Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Inverse Transformations and Preimages

Inverse Transformations and Preimages | APPC-U04-P19

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for inverse existence and two inverse images, conclusion and example matrices, qualified conclusion, existence and uniqueness qualifications, all preimages and all inputs and geometric set.

8 PDFs and a README; 70 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

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Name: ______________________ **Date:** ______________________ **Class:** ______________________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Recover the input from the mixed-sign image and verify it with the forward map.

The image point is $(-2, 3)$.

Required response

Supplied inverse matrix

1	−1
0	1

Forward matrix

1	1
0	1

Bank label: ______

Identify the inverse map without recomputing an adjugate.

Required response

0	1
1	0

Bank label: ______

Apply the inverse turn to recover every labeled original vertex.

The image vertex A' is $(0, 1)$.

The image vertex B' is $(-2, 1)$.

The image vertex C' is $(-2, 3)$.

The image vertex D' is $(0, 3)$.

Required response

Forward matrix

0	−1
1	0

Bank label: ______

Find every input mapped to the target and describe the fiber.

The target point is $(3, -6)$.

Required response

Forward matrix

2	−1
−4	2

Bank label: ______

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Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recover the original rectangle and state its width and height.

Required response

2	0
0	$\frac{1}{2}$

-4	-1
4	-1
4	1
-4	1

Determine whether the target has any preimage.

The target point is $(4, 1)$.

Required response

Forward matrix

1	0
0	0

Decide separately whether the map is determined and whether it has an inverse.

Use $T(e_1) = (1, 2)$.

Use $T(e_2) = (1, 2)$.

Assume that T is linear.

Explain why these four checks are sufficient for mutual inverses.

Assume that F and G are linear.

The supplied checks are $G(F(e_1)) = e_1$; $G(F(e_2)) = e_2$; $F(G(e_1)) = e_1$; $F(G(e_2)) = e_2$.

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Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The preimage vertices are $(2, 1), (-1, 0), (0, -3)$.

Use the supplied inverse to reconstruct the labeled triangle.

Required response

Forward matrix

2	1
1	1

Image vertices (one coordinate pair per row)

0	0
2	1
2	2

Supplied inverse matrix

1	-1
-1	2

Your response: ______________________

Incoming: Map (X, Y) to $(X, Y + 2X)$, using [[1, 0], [2, 1]].

Find when the target lies on the image line and then parameterize its fiber.

The target point is $(k, 9)$.

Required response

Forward matrix

1	2
3	6

Your response: ______________________

Incoming: Reachability requires k = 3. Then $x + 2y = 3$, so the preimages are $(3 - 2t, t)$.

Compute both compositions on their proper domains and classify the relationship.

Let $F(x, y) = (x, 0)$ from R^2 to the x-axis.

Let $G(t, 0) = (t, 0)$ from the x-axis to R^2.

Your response: ______________________

Incoming: The preimage is $x^2 + y^2 \geq 1$ with $y > 0$. The circle is included, while the x-axis is excluded.

Prove the candidate is an inverse and explain exactly where the nonzero condition is needed.

Required response

1	a
0	b

1	$-\frac{a}{b}$
0	$\frac{1}{b}$

Your response: ______________________

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Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

A student claims: “A shear matrix cannot be invertible unless it is diagonal.” Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Verify the inverse-function claim on all vectors by checking both compositions.

Required response

1	1
0	1

1	-1
0	1

Repair the claim that one recovered input proves a global inverse function.

Use the map $F(x, y) = (x, 0)$.

The student proposes the preimage $(3, 0)$.

The target point is $(3, 0)$.

Inverse Transformations and Preimages

Full Worked Solutions - excerpt

Worked example

Problem. Prove the product-inverse rule by checking recovery in both directions and naming the undoing order.

Answer. Undo A first and B second. Composing this undoing map before or after the original product cancels adjacent inverse pairs and gives the identity in both directions.

Explanation and check.

A composite inverse reverses the original factor order so that each action meets its own undoing action.

Worked example

Problem. Prove that the inverse function of B is the original matrix function A.
Assume that $B = A^{-1}$.

Answer. AB = BA=I, so A satisfies both defining equations for B^{-1}. Therefore $(A^{-1})^{-1} = A$.

Explanation and check.

The two-sided inverse equations are symmetric in A and B.