

AP Precalculus

Decomposing Functions

Decomposing Functions | APPC-U02-P12

96 problems · Four activity formats

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Topic focus

Across 96 tasks this topic asks for actual composite domains, domain-faithful decomposition, validity and domain dependence, validity and intermediate images, validity of both pairs and validity relative to H domain.

8 PDFs and a README; 87 buyer pages.

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Name: _____ Date: _____ Class: _____

Answer each question and record the matching bank letter. Give exact values. A value without its unit is not a complete answer.

Split H at the affine expression inside the square. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Use the input condition real x .

Give two nonidentity components.

Let $H(x) = (5x + 1)^2 - 7$.

Bank label: _____

Use the prescribed outer rule to recover the missing g -values at the listed inputs. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given f rule: $f(u) = 2u + 1$

input	H(input)
-3	7
0	-1
4	11

Bank label: _____

Write the visible two-stage rule as $H = f$ composed with g . For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Use the input condition real x .

Give affine and absolute components.

Let $H(x) = |3x - 5| + 2$.

Bank label: _____

List the outer pairs forced by the two aligned tables. For the answer bank, record the complete response required by the original task; use the given context to distinguish responses that share a numerical value.

Given g table:

input	H(input)	input	g(input)
-6	0	-6	3
0	5	0	-1
2	12	2	7

Bank label: _____

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Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. A value without its unit is not a complete answer.

Recompose both proposed pairs and decide whether each is a substantive decomposition of H. Use the input condition real x. Give validity of both pairs. Let $H(x) = (x + 2)^4$. For pair A, use $g(x) = x + 2$, $f(u) = u^4$. For pair B, use $q(x) = (x + 2)^2$, $p(v) = v^2$. _____ _____	Recover the outer table values required for $H(x) = f(g(x))$. Given g table: <table border="1" style="display: inline-table; margin-right: 20px;"> <thead> <tr> <th>input</th> <th>H(input)</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>7</td> </tr> <tr> <td>0</td> <td>2</td> </tr> <tr> <td>4</td> <td>9</td> </tr> </tbody> </table> <table border="1" style="display: inline-table;"> <thead> <tr> <th>input</th> <th>g(input)</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>1</td> </tr> <tr> <td>0</td> <td>3</td> </tr> <tr> <td>4</td> <td>5</td> </tr> </tbody> </table> _____ _____	input	H(input)	-2	7	0	2	4	9	input	g(input)	-2	1	0	3	4	5
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Explain why the proposed f is not a function of its placeholder alone and repair it. Use the input condition real x. Give correct outer rule. Let $H(x) = (x^2 + 1)^3 + 2(x^2 + 1)$. The student proposes $f(u) = u^3 + 2x$. The student proposes $g(x) = x^2 + 1$. _____ _____	Construct the smallest outer table needed to express H as f after g. Given g table: <table border="1" style="display: inline-table; margin-right: 20px;"> <thead> <tr> <th>input</th> <th>H(input)</th> </tr> </thead> <tbody> <tr> <td>-3</td> <td>6</td> </tr> <tr> <td>1</td> <td>-1</td> </tr> <tr> <td>5</td> <td>8</td> </tr> </tbody> </table> <table border="1" style="display: inline-table;"> <thead> <tr> <th>input</th> <th>g(input)</th> </tr> </thead> <tbody> <tr> <td>-3</td> <td>0</td> </tr> <tr> <td>1</td> <td>2</td> </tr> <tr> <td>5</td> <td>4</td> </tr> </tbody> </table> _____ _____	input	H(input)	-3	6	1	-1	5	8	input	g(input)	-3	0	1	2	5	4
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Find an inner rule whose reciprocal gives H, and retain every stated exclusion. Use the input condition real x except $x = 1$ and $x = -2$. Give inner g and domain. Let $H(x) = \frac{x+2}{x-1}$. Use the prescribed outer function $f(u) = \frac{1}{u}$. _____ _____	Compare the recomposed rules and explain why domain declarations are part of the comparison. Use the input condition $x \geq 0$. Give validity relative to H domain. Let $H(x) = x$ on $x \geq 0$. For pair A, use $g(x) = \sqrt{x}$, $f(u) = u^2$. For pair B, use $q(x) = x$, $p(v) = v$. _____ _____																

