, 2, 5]$; machine columns: [press, lathe, dryer]; shift rows: Evening: $[5, 0, 3]$; Morning: $[2, 4, 1]$.

Your response: _____

Incoming: BA = CA=$\begin{pmatrix} 2 & 0 \\ 4 & 0 \end{pmatrix}$, but $B \neq C$. Right cancellation fails because multiplying by A erases the second column, exactly where B and C differ.

Find $(AB)_{23}$ by recognizing the selector row. A row 2: $[0, 1, 0]$; B column 3: $[-4, 7, 2]$; domain: standard basis row selection; target: $(AB)_{2}3$.

Your response: _____

Incoming: AA^{T} is 2×2 and $A^{\mathsf{T}}A$ is 3×3, so they cannot be equal as matrices.

State the dimensions of c and give c_{31}.

matrix =

-4
7
2

Your response: _____

Incoming: The product is $\begin{pmatrix} 0 \end{pmatrix}$.

Multiply the unit-count matrix by the mass column and interpret the two outputs. domain: factory package mass; factory rows: East: $[5, 2]$; West: $[1, 6]$; mass per unit: $[3, 8]$; product columns: [box, crate].

Your response: _____

Incoming: Morning uses 25 kWh and Evening uses 45 kWh. Each dot product sums machine-hours times kWh per machine-hour over the shared machine labels.

Find $(AB)_{12}$. A row 1: $[5, 0]$; B column 2: $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(AB)_{1}2$.

Your response: _____

AP Precalculus

Matrix Operations and Multiplication

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Compute both orders and use them to assess the scalar-style claim that a zero product forces one matrix factor to be zero.

A =

1	−1
1	−1

B =

1	1
1	1

The inner labels match and the numeric dimensions conform. Does quantity-in-kilograms times dollars-per-item yield a valid total cost? State a minimal unit repair.

Matrix Operations and Multiplication

Full Worked Solutions - excerpt

Worked example

Problem. Compute the row-by-column matrix product. column: $[7, -2, 5]$; domain: zero row times a column; row: $[0, 0, 0]$.

Answer. The product is $\begin{pmatrix} 0 \end{pmatrix}$.

Explanation and check.

All three row-column contributions are zero.

Worked example

Problem. Find $(\mathrm{AB})_{12}$. A row 1: $[5, 0]$; B column 2: $[-2, 3]$; domain: selected entry with one zero coordinate; target: $(\mathrm{AB})_{12}$.

Answer. $(\mathrm{AB})_{12} = -10$.

Explanation and check.

The dot product is $5(-2) + 0 \cdot 3$, so only the first contribution remains.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Determinants and Area

Determinants and Area | APPC-U04-P15

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for dot product, determinant, area, singular set and dependence explanation, singular set, determinant and conclusion, empty real solution set with reason and all parameter values and dependence.

8 PDFs and a README; 55 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Determinants and Area

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the lower-triangular structure to evaluate the determinant without an unnecessary cross-product calculation.

Required response

-3	0
4	2

Bank label: _____

Use the triangular structure to find the determinant of the matrix.

Required response

7	0
-2	-3

Bank label: _____

Evaluate the determinant.

Required response

7	0
0	1

Bank label: _____

Read the determinant from the triangular form.

Required response

1	5
0	3

Bank label: _____

Find the determinant of the coordinate-swap matrix.

Required response

0	1
1	0

Bank label: _____

Find the determinant after the row swap.

Apply this change: interchange row 1 and row 2. The original determinant is 13.

Bank label: _____

Without reconstructing the entries, find the determinant after the columns are interchanged.

Apply this change: interchange the two columns. The original determinant is -5.

Bank label: _____

Find the determinant after the stated row scaling.

Apply this change: multiply row 2 by $\frac{1}{5}$. The original determinant is 20.

