Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Matrices for Linear Transformations

Matrices for Linear Transformations | APPC-U04-P17

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for consistency, uniqueness, and two example matrices, determination and nonlinear compatible rule, determination and two compatible rules, classification and decisive test, classification and matrix and classification.

8 PDFs and a README; 60 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

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Name: ____________ **Date:** ____________ **Class:** ____________

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Construct the matrix and use it to map $(2, -1)$ as a check. $T(e_1) = (0,\ 2)$ $T(e_2) = (3,\ 0)$ Bank label: ______	Project the point with the displayed matrix. Use the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Vectors are column vectors. The given point is $(-3,\ 5)$. Bank label: ______
Determine the complete image of the unit disk under the diagonal projection, including exact segment endpoints. Use the matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Vectors are column vectors. Use the disk with center $(0,\ 0)$ and radius 1. Bank label: ______	Solve the two component systems to recover the matrix from these nonbasis correspondences. Assume T is linear. The input-output observations are $(1,\ 2) \mapsto (5,\ 1)$; $(2,\ -1) \mapsto (1,\ 8)$. Bank label: ______

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Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Construct the projection matrix by tracking both basis vectors onto the line $y = x$. The map acts by orthogonally project onto the line $y = x$.	Normalize each scaled-axis correspondence to obtain the two basis images. Assume T is linear. The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(3,\ 6)$.
Classify the rule using the origin test and distinguish affine from linear behavior. Use the rule $F(x, y) = (x + 2, y - 1)$.	Do the axis-only samples establish linearity of an unspecified plane map? Exhibit contrasting compatible rules. The input-output observations are $(0,\ 0)\mapsto(0,\ 0)$; $(1,\ 0)\mapsto(1,\ 0)$; $(2,\ 0)\mapsto(2,\ 0)$.
Construct the matrix and explain geometrically why its image is one-dimensional. $T(e_1) =(2,\ 1)$ $T(e_2) =(-4,\ -2)$	Map the side generators and determine the image area and orientation change. Use the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Vectors are column vectors. The two side vectors are $u =(1,\ 0)$ and $v =(0,\ 2)$.

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Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: Because $(1,1)+(1,-1)=(2,0)$, outputs require $(2+k,4)=(5,4)$, so $k=3$. Then $T(e1)=(\frac{5}{2},2), T(e2)=(-\frac{1}{2},2)$, and the matrix is $[[\frac{5}{2},-\frac{1}{2}],[2,2]]$.

Construct the exact reflection matrix using the double-angle form.

The map acts by reflect across the line through the origin making a30-degree angle with the x-axis.

Your response: ____________________

Incoming: The image generators are $(2,0)$ and $(1,2)$. Their determinant has magnitude 4, so the image area is 4.

Use the two directed basis images to assemble the standard matrix.

$T(e_1)=(0,\ -2)$

$T(e_2)=(3,\ 0)$

Your response: ____________________

Incoming: Under a linearity assumption they determine matrix $[[2,-1],[1,3]]$. Without that assumption they do not prove the map is linear, because nonlinear rules can share the same two values.

Map both points and verify that their known scalar relation survives the transformation.

Use the matrix $A=\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$. Vectors are column vectors.

The given points are $P=(2,\ 1)$; $Q=(-6,\ -3)$.

The relation is $Q=-3P$.

Your response: ____________________

Incoming: Every member is linear, with matrix $[[a,b],[c,d]]$.

Normalize the two scaled basis correspondences and construct the standard matrix.

Assume T is linear.

The input-output observations are $(2,\ 0)\mapsto(6,\ -2)$; $(0,\ -3)\mapsto(-6,\ -9)$.

Your response: ____________________

Incoming: The matrix is $[[0,1],[-1,0]]$.

Complete the image table for all three input rows.

Use the matrix $A=\begin{pmatrix}-1 & 2\\ 3 & 1\end{pmatrix}$. Vectors are column vectors.

The given points are $(0,\ 1)$, $(2,\ 0)$, $(-1,\ -2)$.

Your response: ____________________

Incoming: The first rows give matrix $[[1,3],[2,4]]$, which sends $(1,2)$ to $(7,10)$, not $(8,10)$. No linear map fits all rows.

Recover $T(e2)$ from the image of $e1+e2$ and construct the matrix.

Assume T is linear.

The input-output observations are $(1,\ 0)\mapsto(-1,\ 4)$; $(1,\ 1)\mapsto(2,\ 2)$.

Your response: ____________________

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Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

The student read the rows as basis images. Correct both images and explain the relevant matrix convention.

Use the matrix $A = \begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix}$. Vectors are column vectors.

Evaluate these student claims: $T(e1) = (2,\ 5)$; $T(e2) = (-1,\ 3)$.

A student claims: " $A90^\circ$ rotation produces a different region because every boundary point moves." Diagnose the first invalid step or assumption, give the corrected conclusion, and justify the correction. Then complete or verify the underlying task: Describe the disk's image and verify the claim with one boundary point.

Use the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Vectors are column vectors.

Use the disk with center $(0,\ 0)$ and radius 2.

Evaluate this student claim: $A90^\circ$ rotation produces a different region because every boundary point moves..

Matrices for Linear Transformations

Full Worked Solutions - excerpt

Worked example

Problem. Write the matrix for a half-turn about the origin.

The map acts by rotate 180 degrees about the origin.

Answer. The matrix is $[[-1, 0], [0, -1]]$.

Explanation and check.

Both basis vectors reverse direction under a half-turn.

Worked example

Problem. Classify the rule and give a matrix representation if one exists.

Use the rule $F(x, y) = (2x - y, 3y)$.

Answer. The rule is linear with matrix $[[2, -1], [0, 3]]$.

Explanation and check.

Each output coordinate is a constant linear combination of the inputs with no added constant or nonlinear term.