

AP Precalculus
Phase Shifts of Sine and Cosine

Phase Shifts of Sine and Cosine | APPC-U03-P10

96 problems · Four activity formats

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Topic focus

Across 96 tasks this topic asks for equivalence and canonical representatives, first error and minimal repair, new phase shift and event verification, shifted anchors and window qualification, phase classes or consistency conclusion and phase class and canonical representative.

8 PDFs and a README; 67 buyer pages.

AP Precalculus

Phase Shifts of
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Name: _____ Date: _____ Class: _____

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \cos x$ horizontally by $h = 3\frac{\pi}{2}$. Complete the shifted coordinates, retain all y -values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$	_____
y -1	_____
x $-\frac{\pi}{2}$	_____
y 0	_____
x 0	_____
y 1	_____
x $\frac{\pi}{2}$	_____
y 0	_____

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: upward midline; x: 1); period: 4; signed amplitude: 1

Required response

event	x
upward midline	1

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (event: minimum; x: 7); period: 10; signed amplitude: 1

Required response

event	x
minimum	7

Bank label: _____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: sin; midline: 0; observation table: (event: downward midline; x: 5); period: 8; signed amplitude: 1

Required response

event	x
downward midline	5

Bank label: _____

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Name: _____ Date: _____ Class: _____

Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

Translate every listed landmark of $y = \sin x$ horizontally by $h = 13\frac{\pi}{4}$. Complete the shifted coordinates, retain all y -values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ y 0	_____
x 0 y 0	_____
x π y 0	_____
x 2π y 0	_____

Amplitude, midline, family, and period are fixed as shown. Determine whether the observations identify one phase class, leave several, or are inconsistent. Treat a midline or nonextreme level without direction as ambiguous.

Given: domain: phase analysis with amplitude and period fixed; family: cos; midline: 0; observation table: (direction: not specified; level: midline; x: 3); period: 8; signed amplitude: 1

Required response

direction	level	x
not specified	midline	3

Translate every listed landmark of $y = \cos x$ horizontally by $h = -4\frac{\pi}{3}$. Complete the shifted coordinates, retain all y -values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x $-\pi$ y -1	_____
x $-\frac{\pi}{2}$ y 0	_____
x 0 y 1	_____
x $\frac{\pi}{2}$ y 0	_____

For $y = -\cos(2\pi(x-h)/10)$, amplitude sign and period are fixed. A labeled maximum occurs at $x=7$. Find h modulo the period and give the representative in $[0,10)$.

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Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

Incoming: First issue: a positive-sine peak occurs a quarter period after h, so $h = 0$; repair/defense: subtract $\frac{T}{4}$ from the peak location. Rewrite the radian argument of $y = \cos((7/3)x - 14\pi/9)$ in the form $b(x-h)$. State h in x-units and distinguish it from the angular constant $-14\pi/9$. Your response: _____	Incoming: not equivalent; representatives $\frac{9}{2}$ and 2 differ modulo 5 Two sine rules have the same signed amplitude and period 7, with shifts $h_1 = -2$ and $h_2 = 12$. Decide whether they define the same graph. Reduce each shift modulo the period; a half-period difference does not count as equivalent when the amplitude sign is fixed. Your response: _____
Incoming: $h \equiv \frac{3}{2} \pmod{5}$; canonical $h = \frac{3}{2}$ Rewrite the radian argument of $y = \sin((-4/3)x + 10\pi/9)$ in the form $b(x-h)$. State h in x-units and distinguish it from the angular constant $10\pi/9$. Your response: _____	Incoming: equivalent; difference $12 = 1$ period(s); common representative 7 For $y = \sin(2\pi(x-h)/(5/2))$, amplitude sign and period are fixed. A labeled minimum occurs at $x = 19/8$. Find h modulo the period and give the representative in $[0, 5/2)$. Your response: _____
Incoming: argument $\frac{7}{3}(x - 2\frac{\pi}{3})$; $h = 2\frac{\pi}{3}$ Rewrite the radian argument of $y = \sin((-9/4)x - 27\pi/8)$ in the form $b(x-h)$. State h in x-units and distinguish it from the angular constant $-27\pi/8$. Your response: _____	Incoming: argument $10(x - 3\frac{\pi}{10})$; $h = 3\frac{\pi}{10}$ Rewrite $y = 2 \cos(2\pi \frac{x - \frac{3}{10}}{9})$ using the sin reference while holding period and midline fixed. Track the signed coefficient and explain the quarter-cycle phase convention; for advanced cases give both same-sign and opposite-sign target representatives. Your response: _____

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Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Audit the proposed phase-shift claim. Normalize the argument, use the named event and signed family, and reduce representatives modulo the fixed period. Identify the first error or defend a valid noncanonical shift.

Given: domain: phase analysis with amplitude and period fixed; rule: $y = \sin(2x - \pi)$; student claim: shift right π

A student applies the horizontal shift to output values instead of translating the input landmarks with the stated sign. Diagnose the first mathematical error and state the correction. Then complete the original task: Translate every listed landmark of $y = \sin x$ horizontally by $h = \frac{\pi}{2}$. Complete the shifted coordinates, retain all y -values, and note that a plotting window may clip a landmark without changing the phase rule.

Required response

parent anchor	shifted x
x 0 y 0	_____
x $\frac{\pi}{2}$ y 1	_____
x π y 0	_____

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Full Worked Solutions - excerpt

Worked example

Problem. Rewrite the radian argument of $y = \cos((-7/2)x + 21\pi/4)$ in the form $b(x-h)$. State h in x -units and distinguish it from the angular constant $21\pi/4$.

Answer. argument $-\frac{7}{2}(x - 3\frac{\pi}{2})$; $h = 3\frac{\pi}{2}$

Explanation and check.

Solve $-\frac{7}{2}(x - h) = (-\frac{7}{2})x + 21\frac{\pi}{4}$. Since $h = -\frac{c}{b}$, $h = 3\frac{\pi}{2}$; the inside constant itself is not the displacement unless $b = -1$.

Worked example

Problem. Rewrite the radian argument of $y = \cos(10x - 3\pi)$ in the form $b(x-h)$. State h in x -units and distinguish it from the angular constant -3π .

Answer. argument $10(x - 3\frac{\pi}{10})$; $h = 3\frac{\pi}{10}$

Explanation and check.

Solve $10(x - h) = 10x - 3\pi$. Since $h = -\frac{c}{b}$, $h = 3\frac{\pi}{10}$; the inside constant itself is not the displacement unless $b = -1$.