Sets & Steps
Math Practice & Worked Solutions

Precalculus

Rational Functions and Equivalent Forms

6-topic bundle | 576 problems | Four activity formats

In every topic

Answer Bank Challenge	24 tasks
Grid Rounds	24 tasks
Match & Move	24 cards
Error Analysis & Strategy	24 tasks
Quick Answer Key and Full Worked Solutions	included
Teacher / Parent Use Guide and Terms of Use	included

Product previews

The following preview pages show the cover, all four activity formats, and worked examples for each included resource, in the listed order.

48 PDFs and 6 READMEs; 494 buyer PDF pages across the 6 products.

Preview pages are not included in the buyer page count. Check the list against resources you already own.

This is a fixed collection of the listed products. Purchase includes the existing individual resource files. To open a product's online preview, follow its title link and select View Preview.

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Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Equivalent Polynomial Forms

Equivalent Polynomial Forms | APPC-U01-P13

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for form choice and exact revealed feature, consistent candidate with decisive evidence, first error, corrected identity, and lost graph feature, complete exact polynomial and requested coefficient, factored form plus newly visible feature and feature meaning with units and domain check.

8 PDFs and a README; 78 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Equivalent Polynomial Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

An answer may be used more than once.

Factor $P(x) = 2x^2 - 16x + 32$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Rewrite $f(x) = 2x^2 + 1$ in vertex form. On the stated domain $-2 \le x \le 3$, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible. Bank label: _______
Factor $P(x) = 2x^2 + 18x$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Rewrite $f(x) = x^2 + 8x + 13$ in vertex form. On the stated domain all real x, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible. Bank label: _______
Factor $P(x) = 3x^2 - 42x + 147$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Rewrite $f(x) = 3x^2 + 12x + 11$ in vertex form. On the stated domain $-1 \le x \le 2$, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible. Bank label: _______
Factor $P(x) = 3x^2 - 18x$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots. Bank label: _______	Two equivalent descriptions of the same polynomial are vertex: $(x+1)^2 + 2$; standard: $x^2 + 2x + 3$. Which named form most directly answers the request for vertex? State the feature it reveals and explain why equivalence alone does not make both forms equally efficient. Bank label: _______

AP Precalculus

Equivalent Polynomial Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Rewrite $f(x) = 2x^2 - 12x + 17$ in vertex form. On the stated domain $1 \leq x \leq 6$, determine the minimum value and where it occurs; justify the bound using square nonnegativity and check whether the vertex is feasible.

Factor $P(x) = 2x^2 + 32x + 128$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots.

Use the supplied complete Pascal table to expand $(4x - 4)^5$ and explain how signs and nonunit factors enter each coefficient.

Context/domain: precalculus

j	C(n,j)	power of x	power of constant term	resulting coefficient
0	1	5	0	1024
1	5	4	1	−5120
2	10	3	2	10240
3	10	2	3	−10240
4	5	1	4	5120
5	1	0	5	−1024

Determine all real parameter values that make the two forms $a(x - 3)^2 + k$ and $x^2 - 6x + 8$ identical for every x. Match only decisive coefficients first, then verify every coefficient.

AP Precalculus

Equivalent Polynomial Forms

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Two equivalent descriptions of the same polynomial are factored: $2(x-2)(x-5)$; standard: $2x^2 - 14x + 20$. Which named form most directly answers the request for initial output $f(0)$? State the feature it reveals and explain why equivalence alone does not make both forms equally efficient.

Your response: ______________

Incoming: The roots t = 0 and $t = 9$ mean the object is at height 0 feet at launch and landing; both are feasible endpoints.

Use the supplied complete Pascal table to expand $(x+1)^2$ and explain how signs and nonunit factors enter each coefficient.

Context/domain: precalculus

j	C(n,j)	power of x	power of constant term	resulting coefficient
0	1	2	0	1
1	2	1	1	2
2	1	0	2	1

Your response: ______________

Incoming: Use the factored form; it directly reveals $x = -1$ and $x = 2$.

Factor $P(x) = x^2$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots.

Your response: ______________

Incoming: $(x-1)^3 = x^3 - 3x^2 + 3x - 1$.