Bank label: _____

AP Precalculus

Determinants and Area

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Find the signed and ordinary areas of the spanned parallelogram. The first column is $(0, 1)$. The second column is $(-3, 0)$.	Find the signed and ordinary areas of the column parallelogram. The first column is $(2, 0)$. The second column is $(0, 5)$.
Compute the signed and ordinary areas and classify the column order. The first column is $(-2, 0)$. The second column is $(0, -7)$.	The second column is a90-degree counterclockwise rotation of the first. Use that structure to determine the signed and ordinary areas of their parallelogram. The first column is $(3, -1)$. The second column is $(1, 3)$.
Find the determinant and ordinary area, then state the exact column relation. The first column is $(-1, -2)$. The second column is $(-3, -6)$.	Find the parallelogram's ordinary area and classify the ordered-column orientation. The first column is $(4, 1)$. The second column is $(1, 3)$. The vertices are $(0, 0)$, $(4, 1)$, $(1, 3)$, $(5, 4)$.

AP Precalculus

Determinants and Area

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Find every real t for which the matrix is singular and explain what happens to its columns then.

Required response

t	2
3	t

Your response: _____

Incoming: The image area is 15 square units, and orientation is preserved.

Verify that the matrices have the same determinant, then show from their columns that they do not define the same map.

Required response

2	0
0	2

1	3
0	4

Your response: _____

Incoming: The determinant is $2(\frac{3}{4}) - 1(-1) = \frac{5}{2}$. Therefore the original area is $\frac{35}{\frac{5}{2}} = 14$.

Find the image area and state whether orientation is preserved.

The determinant is $\frac{1}{4}$. The original area is 20.

Your response: _____

Incoming: It is singular for every real t because the second column is always twice the first, and the determinant $2t - 2t$ is identically zero.

Verify the common determinant and give a column-based reason the maps differ.

Required response

2	0
0	1

1	1
-1	1

Your response: _____

Incoming: The determinant is $k^2 + 6$, which is positive for every real k. Therefore there are no real singular parameter values.

Find the square's original area and its image area.

The determinant is 2. The square has side length 3.

Your response: _____

Incoming: The original area is 9 and the image area is 18 square units.

Find all real t that make the matrix singular and explain why the off-diagonal 1 does not change the condition.

Required response

t	1
0	t

Your response: _____

AP Precalculus

Determinants and Area

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Correct the area statement while preserving the sign's geometric information.

The determinant is -7. The student claims: The parallelogram area is -7 square units..

Repair the statement by identifying the actual column relationship and its area consequence.

Use the matrix $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$. The student claims: The determinant is zero, so both columns must be zero..

Repair the claim by stating what equality of determinants does guarantee and one fact showing the maps differ.

Use matrix A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix B = $\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$. The student claims: Both determinants are 4, so A and B are the same map..

Assess the student's claim using the displayed columns. Then replace it with a correct geometric statement.

The proposed first and second columns, respectively, are $(2, -3)$, $(-4, 6)$. The student claims: If $a2 \times 2$ determinant is zero, its two columns must be equal..

Determinants and Area

Full Worked Solutions - excerpt

Worked example

Problem. Give two distinct diagonal matrices with determinant 9 and verify them.

Answer. For example, $\begin{pmatrix} 1 & 0 \\ 0 & 9 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ both have determinant 9, but their diagonal scalings are different.

Explanation and check.

Choose two different factor pairs whose product is 9.

Worked example

Problem. Correct the area statement while preserving the sign's geometric information.

The determinant is -7. The student claims: The parallelogram area is -7 square units..

Answer. The ordinary area is 7 square units, and the negative determinant indicates reversed orientation.

Explanation and check.

Take an absolute value for area but not for the orientation classification.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Matrices and Linear Systems

Inverse Matrices and Linear Systems | APPC-U04-P16

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for values in example and valid general conclusion, conclusion and valid multiplication side, corrected table and verdict, valid and invalid cases, invertible and singular lists and decision and determinant evidence.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Find the inverse by undoing the two independent coordinate scalings.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

2	0
0	5

Bank label: _____

Treat the two right-hand sides as columns of one table and use the same inverse once to solve both systems.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

Coefficient matrix

2	1
1	1

Right-hand-side column 1

5
3

Right-hand-side column 2

1
−1

Supplied inverse

1	−1
−1	2

Bank label: _____

Use the bottom-right identity entry to find q.

Solve the problem, then match the entire requested response to one unused entry in the Answer Bank.

4	0
0	−2

$\frac{1}{4}$	0
0	q

Bank label: _____

Find the inverse of the three-factor expression $AB{A}^{-1}$ and verify the result without commuting A and B.

