Sets & Steps
Math Practice & Worked Solutions

AP Precalculus

Comparing Average Rates of Change

Comparing Average Rates of Change | APPC-U01-P03

96 problems · Four activity formats

Topic focus

Across 96 tasks this topic asks for family with difference evidence, secant, interior comparison, curvature explanation, simplified rule in s, concavity and signed interval changes, secant rates, midpoint comparison, diagnosis and missing output or incompatibility proof.

8 PDFs and a README; 75 buyer pages.

The following pages use the same questions, given data, and notation as the final files. Question/card serial numbers and internal IDs are omitted in this preview only. Table values and matching responses are retained. Full answer keys are not included.

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AP Precalculus

Comparing Average Rates of Change

Name: ______________ **Date:** ______________ **Class:** ______________

Answer each question and record the matching bank letter. Give exact values. Give the whole requested response, including any reasoning the question asks for.

For $f(x) = -x + 29$, derive the average rate of change on $[s, s+3]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Bank label: ______

For the unknown quadratic $f(x) = ax^2 + bx + c$, two width-2 secants begin at $x = -2$ and $x = 0$ and have rates 2 and -6, respectively. Also $f(-1) = 6$. Recover the coefficients supported by these constraints.

Bank label: ______

A function is declared linear on the equally spaced inputs in this exact table: $x = -2, y = -19$; $x = -1, y = -16$; $x = 0, y = ?$; $x = 1, y = -10$; $x = 2, y = -5$. Find the missing output if the declaration is consistent; otherwise explain why no value can make all entries fit.

Required output: missing output or incompatibility proof

x	y
-2	-19
-1	-16
0	______
1	-10
2	-5

Bank label: ______

For $f(x) = x^2 + 4x + 17$, derive the average rate of change on $[s, s+4]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Bank label: ______

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Answer each question and show your reasoning in the workspace. Give exact values rather than decimals unless the question supplies a decimal. Give the whole requested response, including any reasoning the question asks for.

For $f(x) = 3x^2 - 2x - 1$, derive the average rate of change on $[s, s+3]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

For $f(x) = -2x^2 - 3x - 4$, derive the average rate of change on $[s, s+2]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

For $f(x) = -2x^2 - 4x + 20$, derive the average rate of change on $[s, s+5]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

A model is known to be linear, quadratic, or outside those two families. Its exact samples are $x = -2, y = 12$; $x = 1, y = 12$; $x = 4, y = 12$; $x = 7, y = 12$; $x = 10, y = 12$. Identify the consistent family and cite the decisive difference pattern.

Required output: family with difference evidence

x	**y**
-2	12
1	12
4	12
7	12
10	12

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Name: ______________ **Date:** ______________ **Class:** ______________

Begin at START. Answer the card, then find the matching final answer at the top of another card. Use the recording sheet for your reasoning. Visit each card once and continue to FINISH.

START

On $[-2, 3]$, $L(x) = -x + 6$ and $Q(x) = L(x) - 2(x + 2)(x - 3)$. Verify that their endpoint secant rates agree, compare $L(0)$ and $Q(0)$, and use curvature to explain the interior difference.

Your response: ______________

Incoming: Both endpoint rates are -1; $L(0) = 6$ and $Q(0) = 18$. Q is concave down.

The inputs are not equally spaced in the exact table $(-1, 17)$; $(1, 17)$; $(4, 47)$; $(5, 65)$. Normalize every output change by its actual width, attach each rate to the interval midpoint, and test the quadratic requirement that those midpoint-rate pairs lie on a line.

Required output: secant rates, midpoint comparison, diagnosis

x	y
-1	17
1	17
4	47
5	65

Your response: ______________

Incoming: Rates: $0, 10, 18$; midpoints: $0, \frac{5}{2}, \frac{9}{2}$. The evidence is consistent with a quadratic.

For the unknown quadratic $f(x) = ax^2 + bx + c$, two width-1 secants begin at $x = -3$ and $x = -1$ and have rates -11 and -3, respectively. No output value is supplied. Recover the coefficients supported by these constraints.

Your response: ______________

Incoming: a = 2 and $b = -1$, while c is undetermined without an output anchor.

The inputs are not equally spaced in the exact table $(2, -2)$; $(3, -15)$; $(6, -78)$; $(10, -218)$. Normalize every output change by its actual width, attach each rate to the interval midpoint, and test the quadratic requirement that those midpoint-rate pairs lie on a line.

Required output: secant rates, midpoint comparison, diagnosis

x	y
2	-2
3	-15
6	-78
10	-218

Your response: ______________

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Name: ____________ **Date:** ____________ **Class:** ____________

Evaluate the supplied statement or method. Say whether it is correct, repair it if it is not, and verify the result exactly. Not every prompt contains an error.

Use the exact equally spaced table and the stated candidate families. A student reports slope 5 because successive outputs differ by 5. Identify the invalid inference and give the corrected conclusion supported by the exact data.

Required output: family with difference evidence

x	**y**
−4	0
−2	5
0	10
2	15
4	20

Use the exact equally spaced table and the stated candidate families. A student insists that a straight-line pattern is a nondegenerate quadratic because its second differences are constant. Identify the invalid inference and give the corrected conclusion supported by the exact data.

Required output: family with difference evidence

x	**y**
−3	−4
−2	−3
−1	−2
0	−1
1	0

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Full Worked Solutions - excerpt

Worked example

Problem. For $f(x) = x^2 + 4x + 17$, derive the average rate of change on $[s, s+4]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Answer. $R(s) = 2s + 8$; it increases with interval location.

Explanation and check.

The endpoint change factors as $4[2s + 8]$. Dividing by the nonzero width 4 gives $R(s) = 2s + 8$. Thus the secant rate increases with interval location.

Worked example

Problem. For $f(x) = 3x^2 + x + 35$, derive the average rate of change on $[s, s+5]$ as a function of s. Then state what the rule shows about linear versus quadratic change.

Answer. $R(s) = 6s + 16$; it increases with interval location.

Explanation and check.

The endpoint change factors as $5[6s + 16]$. Dividing by the nonzero width 5 gives $R(s) = 6s + 16$. Thus the secant rate increases with interval location.