Two equivalent descriptions of the same polynomial are vertex: $3(x+2)^2 - 1$; standard: $3x^2 + 12x + 11$. Which named form most directly answers the request for quadratic extremum and where it occurs? State the feature it reveals and explain why equivalence alone does not make both forms equally efficient.

Your response: ______________

AP Precalculus

Equivalent Polynomial Forms

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims $3(x-2)^2-1$ is identically equal to x^2-4x+3. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the missing factor of 3 in expansion; compare the values at $x=0$. Give the corrected identity and explain why the claimed expressions are not equivalent.

A student claims $3x^2+2$ is identically equal to x^2+2. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the omitted coefficient 3; compare the values at $x=1$. Give the corrected identity and explain why the claimed expressions are not equivalent.

A student claims $(x+2)^2$ is identically equal to x^2+4. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the omitted cross term; compare both vertices and the values at $x=1$. Give the corrected identity and explain why the claimed expressions are not equivalent.

A student claims $(x+5)^2$ is identically equal to x^2+25. Locate the algebraic error, write the corrected identity, and name one graph feature or value changed by the false rewrite. Diagnose the omitted cross term; compare the values at $x=1$. Give the corrected identity and explain why the claimed expressions are not equivalent.

Equivalent Polynomial Forms

Full Worked Solutions - excerpt

Worked example

Problem. Factor $P(x) = x^2$ to reveal useful structure. Explain the feature made visible by the factorization rather than merely listing roots.

Answer. $P(x) = xx = x^2$; the repeated factor shows a double zero at $x = 0$.

Explanation and check.

The factorization $xx = x^2$ is an identity. The repeated factor shows a double zero at $x = 0$.

Worked example

Problem. Determine all real parameter values that make the two forms $a(x+3)^2 + k$ and $x^2 + 6x + 7$ identical for every x. Match only decisive coefficients first, then verify every coefficient.

Answer. a = $1, k = -2$.

Explanation and check.

Coefficient matching yields the unique pair $a = 1, k = -2$, and substitution reproduces $x^2 + 6x + 7$.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Rational Expression Operations and Restrictions

Rational Expression Operations and Restrictions | APPC-U01-P14

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for recommended representation with preserved domain, function equality and exact agreement domain, verified original ratio and exact exclusion set, correct rewrite, exclusion, and test-input evidence, simplified rule plus every original exclusion and single ratio, reduced rule, and sourced exclusions.

8 PDFs and a README; 55 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Rational Expression Operations and Restrictions

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Rewrite $R(x) = \frac{(x+2)(x-6)(2x+3)}{(x+2)(x-6)(x-2)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: _______	Choose the more useful of the two supplied forms to determine the horizontal asymptote and the nonzero remainder in a quotient-remainder form. A (single ratio): $\frac{-3(x+2)+1}{x+2}$; B (supplied quotient-remainder): $-3 + \frac{1}{x+2}$. State the feature, explain why that form exposes it, and retain every original excluded input. Bank label: _______
Rewrite $R(x) = 6(x+1)\frac{x-5}{(x+1)(x-5)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: _______	Determine all real k for which $\frac{x+k}{x-7}$ and $\frac{x+2}{x-7}$ define identical functions. First compare the algebraic identity, then check zero denominators and whether the domains coincide. Bank label: _______
Rewrite $R(x) = 5(x+7)\frac{x+1}{(x+7)(x+1)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: _______	Determine all real k for which $\frac{(x+2)(x+k)}{(x+2)(x-3)}$ and $\frac{x+k}{x-3}$ define identical functions. First compare the algebraic identity, then check zero denominators and whether the domains coincide. Bank label: _______
Rewrite $R(x) = 4(x-2)\frac{x-8}{(x-2)(x-8)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator. Bank label: _______	Choose the more useful of the two supplied forms to determine the horizontal asymptote and the nonzero remainder in a quotient-remainder form. A (single ratio): $\frac{3(x+5)+3}{x+5}$; B (supplied quotient-remainder): $3 + \frac{3}{x+5}$. State the feature, explain why that form exposes it, and retain every original excluded input. Bank label: _______