Bank label: _____

AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Use the supplied inverse rather than elimination, then substitute the result into both original equations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Coefficient matrix

3	1
1	1

Right-hand-side column

8
4

Supplied inverse

$\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{1}{2}$	$\frac{3}{2}$

Compute the exact inverse, preserving the sign of the determinant throughout the scalar division.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

1	3
2	4

Test the candidate and determine whether a row-column transposition caused the failure.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Proposed inverse

3	−2
−1	1

Given matrix

1	1
2	3

Find the inverse with a single common denominator and verify one diagonal product entry.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

4	3
1	2

AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The determinant is $k^2 + 1$, which is positive for every real k. Every matrix in the family is invertible.

Square the matrix to determine whether using the original matrix as its own inverse is valid.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	0
2	-1

1	0
2	-1

Your response: _____

START

Verify algebraically whether the proposed matrix is an inverse for every real k, rather than checking sample values.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	-1
1-k	k

k	1
$k-1$	1

Your response: _____

Incoming: The determinant is $8k - 8 = 8(k - 1)$. Only $k = 1$ is singular; $k = -2, -1, 0, 2, 3$ give invertible matrices.

Use the bottom-right identity constraint to find q.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

1	2
2	5

5	-2
-2	q

Your response: _____

Incoming: The inverse is $[[\frac{1}{3}, -\frac{5}{6}], [0, \frac{1}{2}]]$.

Use A's inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Your response: _____

AP Precalculus

Inverse Matrices and Linear Systems

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “The proposed matrix cannot be an inverse because its off-diagonal entries are nonzero.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Multiply the matrix by the proposed inverse and decide whether the proposal satisfies the defining identity relation.

Identify the stated incorrect claim or first invalid step, give the corrected result, and state a brief mathematical reason.

1	-1
-1	2

2	1
1	1

Recompute the product instead of trusting the student’s table. Decide whether the error lies in the candidate or in the verification arithmetic.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

$\frac{1}{3}$	$-\frac{1}{3}$
$\frac{1}{3}$	$\frac{2}{3}$

2	1
-1	1

1	0
1	1

Inverse Matrices and Linear Systems

Full Worked Solutions - excerpt

Worked example

Problem. Find the inverse of the three-factor expression AB A^{-1} and verify the result without commuting A and B.

Answer. The inverse is A B^{-1} A^{-1}. Multiplication gives A B A^{-1} A B^{-1} A^{-1} =I after adjacent cancellations.

Explanation and check.

Reverse all three factors and invert each one; the inverse of A inverse is A itself.

Worked example

Problem. Use A's inverse to determine what follows from BA=CA, making the cancellation on the correct side.

Answer. Multiplying both sides on the right by A^{-1} gives $B = C$.

Explanation and check.

The common factor is on the right, so its inverse must also be placed on the right to cancel it.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Matrices for Linear Transformations

Matrices for Linear Transformations | APPC-U04-P17

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for consistency, uniqueness, and two example matrices, determination and nonlinear compatible rule, determination and two compatible rules, classification and decisive test, classification and matrix and classification.

8 PDFs and a README; 60 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Matrices for Linear Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Construct the matrix and use it to map $(2, -1)$ as a check. $T(e_1) = (0,\ 2)$ $T(e_2) = (3,\ 0)$ Bank label: _____	Project the point with the displayed matrix. Use the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Vectors are column vectors. The given point is $(-3,\ 5)$. Bank label: _____
Determine the complete image of the unit disk under the diagonal projection, including exact segment endpoints. Use the matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Vectors are column vectors. Use the disk with center $(0,\ 0)$ and radius 1. Bank label: _____	Solve the two component systems to recover the matrix from these nonbasis correspondences. Assume T is linear. The input-output observations are $(1,\ 2) \mapsto (5,\ 1)$; $(2,\ -1) \mapsto (1,\ 8)$. Bank label: _____

AP Precalculus

Matrices for Linear Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the projection matrix by tracking both basis vectors onto the line $y = x$. The map acts by orthogonally project onto the line $y = x$.	Normalize each scaled-axis correspondence to obtain the two basis images. Assume T is linear. The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(3,\ 6)$.
Classify the rule using the origin test and distinguish affine from linear behavior. Use the rule $F(x, y) = (x + 2, y - 1)$.	Do the axis-only samples establish linearity of an unspecified plane map? Exhibit contrasting compatible rules. The input-output observations are $(0,\ 0)\mapsto(0,\ 0)$; $(1,\ 0)\mapsto(1,\ 0)$; $(2,\ 0)\mapsto(2,\ 0)$.
Construct the matrix and explain geometrically why its image is one-dimensional. $T(e_1) = (2,\ 1)$ $T(e_2) = (-4,\ -2)$	Map the side generators and determine the image area and orientation change. Use the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Vectors are column vectors. The two side vectors are $u = (1,\ 0)$ and $v = (0,\ 2)$.

