

AP Precalculus

Determinants and Area

Determinants and Area | APPC-U04-P15

96 problems · Four activity formats

01	Answer Bank Challenge	24 tasks
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Topic focus

Across 96 tasks this topic asks for dot product, determinant, area, singular set and dependence explanation, singular set, determinant and conclusion, empty real solution set with reason and all parameter values and dependence.

8 PDFs and a README; 55 buyer pages.

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Name: _____ Date: _____ Class: _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Use the lower-triangular structure to evaluate the determinant without an unnecessary cross-product calculation. Required response <table border="1"> <tbody> <tr> <td>-3</td> <td>0</td> </tr> <tr> <td>4</td> <td>2</td> </tr> </tbody> </table> _____ Bank label: _____	-3	0	4	2	Use the triangular structure to find the determinant of the matrix. Required response <table border="1"> <tbody> <tr> <td>7</td> <td>0</td> </tr> <tr> <td>-2</td> <td>-3</td> </tr> </tbody> </table> _____ Bank label: _____	7	0	-2	-3
-3	0								
4	2								
7	0								
-2	-3								
Evaluate the determinant. Required response <table border="1"> <tbody> <tr> <td>7</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> </tr> </tbody> </table> _____ Bank label: _____	7	0	0	1	Read the determinant from the triangular form. Required response <table border="1"> <tbody> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>0</td> <td>3</td> </tr> </tbody> </table> _____ Bank label: _____	1	5	0	3
7	0								
0	1								
1	5								
0	3								
Find the determinant of the coordinate-swap matrix. Required response <table border="1"> <tbody> <tr> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> </tr> </tbody> </table> _____ Bank label: _____	0	1	1	0	Find the determinant after the row swap. Apply this change: interchange row 1 and row 2. The original determinant is 13. _____ Bank label: _____				
0	1								
1	0								
Without reconstructing the entries, find the determinant after the columns are interchanged. Apply this change: interchange the two columns. The original determinant is -5. _____ Bank label: _____	Find the determinant after the stated row scaling. Apply this change: multiply row 2 by $\frac{1}{5}$. The original determinant is 20. _____ Bank label: _____								

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Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Find the signed and ordinary areas of the spanned parallelogram. The first column is $(0, 1)$. The second column is $(-3, 0)$. 	Find the signed and ordinary areas of the column parallelogram. The first column is $(2, 0)$. The second column is $(0, 5)$.
Compute the signed and ordinary areas and classify the column order. The first column is $(-2, 0)$. The second column is $(0, -7)$. 	The second column is a 90°-degree counterclockwise rotation of the first. Use that structure to determine the signed and ordinary areas of their parallelogram. The first column is $(3, -1)$. The second column is $(1, 3)$.
Find the determinant and ordinary area, then state the exact column relation. The first column is $(-1, -2)$. The second column is $(-3, -6)$. 	Find the parallelogram's ordinary area and classify the ordered-column orientation. The first column is $(4, 1)$. The second column is $(1, 3)$. The vertices are $(0, 0)$, $(4, 1)$, $(1, 3)$, $(5, 4)$.

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Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: The image area is 15 square units, and orientation is preserved. Verify that the matrices have the same determinant, then show from their columns that they do not define the same map. Required response <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px 10px;">2</td> <td style="border: 1px solid black; padding: 2px 10px;">0</td> <td style="border: 1px solid black; padding: 2px 10px;">1</td> <td style="border: 1px solid black; padding: 2px 10px;">3</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px 10px;">0</td> <td style="border: 1px solid black; padding: 2px 10px;">2</td> <td style="border: 1px solid black; padding: 2px 10px;">0</td> <td style="border: 1px solid black; padding: 2px 10px;">4</td> </tr> </table> Your response: _____	2	0	1	3	0	2	0	4	START Find every real t for which the matrix is singular and explain what happens to its columns then. Required response <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px 10px;">t</td> <td style="border: 1px solid black; padding: 2px 10px;">2</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px 10px;">3</td> <td style="border: 1px solid black; padding: 2px 10px;">t</td> </tr> </table> Your response: _____	t	2	3	t
2	0	1	3										
0	2	0	4										
t	2												
3	t												
Incoming: It is singular for every real t because the second column is always twice the first, and the determinant $2t - 2t$ is identically zero. Verify the common determinant and give a column-based reason the maps differ. Required response <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px 10px;">2</td> <td style="border: 1px solid black; padding: 2px 10px;">0</td> <td style="border: 1px solid black; padding: 2px 10px;">1</td> <td style="border: 1px solid black; padding: 2px 10px;">1</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px 10px;">0</td> <td style="border: 1px solid black; padding: 2px 10px;">1</td> <td style="border: 1px solid black; padding: 2px 10px;">-1</td> <td style="border: 1px solid black; padding: 2px 10px;">1</td> </tr> </table> Your response: _____	2	0	1	1	0	1	-1	1	Incoming: The determinant is $2(\frac{3}{4}) - 1(-1) = \frac{5}{2}$. Therefore the original area is $\frac{35}{\frac{5}{2}} = 14$. Find the image area and state whether orientation is preserved. The determinant is $\frac{1}{4}$. The original area is 20. Your response: _____				
2	0	1	1										
0	1	-1	1										
Incoming: The determinant is $k^2 + 6$, which is positive for every real k. Therefore there are no real singular parameter values. Find the square's original area and its image area. The determinant is 2. The square has side length 3. Your response: _____	Incoming: The original area is 9 and the image area is 18 square units. Find all real t that make the matrix singular and explain why the off-diagonal 1 does not change the condition. Required response <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px 10px;">t</td> <td style="border: 1px solid black; padding: 2px 10px;">1</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px 10px;">0</td> <td style="border: 1px solid black; padding: 2px 10px;">t</td> </tr> </table> Your response: _____	t	1	0	t								
t	1												
0	t												

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Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Correct the area statement while preserving the sign's geometric information.

The determinant is -7 . The student claims: The parallelogram area is -7 square units..

Repair the statement by identifying the actual column relationship and its area consequence.

Use the matrix $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$. The student claims: The determinant is zero, so both columns must be zero..

Repair the claim by stating what equality of determinants does guarantee and one fact showing the maps differ.

Use matrix $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Use matrix $B = \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$. The student claims: Both determinants are 4, so A and B are the same map..

Assess the student's claim using the displayed columns. Then replace it with a correct geometric statement.

The proposed first and second columns, respectively, are $(2, -3)$, $(-4, 6)$. The student claims: If 2×2 determinant is zero, its two columns must be equal..

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Full Worked Solutions - excerpt

Worked example

Problem. Give two distinct diagonal matrices with determinant 9 and verify them.

Answer. For example, $\begin{pmatrix} 1 & 0 \\ 0 & 9 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ both have determinant 9, but their diagonal scalings are different.

Explanation and check.

Choose two different factor pairs whose product is 9.

Worked example

Problem. Correct the area statement while preserving the sign's geometric information.

The determinant is -7 . The student claims: The parallelogram area is -7 square units..

Answer. The ordinary area is 7 square units, and the negative determinant indicates reversed orientation.

Explanation and check.

Take an absolute value for area but not for the orientation classification.