AP Precalculus

Rational Expression Operations and Restrictions

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Construct an original rational ratio that reduces to $x-1$ but has requested original exclusions $\{x=-1\}$. The reduced rule already excludes none; do not add redundant common factors or unintended restrictions.	Rewrite $(\frac{x+7}{x+4}) \div (\frac{x-3}{x})$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.
Rewrite $R(x)=\frac{(x+8)x(2x-3)}{(x+8)x(x+2)}$ in reduced form and state its original domain. Cancel only common nonzero factors; do not restore an input merely because it disappears from the denominator.	Given $f(x)=\frac{(x+6)(2x-3)}{(x+6)(x+1)}$ with excluded inputs $-6,-1$, and $g(x)=\frac{2x-3}{x+1}$ with $x\neq-6,-1$ with excluded inputs $-6,-1$. Decide whether they are identical functions and state exactly where their values agree. A matching simplified formula is not sufficient without matching domains.
Construct an original rational ratio that reduces to $x-2$ but has requested original exclusions $\{x=-7\}$. The reduced rule already excludes none; do not add redundant common factors or unintended restrictions.	Rewrite $(\frac{x+5}{x+2})(\frac{x+2}{x-2})$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

AP Precalculus

Rational Expression Operations and Restrictions

Name: ____________________ **Date:** ____________________ **Class:** ____________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Rewrite $(\frac{x-2}{x-5}) \div (\frac{x-12}{x-9})$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Your response: ____________________

Incoming: The single ratio is $\frac{(x-2)(x-9)}{(x-5)(x-12)}$, reducing to $\frac{(x-2)(x-9)}{(x-5)(x-12)}$, with all real x except $x = 5, x = 9, x = 12$.

Determine all real k for which $\frac{x+k}{x+2}$ and $\frac{x}{x+2}$ define identical functions. First compare the algebraic identity, then check zero denominators and whether the domains coincide.

Your response: ____________________

Incoming: k = 0.

Rewrite $(\frac{x+10}{x+7}) \div (\frac{x}{x+3})$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Your response: ____________________

Incoming: The single ratio is $\frac{(x+10)(x+3)}{(x+7)x}$, reducing to $\frac{(x+10)(x+3)}{(x+7)x}$, with all real x except $x = -7, x = -3, x = 0$.

Construct an original rational ratio that reduces to $\frac{3x-2}{x+6}$ but has requested original exclusions $\{x = -6, x = -2\}$. The reduced rule already excludes $\{x = -6\}$; do not add redundant common factors or unintended restrictions.

Your response: ____________________

Incoming: One valid predecessor is $\frac{(x+2)(3x-2)}{(x+2)(x+6)}$, with all real x except $x = -6, x = -2$.

Rewrite $(\frac{x+3}{x}) + (\frac{4}{x})$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Your response: ____________________

Incoming: The single ratio is $\frac{x+7}{x}$, reducing to $\frac{x+7}{x}$, with all real x except $x = 0$.

Choose the more useful of the two supplied forms to determine the horizontal asymptote and the nonzero remainder in a quotient-remainder form. A (single ratio): $\frac{-2(x-6)+4}{x-6}$; B (supplied quotient-remainder): $-2 + \frac{4}{x-6}$. State the feature, explain why that form exposes it, and retain every original excluded input.

Your response: ____________________

AP Precalculus

Rational Expression Operations and Restrictions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student rewrites $(x-3)\,\frac{x+2}{x-3}$ as $x+2$ for every real x. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = 3$.

A student rewrites $\frac{x-1}{x-4}$ as $-\frac{1}{-4}$. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = 1$.

A student rewrites $(x+3)\,\frac{x+2}{x+3}$ as $x+2$ for every real x. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = -3$.

A student rewrites $\frac{4-x}{x-4}$ as 1. Identify the first invalid inference. State the corrected rule and its original domain, then test the designated input when it is allowed.

Designated test input: $x = 5$.

Rational Expression Operations and Restrictions

Full Worked Solutions - excerpt

Worked example

Problem. Rewrite $\left(\frac{x+3}{x}\right) + \left(\frac{4}{x}\right)$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Answer. The single ratio is $\frac{x+7}{x}$, reducing to $\frac{x+7}{x}$, with all real x except $x = 0$.

Explanation and check.

Combining the operation gives $\frac{x+7}{x}$. The common denominator supplies the sole restriction. Therefore the complete exclusion set is 0.

Worked example

Problem. Rewrite $\left(\frac{x-9}{x-12}\right) + \left(\frac{6}{x-12}\right)$ as one rational expression and state the domain. Track every component denominator before simplifying; for division, also exclude inputs where the divisor equals zero.