AP Precalculus

Matrices for Linear Transformations

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Because $(1,1)+(1,-1)=(2,0)$, outputs require $(2+k,4)=(5,4)$, so $k=3$. Then $T(e1)=(\frac{5}{2},2)$, $T(e2)=(-\frac{1}{2},2)$, and the matrix is $[[\frac{5}{2},-\frac{1}{2}],[2,2]]$.

Construct the exact reflection matrix using the double-angle form.

The map acts by reflect across the line through the origin making a30-degree angle with the x-axis.

Your response: _____

Incoming: The image generators are $(2,0)$ and $(1,2)$. Their determinant has magnitude 4, so the image area is 4.

Use the two directed basis images to assemble the standard matrix.

$T(e_1)=(0,\ -2)$

$T(e_2)=(3,\ 0)$

Your response: _____

Incoming: Under a linearity assumption they determine matrix $[[2,-1],[1,3]]$. Without that assumption they do not prove the map is linear, because nonlinear rules can share the same two values.

Map both points and verify that their known scalar relation survives the transformation.

Use the matrix $A=\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$. Vectors are column vectors.

The given points are $P=(2,\ 1)$; $Q=(-6,\ -3)$.

The relation is $Q=-3P$.

Your response: _____

Incoming: Every member is linear, with matrix $[[a,b],[c,d]]$.

Normalize the two scaled basis correspondences and construct the standard matrix.

Assume T is linear.

The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(-6,\ -9)$.

Your response: _____

Incoming: The matrix is $[[0,1],[-1,0]]$.

Complete the image table for all three input rows.

Use the matrix $A=\begin{pmatrix}-1 & 2\\ 3 & 1\end{pmatrix}$. Vectors are column vectors.

The given points are $(0,\ 1)$, $(2,\ 0)$, $(-1,\ -2)$.

Your response: _____

Incoming: The first rows give matrix $[[1,3],[2,4]]$, which sends $(1,2)$ to $(7,10)$, not $(8,10)$. No linear map fits all rows.

Recover $T(e2)$ from the image of $e1+e2$ and construct the matrix.

Assume T is linear.

The input-output observations are $(1,\ 0)\mapsto(-1,\ 4)$; $(1,\ 1)\mapsto(2,\ 2)$.

Your response: _____

AP Precalculus

Matrices for Linear Transformations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The student read the rows as basis images. Correct both images and explain the relevant matrix convention.

Use the matrix $A = \begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix}$. Vectors are column vectors.

Evaluate these student claims: $T(e1) = (2,\ 5)$; $T(e2) = (-1,\ 3)$.

A student claims: "$A90^\circ$ rotation produces a different region because every boundary point moves." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Describe the disk's image and verify the claim with one boundary point.

Use the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Vectors are column vectors.

Use the disk with center $(0,\ 0)$ and radius 2.

Evaluate this student claim: $A90^\circ$ rotation produces a different region because every boundary point moves..

Matrices for Linear Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Write the matrix for a half-turn about the origin.

The map acts by rotate 180 degrees about the origin.

Answer. The matrix is $[[-1, 0], [0, -1]]$.

Explanation and check.

Both basis vectors reverse direction under a half-turn.

Worked example

Problem. Classify the rule and give a matrix representation if one exists.

Use the rule $F(x, y) = (2x - y, 3y)$.

Answer. The rule is linear with matrix $[[2, -1], [0, 3]]$.

Explanation and check.

Each output coordinate is a constant linear combination of the inputs with no added constant or nonlinear term.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Composing Matrix Transformations

Composing Matrix Transformations | APPC-U04-P18

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for projection and product, second matrix and product, all valid ordered pairs and verification principle, all valid pairs, all sequences and exclusions and all valid sequences and exclusion.