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Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START Replace the prescribed inner expression consistently to recover f. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the input condition real x. Give outer polynomial. Let $H(x) = (x - 4)^3 - 2(x - 4) + 6$. Use the prescribed inner function $g(x) = x - 4$. Your response: _____	Incoming: $f(u) = u^3 - 2u + 6$. Undo the outer translation to recover g. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the input condition real x. Give inner function. Let $H(x) = \sqrt{x^2 + 4} + 7$. Use the prescribed outer function $f(u) = u + 7$. Your response: _____
Incoming: $g(x) = \sqrt{x^2 + 4}$. Recover f and state the intermediate inputs on which it must be defined. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the input condition real x. Give outer rule on image. Let $H(x) = 4x^2 - 12$. Use the prescribed inner function $g(x) = x^2$. Your response: _____	Incoming: $f(u) = 4u - 12$ for $u \geq 0$. Recover the pre-tax subtotal and tax stages. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the input condition $0 \leq n \leq 80$ tickets. Give subtotal and taxed-total maps. Use the compound model $T(n) = 1.075(14n + 5)$ dollars. Your response: _____
Incoming: $S(n) = 14n + 5$ dollars and $T(S) = 1.075S$ dollars. Treat the recurring logarithm as one intermediate variable. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the input condition $x > 0$. Give log inner and polynomial outer. Let $H(x) = (\ln(x))^2 - 6 \ln(x) + 2$. Your response: _____	Incoming: $g(x) = \ln(x)$, $x > 0$, and $f(u) = u^2 - 6u + 2$. Remove the outer scaling to identify g. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task. Use the input condition real x. Give inner rule. Let $H(x) = \frac{3}{x^2 + 1}$. Use the prescribed outer function $f(u) = 3u$. Your response: _____

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Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Check whether pulling the positive factor outside absolute value gives a second valid decomposition. Student work: Both are invalid for all real x because $|2(x + 3)| = 2|x + 3|$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use the input condition real x .

Give validity.

Let $H(x) = |2x + 6|$.

For pair A, use $g(x) = 2x + 6$, $f(u) = |u|$.

For pair B, use $q(x) = x + 3$, $p(v) = 2|v|$.

Isolate the quadratic expression evaluated before absolute value. Student work: $g(x) = x^2 + 6x + 2$ and $f(u) = |u| + 5$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Use the input condition real x .

Give quadratic inner and absolute outer.

Let $H(x) = |x^2 - 6x + 2| + 5$.

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Full Worked Solutions - excerpt

Worked example

Problem. Can a single f satisfy $x = f(\sin(x))$ for every real x ? Give a decisive pair of inputs. Use the complete final response to identify the next Match & Move card; show the reasoning required by the original task.

Answer. No. $\sin(0) = \sin(2\pi) = 0$, but $H(0) = 0$ and $H(2\pi) = 2\pi$.

Explanation and check.

The periodic inner map discards which cycle produced the value, while H retains that information.

Worked example

Problem. Formally undo the logarithm and verify whether the resulting inner function is admissible. Student work: $g(x) = e^{-x^2}$, which is negative for every real x and satisfies $\ln(g(x)) = -x^2$. Identify the precise mathematical error or omitted condition, then complete the original task and verify the correction from the supplied givens.

Answer. $g(x) = e^{-x^2}$, which is positive for every real x and satisfies $\ln(g(x)) = -x^2$.

Explanation and check.

A logarithm may output negative values; only its input must be positive, which the exponential guarantees.