Answer. The single ratio is $\frac{x-3}{x-12}$, reducing to $\frac{x-3}{x-12}$, with all real x except $x = 12$.

Explanation and check.

Combining the operation gives $\frac{x-3}{x-12}$. The common denominator supplies the sole restriction. Therefore the complete exclusion set is 12.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Rational Function Domains and Sign Charts

Rational Function Domains and Sign Charts | APPC-U01-P10

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for compatible graph with decisive feature, verified formula or impossibility proof, exact real domain, repaired solution with domain/sign reason, qualified exact zero conclusion and complete real parameter set.

8 PDFs and a README; 59 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Rational Function Domains and Sign Charts

Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Determine the real domain of the original rule $f(x) = \frac{x+40}{(x-6)^2(x-10)(x^2+23)}$. Factor the denominator as needed and preserve exclusions even if a factor cancels.

Bank label: ____

Find all real zeros and x-intercepts of $f(x) = \frac{(x+3)(x+1)(x-2)}{(x+1)(x-5)(x^2+15)}$. List numerator roots first, then enforce the original denominator restrictions.

Bank label: ____

The table rows (x, reported $f(x)$) are $(-9, 0.03), (-8, 0.00), (-7, -0.02)$. Additional exact information: $f(-8) = \frac{1}{500}$ exactly. Decide what zero claim is warranted, using the stated precision and definition status.

Required output: qualified exact zero conclusion

x	**reported $f(x)$**
-9	0.03
-8	0.00
-7	-0.02

Bank label: ____

Find all real zeros and x-intercepts of $f(x) = \frac{(x+9)(x+7)(x+4)}{(x+7)(x+1)(x^2+9)}$. List numerator roots first, then enforce the original denominator restrictions.

Bank label: ____

AP Precalculus

Rational Function Domains and Sign Charts

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Determine the real domain of the original rule $f(x) = \frac{(x+2)x}{(x+2)(x-4)}$. Factor the denominator as needed and preserve exclusions even if a factor cancels.	Construct one real rational function with zeros $\{-6, -3\}$, exclusions $\{0, 2\}$, and common factor $(x - 2)$, or prove the conditions incompatible. No hole height is prescribed.
A student argues: "For $\frac{(x+3)x}{x} = 0$, cancel to $(x + 3) = 0$ and include both $x = -3$ and $x = 0$ as zeros." Diagnose the exact error and repair the solution while preserving original-domain and sign conditions.	Determine the real domain of the original rule $f(x) = \frac{(x-14)}{(x-11)(x^2+27)}$. Factor the denominator as needed and preserve exclusions even if a factor cancels.
Solve $f(x) = \frac{0}{(x-11)(x-13)} \leq 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly.	Construct one real rational function with zero set containing 5 and excluded set also containing 5, or prove the conditions incompatible. No hole height is prescribed.

AP Precalculus

Rational Function Domains and Sign Charts

Name: __________ **Date:** __________ **Class:** __________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START Solve $f(x) = \frac{(x+8)}{(x+6)} > 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly. Your response: __________	**Incoming:** Solution: $(-\infty, -8) \cup (-6, \infty)$. Find all real zeros and x-intercepts of $f(x) = \frac{(x+13)(x+11)}{(x+13)(x+11)(x^2+5)}$. List numerator roots first, then enforce the original denominator restrictions. Your response: __________
Incoming: Real zeros: none; there are no x-intercepts. Solve $f(x) = \frac{0}{(x+9)(x+7)} \leq 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly. Your response: __________	**Incoming:** Solution: $(-\infty, -9) \cup (-9, -7) \cup (-7, \infty)$. Determine all real k such that $x = -5$ is a real zero of $f_k(x) = \frac{x-k}{x}$. Enforce numerator zero and original denominator nonzero simultaneously. Your response: __________
Incoming: k = −5. Solve $f(x) = \frac{(x+6)(x+4)}{(x+4)(x+1)} < 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly. Your response: __________	**Incoming:** Solution: $(-6, -4) \cup (-4, -1)$. Solve $f(x) = \frac{(x+10)(x+8)}{(x+8)(x+5)} < 0$. Use the original factored rule to build a sign chart and state every endpoint membership explicitly. Your response: __________

AP Precalculus

Rational Function Domains and Sign Charts

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student argues: “For $\frac{x(x-3)}{x-3} = 0$, cancel to $x = 0$ and include both $x = 0$ and $x = 3$ as zeros.” Diagnose the exact error and repair the solution while preserving original-domain and sign conditions. Identify the first invalid inference, give the corrected solution set, and justify it using the original equation.