8 PDFs and a README; 156 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Composing Matrix Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Compute both orders and decide whether these maps commute.

Use the reflection matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Use the scaling matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.

Bank label: _____

AP Precalculus

Composing Matrix Transformations

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the ordered composite and verify it stage by stage.

First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The original input vector is $(1, 2)$.

Classify all allowed two-stage sequences that produce reflection across $y = x$.

The allowed transformations are exactly one 90-degree counterclockwise rotation and one coordinate-axis reflection. The target matrix is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

AP Precalculus

Composing Matrix Transformations

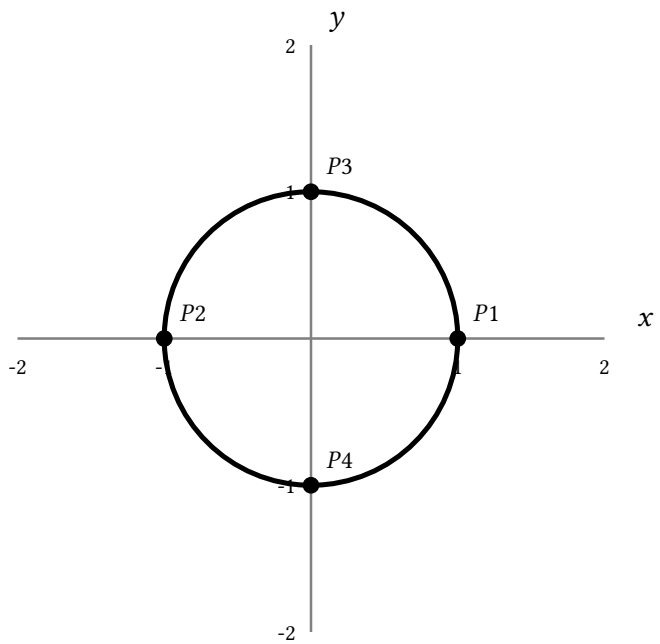
Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The first two stages form a half-turn; the final product is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, reflection across the y-axis.

Track the four key boundary points and state the major-axis direction after each stage.

The boundary points are $(1, 0)$, $(-1, 0)$, $(0, 1)$, $(0, -1)$. First apply $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Then apply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.



Your response: _____

AP Precalculus

Composing Matrix Transformations

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student multiplies the two matrices in the written transformation order and claims the composite is the first matrix times the second. Identify the order error, construct the correct composite, and verify it on the test vector.

First apply $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Then apply $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. The original input vector is $(2, 1)$.

Composing Matrix Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Compare both orders and identify their common net action.

Use matrix A = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix B = $\begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix}$.

Answer. Both products equal -I, so either order is a half-turn with no change in length.

Explanation and check.

Scalar matrices commute, and the scale factors multiply to -1.

Worked example

Problem. Complete the table and explain why the final column returns to the original data.

First apply $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Use the points A: $(1, 2)$; B: $(-2, 1)$; C: $(0, -3)$. Then apply $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

Answer. Intermediate: $A = (5, 2), B = (0, 1), C = (-6, -3)$. Final: $A = (1, 2), B = (-2, 1), C = (0, -3)$, the original points.

Explanation and check.

The second shear subtracts the same vertical-coordinate multiple that the first added.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Transformations and Preimages

Inverse Transformations and Preimages | APPC-U04-P19

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for inverse existence and two inverse images, conclusion and example matrices, qualified conclusion, existence and uniqueness qualifications, all preimages and all inputs and geometric set.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Inverse Transformations and Preimages

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Recover the input from the mixed-sign image and verify it with the forward map.

The image point is $(-2, 3)$.

Required response

Supplied inverse matrix

1	-1
0	1

Forward matrix

1	1
0	1

Bank label: _____

Identify the inverse map without recomputing an adjugate.

Required response

0	1
1	0

Bank label: _____

Apply the inverse turn to recover every labeled original vertex.

The image vertex A' is $(0, 1)$.

The image vertex B' is $(-2, 1)$.

The image vertex C' is $(-2, 3)$.

The image vertex D' is $(0, 3)$.