A student argues: “For $\frac{(x-3)(x-6)}{x-6} = 0$, cancel to $(x-3) = 0$ and include both $x = 3$ and $x = 6$ as zeros.” Diagnose the exact error and repair the solution while preserving original-domain and sign conditions. Identify the first invalid inference, give the corrected solution set, and justify it using the original equation.

A student argues: “For $\frac{x-1}{x-4} = 1$, cross-multiply and include $x = 4$ because both sides become 0.” Diagnose the exact error and repair the solution while preserving original-domain and sign conditions. Identify the first invalid inference, give the corrected solution set, and justify it using the original equation.

Rational Function Domains and Sign Charts

Full Worked Solutions - excerpt

Worked example

Problem. Construct one real rational function with zero set containing 5 and excluded set also containing 5, or prove the conditions incompatible. No hole height is prescribed.

Answer. No such rational function exists.

Explanation and check.

Impossible: a rational zero must be in the domain, while $x = 5$ is required to be outside the domain.

Worked example

Problem. Determine all real k such that $x = 2$ is a real zero of $f_k(x) = \frac{x-k}{x-k}$. Enforce numerator zero and original denominator nonzero simultaneously.

Answer. No real value of k works.

Explanation and check.

The numerator condition at $x = 2$ forces $k = 2$, but then the denominator is also zero at $x = 2$. The only candidate is forbidden.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus
Holes in Rational Functions

Holes in Rational Functions | APPC-U01-P12

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for verified original ratio or impossibility proof, height membership conclusion with alternate-preimage check, corrected value-limit-hole statement, separate limit and point-value claims, exact removable point coordinates and required point value or impossibility.

8 PDFs and a README; 116 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Holes in Rational Functions

Name: ________________ **Date:** ________________ **Class:** ________________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Locate every removable hole of $f(x) = \frac{(x+6)(x+2)(-x-1)}{(x+6)(x+2)}$. The reduced rule is not the original value at an excluded input; report full coordinates and ignore any residual pole except to keep it distinct.

Bank label: ______

The exact approach table follows the continuation $g(x) = -x + 1$ near $x = -8$. The point definition says: $f(c)$ is undefined. State the two-sided limit when justified and state $f(-8)$ separately.

Given original rule: $f(x) = \frac{(x+8)(-x+1)}{(x+8)}, x \neq -8$

x	reported f(x)
-9	10
$-\frac{17}{2}$	$\frac{19}{2}$
$-\frac{81}{10}$	$\frac{91}{10}$
$-\frac{79}{10}$	$\frac{89}{10}$
$-\frac{15}{2}$	$\frac{17}{2}$
-7	8

Bank label: ______

AP Precalculus

Holes in Rational Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Locate every removable hole of $f(x) = \frac{x(-x-2)}{x(x-5)}$. The reduced rule is not the original value at an excluded input; report full coordinates and ignore any residual pole except to keep it distinct.

The exact approach table follows the continuation $g(x) = 2x - 1$ near $x = 5$. The point definition says: $f(5) = 11$ is separately defined. State the two-sided limit when justified and state $f(5)$ separately.

Given original rule: $f(x) = \frac{(x-5)(2x-1)}{(x-5)}, x \neq 5$

x	**reported f(x)**
4	7
$\frac{9}{2}$	8
$\frac{49}{10}$	$\frac{44}{5}$
$\frac{51}{10}$	$\frac{46}{5}$
$\frac{11}{2}$	10
6	11

AP Precalculus

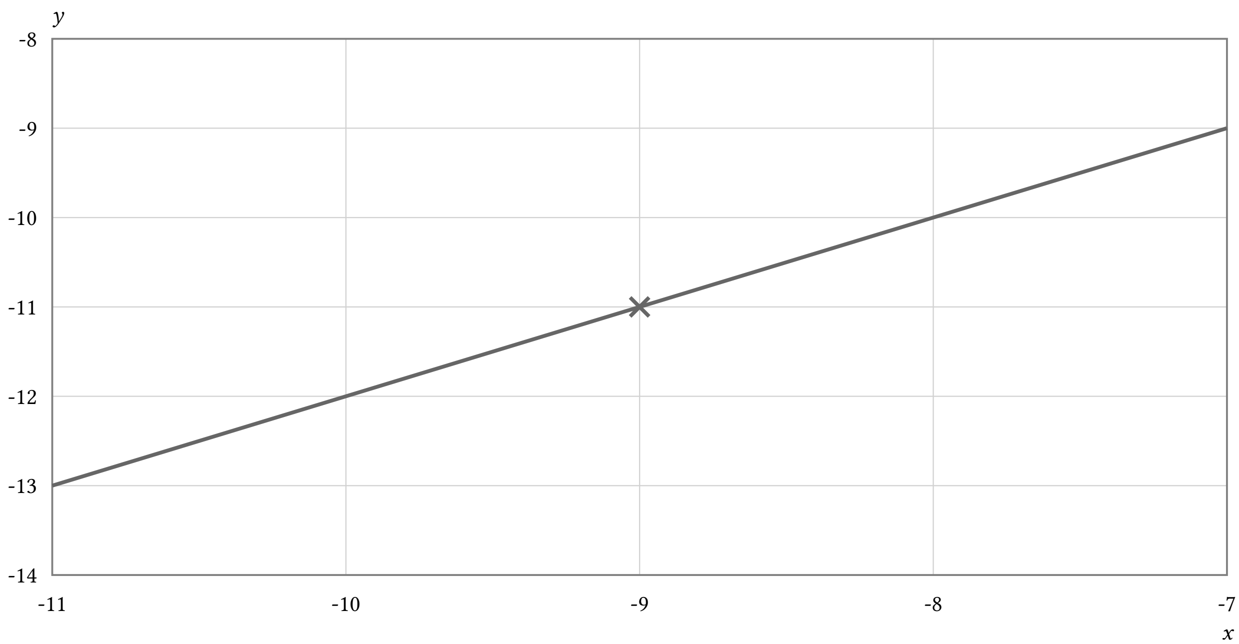
Holes in Rational Functions

Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

Construct an original rational function satisfying reduced graph $y = x - 2$ with one hole at $(-9, -11)$, or prove it impossible. Verify every target height against the reduced graph.



Your response: ______________

AP Precalculus

Holes in Rational Functions

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: "For $f(x) = \frac{(x-2)(x-1)}{x-2}$, $\frac{0}{0}$ means $f(2) = 0$." Diagnose the implication and distinguish nearby equality, the limit, and the original point value. Identify the invalid inference. Give the nearby simplified rule, the two-sided limit when justified, and the original point value separately.

A student claims: "For $f(x) = \frac{(x+1)(x-1)}{x+1}$, $\frac{0}{0}$ means $f(-1) = 0$." Diagnose the implication and distinguish nearby equality, the limit, and the original point value. Identify the invalid inference. Give the nearby simplified rule, the two-sided limit when justified, and the original point value separately.

Holes in Rational Functions

Full Worked Solutions - excerpt

Worked example

Problem. Determine the point value, if any, that makes this function continuous at $x = 9$: $f(x) = \frac{1}{x-9}$ for $x \neq 9$, with a proposed finite $f(9)$. Verify a finite two-sided limit before assigning the point.

Answer. No finite point value can make the function continuous.

Explanation and check.

The one-sided outputs diverge with opposite signs at $x = 9$, so no finite two-sided limit exists to use as a fill value.

Worked example

Problem. Determine the point value, if any, that makes this function continuous at $x = 0$: $f(x) = \frac{1}{x}$ for $x \neq 0$, with a proposed finite $f(0)$. Verify a finite two-sided limit before assigning the point.

Answer. No finite point value can make the function continuous.

Explanation and check.

The one-sided outputs diverge with opposite signs at $x = 0$, so no finite two-sided limit exists to use as a fill value.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Vertical Asymptotes and One-Sided Behavior | APPC-U01-P11

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for two signed one-sided limits, complete generic and exceptional classification, verified formula or impossibility proof, evidence-based pole decision, qualified local limit conclusion and exact pole set with residual orders.