Required response

Forward matrix

0	-1
1	0

Bank label: _____

Find every input mapped to the target and describe the fiber.

The target point is $(3, -6)$.

Required response

Forward matrix

2	-1
-4	2

Bank label: _____

AP Precalculus

Inverse Transformations and Preimages

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recover the original rectangle and state its width and height.

Required response

2	0
0	$\frac{1}{2}$

-4	-1
4	-1
4	1
-4	1

Determine whether the target has any preimage.

The target point is $(4, 1)$.

Required response

Forward matrix

1	0
0	0

Decide separately whether the map is determined and whether it has an inverse.

Use $T(e_1) = (1, 2)$.

Use $T(e_2) = (1, 2)$.

Assume that T is linear.

Explain why these four checks are sufficient for mutual inverses.

Assume that F and G are linear.

The supplied checks are $G(F(e_1)) = e_1$; $G(F(e_2)) = e_2$; $F(G(e_1)) = e_1$; $F(G(e_2)) = e_2$.

AP Precalculus

Inverse Transformations and Preimages

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The preimage vertices are $(2,1),(-1,0),(0,-3)$.

Use the supplied inverse to reconstruct the labeled triangle.

Required response

Forward matrix

2	1
1	1

Image vertices (one coordinate pair per row)

0	0
2	1
2	2

Supplied inverse matrix

1	-1
-1	2

Your response: _____

Incoming: Map (X,Y) to $(X, Y+2X)$, using $[[1,0],[2,1]]$.

Find when the target lies on the image line and then parameterize its fiber.

The target point is $(k,9)$.

Required response

Forward matrix

1	2
3	6

Your response: _____

Incoming: Reachability requires k = 3. Then $x+2y=3$, so the preimages are $(3-2t,t)$.

Compute both compositions on their proper domains and classify the relationship.

Let $F(x,y)=(x,0)$ from R^2 to the x-axis.

Let $G(t,0)=(t,0)$ from the x-axis to R^2.

Your response: _____

Incoming: The preimage is $x^2+y^2 \geq 1$ with $y>0$. The circle is included, while the x-axis is excluded.

Prove the candidate is an inverse and explain exactly where the nonzero condition is needed.

Required response

1	a
0	b

1	$-\frac{a}{b}$
0	$\frac{1}{b}$

Your response: _____

AP Precalculus

Inverse Transformations and Preimages

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “A shear matrix cannot be invertible unless it is diagonal.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Verify the inverse-function claim on all vectors by checking both compositions.

Required response

1	1
0	1

1	-1
0	1

Repair the claim that one recovered input proves a global inverse function.

Use the map $F(x, y) = (x, 0)$.

The student proposes the preimage $(3, 0)$.

The target point is $(3, 0)$.

Inverse Transformations and Preimages

Full Worked Solutions - excerpt

Worked example

Problem. Prove the product-inverse rule by checking recovery in both directions and naming the undoing order.

Answer. Undo A first and B second. Composing this undoing map before or after the original product cancels adjacent inverse pairs and gives the identity in both directions.

Explanation and check.

A composite inverse reverses the original factor order so that each action meets its own undoing action.

Worked example

Problem. Prove that the inverse function of B is the original matrix function A.

Assume that $B = A^{-1}$.

Answer. AB = BA=I, so A satisfies both defining equations for B^{-1}. Therefore $(A^{-1})^{-1} = A$.

Explanation and check.

The two-sided inverse equations are symmetric in A and B.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Transition Matrices and Long-Term Behavior

Transition Matrices and Long-Term Behavior | APPC-U04-P20

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for matrix family and parameter interval, terminal states and decision, switch counts, terminal states, and decision, complete set, fixed states for t=0 and t>0 and all possible priors.