8 PDFs and a README; 120 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Determine every vertical asymptote of $f(x) = \frac{(x-2)^2(x-6)}{(x-2)^2(x-6)^2(x^2+19)}$. Collect common factors and compare multiplicities; do not call every original denominator zero a pole.

Bank label: ______

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Determine every vertical asymptote of $f(x) = \frac{(x-11)^2(x-15)}{(x-11)^2(x-15)^2(x^2+28)}$. Collect common factors and compare multiplicities; do not call every original denominator zero a pole.

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ____________ **Date:** ____________ **Class:** ____________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

For $f(x) = \frac{3}{(x-1)(x^2+18)}$, analyze the two one-sided limits at $x = 1$. Use the residual denominator parity and the sign of the nonzero local cofactor.

Your response: ____________

AP Precalculus

Vertical Asymptotes and One-Sided Behavior

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

An exact display uses $f(x) = \frac{1}{x-10}$ on window $[0, 4]$. A student claims: “No spike appears in the window $[0, 4]$, so the function has no vertical asymptote.” Diagnose the claim using residual denominator factors rather than visual height alone. Diagnose the no-spike claim by locating the actual pole relative to the displayed window.

Vertical Asymptotes and One-Sided Behavior

Full Worked Solutions - excerpt

Worked example

Problem. Construct a simple rational example satisfying one pole at $x = -6$ with left limit $+\infty$ and right limit $-\infty$, or prove it impossible. Verify pole order and every prescribed one-sided sign.

Answer. One example is $f(x) = -\frac{1}{x+6}$.

Explanation and check.

The denominator power 1 has the required parity and numerator sign -1; direct local sign analysis verifies both branches.

Worked example

Problem. Construct a simple rational example satisfying one pole at $x = 0$ with left limit $-\infty$ and right limit $+\infty$, or prove it impossible. Verify pole order and every prescribed one-sided sign.

Answer. One example is $f(x) = \frac{1}{x}$.

Explanation and check.

The denominator power 1 has the required parity and numerator sign 1; direct local sign analysis verifies both branches.

Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Polynomial Division and Slant Asymptotes

Polynomial Division and Slant Asymptotes | APPC-U01-P15

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for quotient line, remainder correction, and exclusion, quotient, remainder, and reconstructed identity, quotient and degree-valid remainder, validity verdict with both requirements, first invalid step and corrected continuation and missing polynomial or inconsistency proof.

8 PDFs and a README; 66 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

Published by Tusils LLC.

AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Divide $12x^4 + 48x^3 + 3x^2 + 9x - 14$ by $3x + 12$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Bank label: ______

Determine the parameter value(s) so division of $k + x^2 + x$ by $x + 8$ has required remainder 0. Use the supplied exact constraint table and verify $p = dq + r$.

Complete response required: complete parameters with verified remainder.

input	**p_k(input)**	**required remainder**
-8	$56 + k$	0

Bank label: ______

Divide $2x^3 - 4x^2 + 2$ by $2x - 2$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Bank label: ______

In the identity $p(x) = d(x)q(x) + r(x), d(x) = x^2 + 3x + 8, q(x) = x - 1, r(x) = 1 - x$ are given. Determine the dividend $p(x)$. Then verify the identity and the strict condition $\deg r < \deg d$.

Bank label: ______

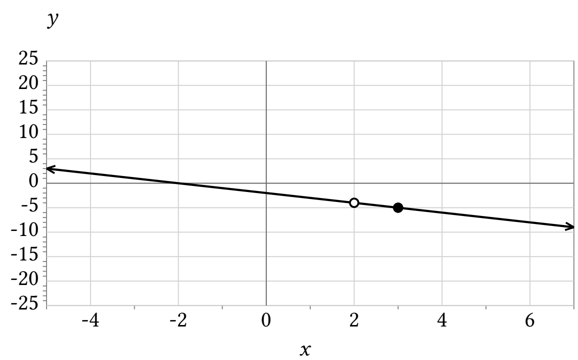
AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ______________ **Date:** ______________ **Class:** ______________

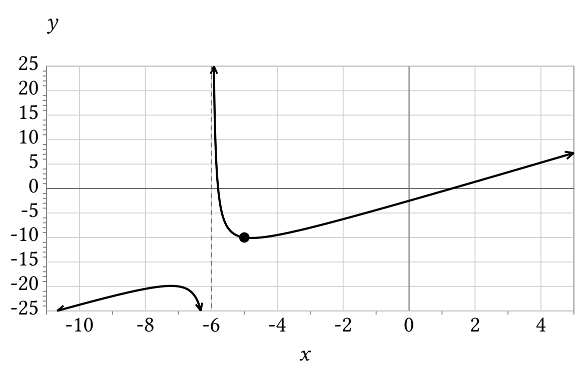
Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Divide the numerator in $\frac{4-x^2}{x-2}$ by its denominator. Write the quotient-remainder form, derive the slant line, and explain why the correction tends to 0 while preserving $x = 2$ as excluded.



Divide $9x^4 + 9x^3 - 9x^2 - 3x + 7$ by $3x + 3$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Divide the numerator in $\frac{2x^2+9x-15}{x+6}$ by its denominator. Write the quotient-remainder form, derive the slant line, and explain why the correction tends to 0 while preserving $x = -6$ as excluded.



A student division of $3x^3 + 3x^2 + 6x$ by $3x$ first fails because the divisor leading coefficient was ignored when choosing the next quotient term. Explain the repair, finish the division, and verify the result.

AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: $q(x) = 2x - 1, r(x) = -3$.

Divide $x^3 - x^2 + 16x - 32$ by $x^2 + x + 17$. Continue only while the current remainder degree is at least 2, then justify the stopping point.

Your response: ______________________

Incoming: $q(x) = x - 1, r(x) = x$.

In the identity $p(x) = d(x)q(x) + r(x)$, $p(x) = 2x^3 - 5x^2 + 7x - 3, q(x) = 2x - 1, r(x) = -x$ are given. Determine the divisor $d(x)$. Then verify the identity and the strict condition $\deg r < \deg d$.

Your response: ______________________

Incoming: $3x - 2 + \frac{4}{x+5}$; slant asymptote $y = 3x - 2$.

Divide $x^3 - 3x^2 + 2x + 22$ by $x^2 - 5x + 11$. Continue only while the current remainder degree is at least 2, then justify the stopping point.

Your response: ______________________

Incoming: $q(x) = 2x + 1, r(x) = -3$.

Divide $4x^3 - 16x^2 - 8x + 12$ by $2x^2 - 10x + 6$. Continue only while the current remainder degree is at least 2, then justify the stopping point.

Your response: ______________________

AP Precalculus

Polynomial Division and Slant Asymptotes

Name: ______________________ **Date:** ______________________ **Class:** ______________________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student is dividing $2x^3 - x^2 + 2x - 2$ by $2x - 1$. Diagnose this specific student statement: The quotient coefficient list $[1, 1]$ represents $x^2 + 1$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

A student is dividing $x^3 - x - 1$ by $x - 1$. Diagnose this specific student statement: After subtracting $x^3 - x^2$ from $x^3 + 0x^2 - x - 1$, the remainder is $-x^2 - x - 1$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

A student is dividing $x^3 - 3x^2 + 2x - 2$ by $x - 2$. Diagnose this specific student statement: Subtracting $x^3 - 2x^2$ from the dividend gives $-5x^2 + 2x - 2$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

A student is dividing $2x^3 + 4$ by $2x + 2$. Diagnose this specific student statement: The descending coefficient list of $2x^3 + 4$ is $[2, 4]$. Correct the line or coefficient list, complete the division, and verify the quotient and remainder.

Polynomial Division and Slant Asymptotes

Full Worked Solutions - excerpt

Worked example

Problem. Divide $x^2 - 7x - 18$ by $x - 9$. Insert zero placeholders for any missing powers, report q and r, and verify $p = dq + r$.

Answer. $q(x) = x + 2, r(x) = 0$.

Explanation and check.

Successive leading-term cancellation gives $q = x + 2$ and $r = 0$. Re-expansion verifies $x^2 - 7x - 18 = (x - 9)(x + 2)$; the zero remainder makes the division exact.

Worked example

Problem. Divide $x^3 - x^2 + 16x - 32$ by $x^2 + x + 17$. Continue only while the current remainder degree is at least 2, then justify the stopping point.

Answer. $q(x) = x - 2, r(x) = x + 2$.

Explanation and check.

Multiplication gives $(x^2 + x + 17)(x - 2)$ and subtraction leaves $x + 2$. Its degree 1 is below 2, so the division is complete.