8 PDFs and a README; 85 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

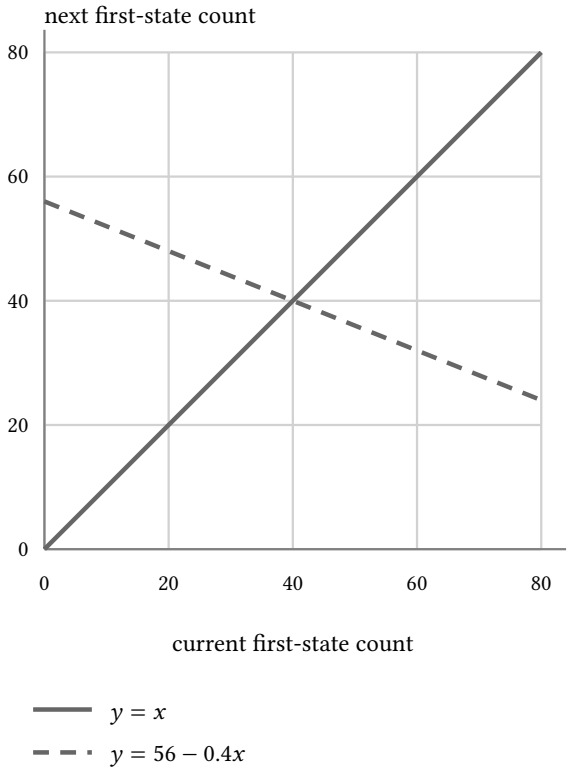
AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the graph equations to find the fixed distribution.



0.3	0.7
0.7	0.3

Bank label: _____

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Compare subscribed counts after two months and recommend the policy for that horizon. The components are ordered subscribed, then paused. The horizon in cycles is 2.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

100
0

Policy A

0.8	0.3
0.2	0.7

Policy B

0.6	0.5
0.4	0.5

Compute tomorrow's proportions and identify both contributions to the online state. The components are ordered online, then offline.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Initial state

0.7
0.3

Transition matrix

0.85	0.4
0.15	0.6

Classify all fixed distributions as t ranges from 0 through 0.5. The conserved total is 90. The allowed parameter interval is $0 \le t \le 0.5$.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Transition matrix

1-t	$2t$
t	$1 - 2t$

Determine the two unknown first-row probabilities from the observations.

Show a complete response in the numbered workspace, preserving requested exactness, units, interval endpoints, and justification.

Matrix pattern

p	q
1-p	1-q

Observation 1: initial state

50
50

Observation 1: later state

55
45

Observation 2: initial state

80
20

Observation 2: later state

70
30

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $[80, 20]$

Convert the shares to counts and calculate the next state. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Initial proportions

0.25
0.75

Transition matrix

0.5	0.1
0.5	0.9

Your response: _____

Incoming: $[28, 72]$

Find the equilibrium counts. The conserved total is 80.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

Transition matrix

0.9	0.3
0.1	0.7

Your response: _____

Incoming: $[[0, 0], [1, 1]]$

Translate the conditional branches into a transition matrix. The components are ordered A, then *B*. The conditional A-to-*B* probability is 0.4; the *B*-to-A probability is 0.15.

Your response: _____

Incoming: $[16, 64]$

Find the fixed distribution with total 120.

Solve the card, use the complete requested response as the outgoing match, and move to the unique card whose incoming header matches it; continue until FINISH.

0.7	0.3
0.3	0.7

Your response: _____

AP Precalculus

Transition Matrices and Long-Term Behavior

Name: _____ **Date:** _____ **Class:** _____

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: "Because every displayed row rounds to $(50.0, 50.0)$, the state sequence converges to $(50, 50)$." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete the stated task: A rounded table displays $(50.0, 50.0)$ repeatedly. Decide whether that proves convergence.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

49.96
50.04

0	1
1	0

Find the state after three periods using the supplied power, and explain why multiplying the one-period matrix by 3 would be invalid.

Identify the error or first invalid step, give the corrected result, and state a brief mathematical reason.

40
60

0.8	0.3
0.2	0.7

0.65	0.525
0.35	0.475

Transition Matrices and Long-Term Behavior

Full Worked Solutions - excerpt

Worked example

Problem. Translate the conditional branches into a transition matrix. The components are ordered A, then B. The conditional A-to-B probability is 0.4; the B-to-A probability is 0.15.

Answer. $[[0.6, 0.15], [0.4, 0.85]]$

Explanation and check.

Each tree root is a current-state source, so its two destination branches become one matrix column.

Worked example

Problem. Encode the deterministic switching rule. The components are ordered left, then right. The transition rule is: every member switches states each cycle.

Answer. $[[0, 1], [1, 0]]$

Explanation and check.

Each source sends probability one to the opposite destination and zero to itself.